CHAPTER 90

ANALYSIS OF NONLINEAR COEFFICIENTS OF REFLECTION AND TRANSMISSION OF WAVES PROPAGATING OVER A RECTANGULAR STEP

Wudhipong Kittitanasuan [1] and Yoshimi Goda [2]

Abstract

A mathematical model for nonlinear wave propagation over a rectangular step is formulated based on the boundary element method. The wave profile computed by this model gives a more realistic wave profile than that of linear wave theory. The calculation of the height of nonlinear waves is studied, and as a results of the study the reflection and transmission coefficients are computed based on the technique which considerd the higher harmonic components in the computation of wave height. The solutions of these coefficients are presented and compared with laboratory experiments.

1 Introduction

A land reclamation has been recommended to provide an artificial shallow ledge around its periphery in order to mitigate adverse effects on the environment. This shallow ledge can be considered as a rectangular step placed in finite depth water. A breakthrough of knowledge of the transformations of waves on this marine structure is necessary so that we can design and construct all related structures properly. Transformations of waves on a rectangular step were originally studied by Lamb (1932), who solved this kind of problem by applying basic continuity requirements at the point of discontinuity. Up to now, Bartolomeusz (1958), Newman (1965), Mei and Black (1969), Ijima (1971), *etc.*, have proposed various solutions for the prediction of wave reflection and transmission coefficients. Most of these solutions were derived based on linear wave theory. In

[1] D. Eng., Lecturer, Department of Water Resources Engineering, Chulalongkorn University, Phayathai Rd., Bangkok 10330, Thailand

[2] D. Eng., Professor, Department of Civil Engineering, Yokohama National University, Hodogaya-ku, Yokohama 240, Japan

many applications, however, it is the waves of large amplitude which are of primary importance. It is because the large amplitude waves exercise vast influence on the aquatic environment, owing to their strong movements of water particle.

As illustrated by Ohyama and Nadaoka (1991) and Kittitanasuan *et al.* (1993) waves on the step exhibit highly nonlinear behavior exemplified by the enhancement of the higher harmonic components, and therefore, the effect of these nonlinear components should be considered in the computation of the coefficients of wave reflection and transmission of rectangular step. In this study, an attempt is made to provide the solutions of wave reflection and transmission coefficients based on the nonlinear computation.

2 Mathematical Formulation

2.1 Governing Equation

The nonlinear boundary value problem based on the velocity potential theory is formulated by assuming that the fluid is inviscid and incompressible, and the fluid motion is irrotational. The two-dimensional continuity equation in the fluid domain Ω can be written as follows :

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (\text{in} \quad \Omega) \tag{1}$$

where ϕ is the velocity potential, x is the horizontal axis, and z is the vertical axis taken upward the mean water level.

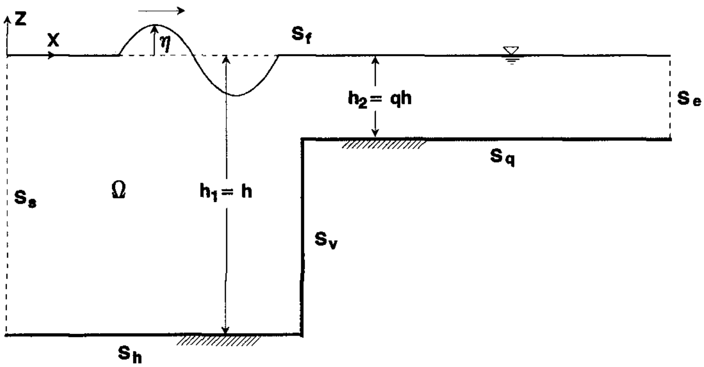

Figure 1: Definition sketch

2.2 Boundary Conditions

Along the solid boundaries, S_h, S_q, and S_v sketched in Fig. 1, the boundary conditions are formulated as shown below.

$$\frac{\partial \phi}{\partial z} = 0 \quad (\text{on} \quad S_h) \tag{2}$$

$$\frac{\partial \phi}{\partial z} = 0 \quad (\text{on} \quad S_q) \tag{3}$$

$$\frac{\partial \phi}{\partial x} = 0 \quad (\text{on} \quad S_v) \tag{4}$$

For the incident boundary, S_s, at the left-hand side of Fig. 1, the velocity normal to this boundary is equal to the moving velocity of a wave paddle.

$$-\frac{\partial \phi}{\partial n} = \frac{\partial \phi}{\partial x} = U \quad (\text{on} \quad S_s) \tag{5}$$

where U is the moving velocity of a wave paddle, which is calculated according to the wave maker theory for a given wave height and period. It should be noted that the depth-dependent velocity computed from the orbital velocity of water particle can also be utilized as the velocity U.

Two free-surface boundary conditions must be satisfied; the first condition is the kinematic boundary condition Eq. (6), and the second condition is the Bernoulli equation Eq. (7) with the assumption of constant pressure everywhere on the free surface.

$$\frac{\partial \phi}{\partial n} = \dot{\eta} \cos \beta \quad (\text{on} \quad S_f) \tag{6}$$

$$\dot{\phi} + \frac{1}{2}\left\{\left(\frac{\partial \phi}{\partial n}\right)^2 + \left(\frac{\partial \phi}{\partial s}\right)^2\right\} + g\eta = 0 \quad (\text{on} \quad S_f) \tag{7}$$

where $\dot{\eta} = \partial\eta/\partial t$, $\dot{\phi} = \partial\phi/\partial t$, g is the acceleration of gravity, n and s show the directions of normal and tangent vectors respectively, and β is the angle between the normal vector and the vertical axis.

Both of the free surface boundary conditions are nonlinear and are applied on the free surface, the elevation of which is not known *a priori*.

In order to simulate wave motions for a long duration, an appropriate boundary condition has to be introduced at the vertical boundary S_e, at the right-hand side of Fig. 1 so that the waves can pass through the boundary without undergoing significant distortion and without influencing the interior solution. A variety of methods have been developed to achieve the non-reflectivity at the boundary for wave propagation problems. Among various methods proposed, use of the Sommerfeld boundary condition appears most appropriate. In this study, we shall utilize the Sommerfeld boundary condition at the boundary S_e; therefore, the boundary condition at S_e can be defined as:

$$\frac{\partial \phi}{\partial x} = -\frac{1}{C}\frac{\partial \phi}{\partial t} \quad (\text{on} \quad S_e) \tag{8}$$

where C is the phase speed of the wave, and is approximated with that derived by the linear wave theory.

3 Numerical Results

3.1 Propagating Wave Profile

A study of wave deformation on the step was made through the numerical analysis by the present model and experiment. A numerical flume composed of 105 elements on the free surface and 63 elements on the bottom and lateral boundaries was created. The water depth was set at 0.376 m in front of the step and 0.113 m on the step, the step height of 26.3 cm was utilized. A wave period of 1.74 s was utilized in the computation. On the free surface, the element size of 20 cm was used in the deep water region and 10 cm was used in the shallow water region; these element sizes were equal to $L/15.3$ and $L/17.9$, respectively. A time step of $T/16$ were utilized in the computation. The spatial profile of wave propagation is computed by the present model and is plotted in Fig. 2. We can see from this figure that when waves enter into the shallow zone, the wave height increases, and the wave crest becomes much sharper than the wave trough, and the secondary crest is developed. The location of this secondary crest on the wave profile changes gradually as the wave propagates into the shallow water zone.

3.2 Comparison of Numerical and Experimental Results

In order to verify the results of the present model, a laboratory experiment was conducted in a 17 m long wave flume, equipped with a computer controlled piston-type wave generator. A rectangular step of the same height 26.3 cm as used in the numerical computation was installed in this flume. At the end of the flume, a flat plate of 2.0 m long was installed with a slope of 1/10. On the top of this plate, a wave absorber was placed to reduce wave reflection at the end of the flume. A wave period of 1.74 s was utilized for this experiment. A single wave gage was used to measure time-history wave profiles at 4 measuring points, at the distances of 1.0 m offshoreward from the tip of the step, and 1.0 m, 2.0 m, and 3.0 m inshoreward from the tip of the step.

The wave profiles computed by the present model are compared with the experimental results for four different locations as mentioned above. The comparisons are shown in Figs. 3. In the comparisons of wave profiles about 20 wave cycles were employed in order to have stable wave profiles. The profile of waves computed by the present model illustrates similar wave profiles in the both regions, in front of the step and on the step. The secondary wave crest is also well simulated having a similar shape compared with the experiments. Therefore, it is clear that the present model can accurately predict the wave profile of wave propagation over the rectangular step.

Both results have indicated that the wave profile on the rectangular step is different from that of sinusoidal waves; *i.e.*, the wave crest is much sharper than the wave trough, and a secondary crest is developed.

3.3 Higher Harmonic Components

The effect of finite amplitude on the amount of energy transferred to higher harmonic components of waves on the step was numerically and experimentally investigated. For this analysis, the water depth was set at 0.376 m in front of the step and 0.113 m on the step. The wave height was varied at three levels,

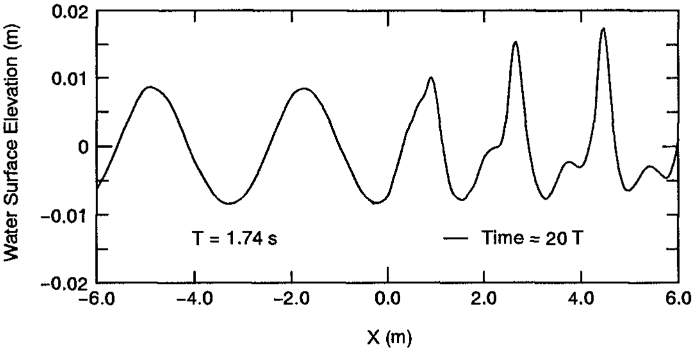

Figure 2: Wave propagating profile

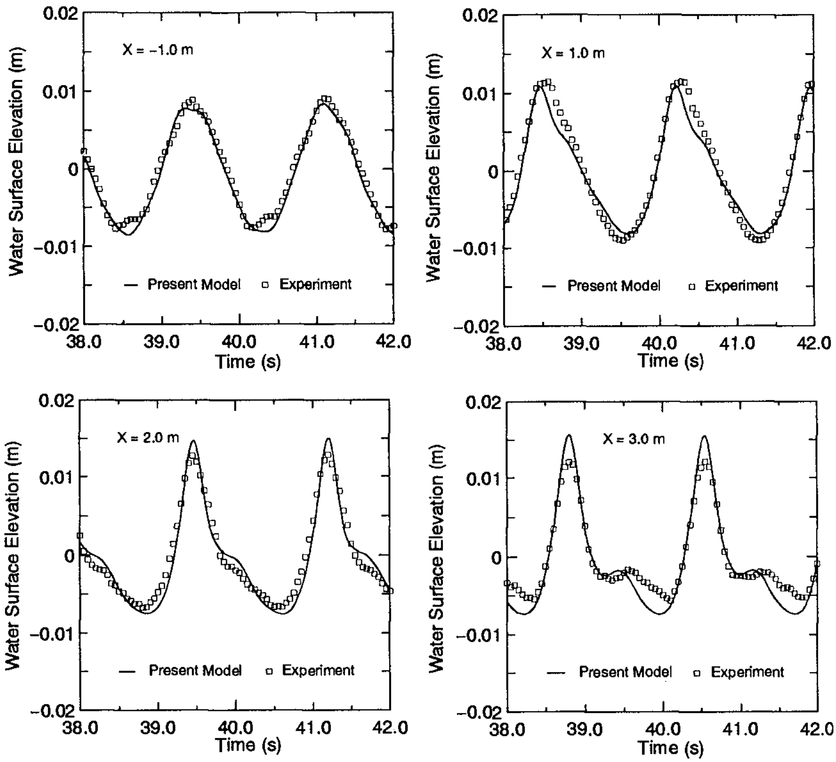

Figure 3: Comparison of wave profiles

while the wave period was fixed at 1.74 s. From the results of wave profiles by numerical model and experiment, the FFT analyis are made to computed the amplitudes of all frequency components. In order to seperate the transmitted waves from the reflected waves at the end of the flume, the resolution technique introduced by Goda and Suzuki (1976) is employed. Then, the potential energy of the first, second and third harmonics are computed. In Fig. 4, the ratios of maximum energy of the second and third harmonics to that of the first harmonic are plotted against the relative wave height, the ratio of the height of incident waves H_{in} to the water depth on the step h_2. The relative energy is found to be 0.2~0.4 for the second harmonic and 0.01~0.09 for the third harmonic. Therefore, the energy tranferred to higher harmonic components is very significant compared with that of the first harmonic.

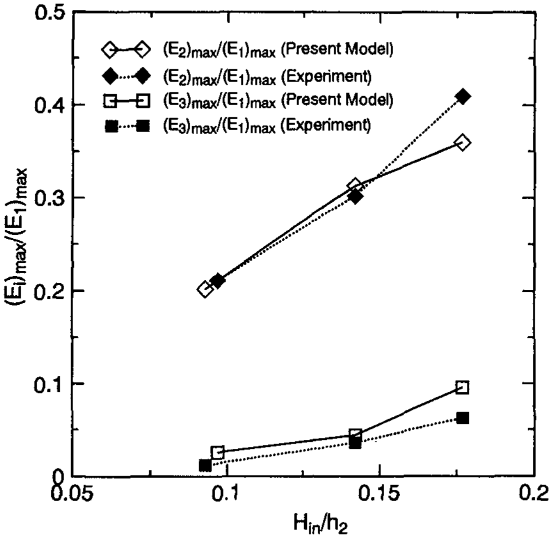

Figure 4: Relative energy of higher harmonic components

4 Analysis of the Height of Nonlinear Waves

4.1 Conventional Methods

The reflection of water waves is known to associate with every problem of wave and structure interaction. In order to estimate wave heights of incident and reflected waves, we need to have an information of reflection coefficient, or the ratio of reflected to incident wave heights. Up to now, many techniques have been proposed by various researchers, Healy (1953), Thornton and Calhoun (1972), Goda and Suzuki (1976), Morden *et al.* (1976), Mansard and Funke (1980), and others for the estimation of wave reflection. The method introduced by Healy is based on a measurement of wave profile with a single wave gage which is traversed over a distance of more than a half wave length. The methods of Thornton and Calhoun, Goda and Suzuki, and Morden *et al.* are based on a simultaneous measurement of wave profile at two positions on a line parrallel to the direction of wave propagation. Meanwhile, the method by Mansard and Funke employs three-points measurements of the wave profile.

The first method to be described herein is based on the assumption that the total energy is equal to the summation of the energies of incident and reflected waves. The method is later referred as the method (a) in the calculation of wave height of experiments. The formulation of this method can be summarized as follow :

$$E = E_{in} + E_{rf} = \frac{1}{8}\rho g(H_{in}^2 + H_{rf}^2) = (1 + K_{rf}^2)\,\frac{1}{8}\rho g H_{in}^2 \tag{9}$$

where the subscripts "*in*" and "*rf*" refer to the incident and reflected waves respectively, and K_{rf} is the reflection coefficient. This reflection coefficient is estimated as the ratio of the amplitude of the foundamental component of reflected waves to that of incident waves which are computed by utilizing the resolution technique of Goda nd Suzuki (1976).

From Eq. (9), the incident wave height can be computed as

$$H_{in} = \sqrt{\frac{8E}{\rho g(1 + K_{rf}^2)}} = \sqrt{\frac{4(E_1 + E_2)}{\rho g(1 + K_{rf}^2)}} = \sqrt{\frac{4}{\rho g}[(E_1)_i + (E_2)_i]} \tag{10}$$

where

$$(E_1)_i = \frac{E_1}{1 + K_{rf}^2}, \quad E_1 = \frac{1}{2}\rho g \sum_{i=1}^{n/2}(a_i^2 + b_i^2)_1$$

$$(E_2)_i = \frac{E_2}{1 + K_{rf}^2}, \quad E_2 = \frac{1}{2}\rho g \sum_{i=1}^{n/2}(a_i^2 + b_i^2)_2$$

and E_1 and E_2 are the energy calculated from all fourier components of the first and second wave gages, respectively. a_i and b_i are the amplitudes of sine and cosine terms obtained form the Fourier analysis of recorded wave profiles, respectively. The subscripts 1 and 2 represent the first and second wave gages, respectively.

The second method, to be referred as the method (b), is the method introduced by Goda and Suzuki (1976) for the calculation of wave heights of incident and reflected waves of irregular waves. The method is based on two main assumptions. The first one is that the energy of composite waves is represented with the sum of the energies of individual wave trains. The second one is that the proportionality of representative wave heights to the square root of wave energy holds for such composite wave too, regardless of the directions of individual wave trains. In general, the basic concept of the approach is similar to that the first method, except that the energy is estimated from the wave height of zero-crossing method. The wave height of incident waves of this method can be computed as

$$H_{in} = \frac{1}{2\sqrt{1+K_{rf}^2}}(\overline{H}_1 + \overline{H}_2) \tag{11}$$

where $\overline{H}_1$ and $\overline{H}_2$ are the average wave heights computed by the zero-downcrossing method of the first and second wave gages, respectively.

4.2 Method by Kittitanasuan et al. (1993)

In the calculation methods of wave heights of incident and reflected waves described above, the reflection of waves is treated as a single value of the reflection of fundamental component, but the harmonic components of incident and reflected waves are not given appropriate consideration. As illustrated by Kittitanasuan *et al.* (1993), the second and third harmonic components play an important role on waves propagating in the shallow water. Therefore, these harmonic components should be considered in the calculation of the height of waves exhibiting nonlinear behavior. In this study, the method proposed by Kittitanasuan *et al.* for the calculation of wave height when waves contain enhanced higher harmonic components will be reviewed. The basic concept of this method is the conservation of energy density over a range of frequencies. The energy is considered to preserve within the range of major frequency components, *i.e.* the first, second and third harmonic components. In this method, the reflection of each harmonic component is treated individually. That means the energy density of incident and reflected waves are computed from the summation of energies of all resolved components of incident and reflected waves, respectively. In the resolution of incident and reflected components, the technique introduced by Goda and Suzuki (1976) is utilized.

According to this method, the wave height is calculated by converting the energy of all resolved incident and reflected components as shown in the following.

$$H_{in}^* = 2\sqrt{\frac{2E_{in}}{\rho g}} \quad ; \quad H_{rf}^* = 2\sqrt{\frac{2E_{rf}}{\rho g}} \tag{12}$$

where

$$E_{in} = \frac{1}{2}\rho g \sum_{i=f_{\min}}^{f_{\max}} (a_{in})_i^2 \quad ; \quad E_{rf} = \frac{1}{2}\rho g \sum_{i=f_{\min}}^{f_{\max}} (a_{rf})_i^2 \tag{13}$$

in which $(a_{in})_i$ is the amplitude of a resolved component of incident waves and $(a_{rf})_i$ is that of reflected waves. $f_{\min}$ and $f_{\max}$ are the minimum and maximum frequencies corresponding to the resolution technique.

Use of the whole energy is made to minimize the effect of energy spreading around the harmonic frequencies by the FFT analysis. In fact, the energy of frequency components outside the three consecutive frequencies around the harmonics was less than 5% in the present analysis.

By knowing these wave heights, the reflection coefficient can be calculated as follow :

$$K_{rf} = \frac{H^*_{rf}}{H^*_{in}} \tag{14}$$

The transmission coefficient, K_{tr}, can also be calculated as the ratio of the representative incident wave height on the step to the representative incident wave height in front of the step.

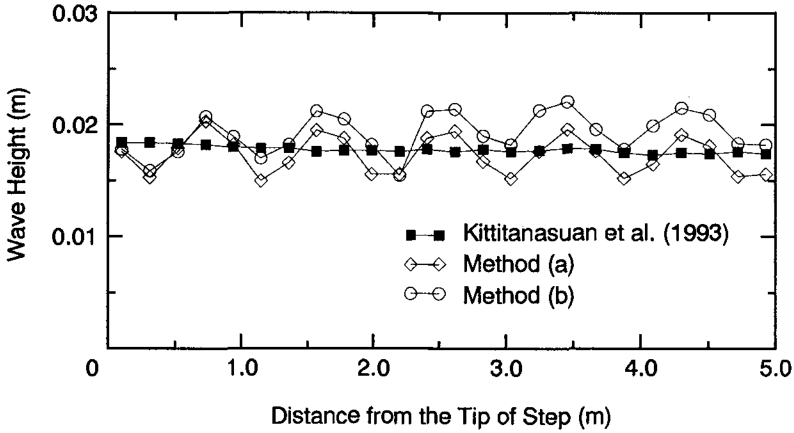

Figure 5: The comparison of computed wave heights

The three methods, methods (a), (b) and method by Kittitanasuan *et al.* (1993), were applied for the experimental data on the step. The water depths were set at 0.376 m in front of the step and 0.113 m on the step. A wave period of 1.74 s are utilized. The comparison of results of these three methods are illustrated in Fig. 5. The wave height computed by the method by Kittitanasuan *et al.* give a slight fluctuation of wave height along the shallow zone, and meanwhile the other two conventional methods give considerable fluctuations of wave heights. Among the three methods, it is evident that the method proposed by Kittitanasuan *et al.* yields the smallest fluctuation of the wave height on the step. Therefore, this method will be utilized for the computation of wave height through out the present study.

If we compare the wave heights computed from the methods (a) and (b) with that of Kittitanasuan *et al.* (1993), the method (a) is seen to under-estimate, and the method (b) tends to over-estimate the wave height. In the partial standing wave system, the maximum and minimum vertical displacements are results of the superposition of the profiles of the incident and reflected waves. When the wave height is measured directly by these maximum and minimum displacements as in the zero-crossing method, there is a possibility of over-estimation of the wave height.

The fluctuation of the wave heights computed by the two conventional methods was investigated. A relatively high reflection of waves from the wave absorber at the end of rectangular step is considered to be the reason of the fluctutation. The maximum wave reflection coefficient computed at all measuring points is found to be 0.26. The variation of wave height shown in Fig. 5 is considered to be corresponding to the wave height envelope in the partial standing wave system. Therefore, the wave length estimated by the wave height envelope was examined to comfirm the assumption. The estimated wave length is 1.80 m, and this length corresponds to the wave length computed from the dispersion relationship of linear wave theory which is 1.79 m for this case. Therefore, we can conclude that the fluctuation of wave heights computed by the two conventional methods is caused by the reflection of waves at the end of rectangular step.

5 Reflection and Transmission Coefficients

5.1 Coefficients of Nonlinear Wave Reflection and Transmission

The numerical model based on the boundary element method described earlier is used to compute the profile of wave propagating over a rectangular step. The free surface boundary was discretized into 104 elements, 36 elements in the deep water side and 68 elements in the shallow water side. Besides the free surface boundary, there were 63 elements on the bottom and lateral boundaries. The element size on the free surface is selected to include the first three major harmonic components in the resolution of reflected waves. This element size is ranged between $L/18.1$ to $L/16.1$. Three levels of water depth ratio q, the ratio of water depth on the step to the water depth in front of the step, were setup. The computations were conducted for 45 cases, 15 cases for each water depth ratio q. Three values of the relative wave depth on the step h_2/L_0, 0.025, 0.05 and 0.1, are utilized in the computations. The incident wave height is varied from 1% of water depth on the step to 26% of water depth on the step for the maximum case. A time step of $T/16$ is used for all the computations. After obtaining time-history displacement of all nodal points, the wave height of incident, reflected and transmitted waves are computed by using the technique proposed by Kittitanasuan *at al.* (1993). Because of the spatial variation of wave heights on the step, the reflection and transmission coefficients are computed based on their average values.

The reflection coefficients of a rectangular step computed by the present model are illustrated in Fig. 6. The reflection coefficients computed by the analytical solutions derived by using Ijima's technique, are also included in these figures. The reflection coefficients computed by the present model give nearly the same values as those computed from the analytical solutions for $q = 0.5$ and 0.3. For $q = 0.1$, the reflection coefficients computed by the model give smaller values than those computed from the analytical solution for h_2/L_0 less than 0.05. From these figures, it is seen that the incident wave height has a rather small effect on the reflection coefficients.

The transmission coefficients of waves propagating over a rectangular step computed by the present model are exhibited in Fig. 7. The transmission coefficients computed by the present model give nearly the same values as those computed by the analytical solution for the water depth ratio q of 0.5 and 0.3.

However, the coefficients by the present model yield lower values than those of the analytical solution for $q=0.1$. From the results of these transmission coefficients, the coefficients increase as the incident wave height increases for the relative depth h_2/L_0 less than 0.05. However, the effect of this wave height is quite small.

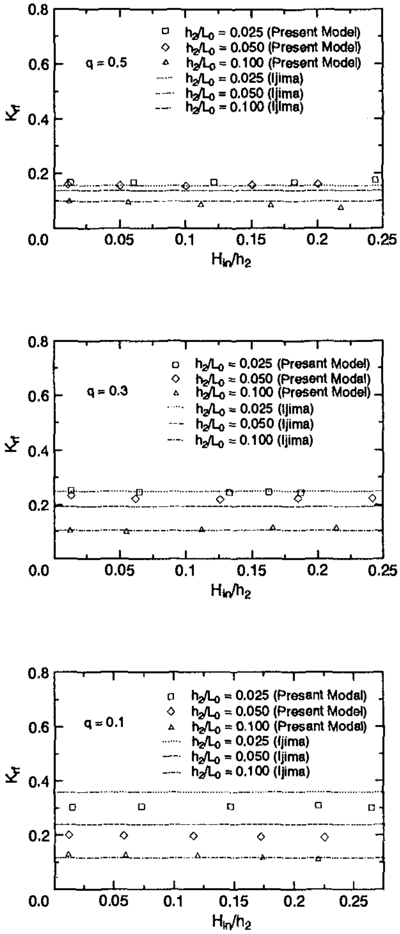

Figure 6: Comparisons of nonlinear and linear reflection coefficients

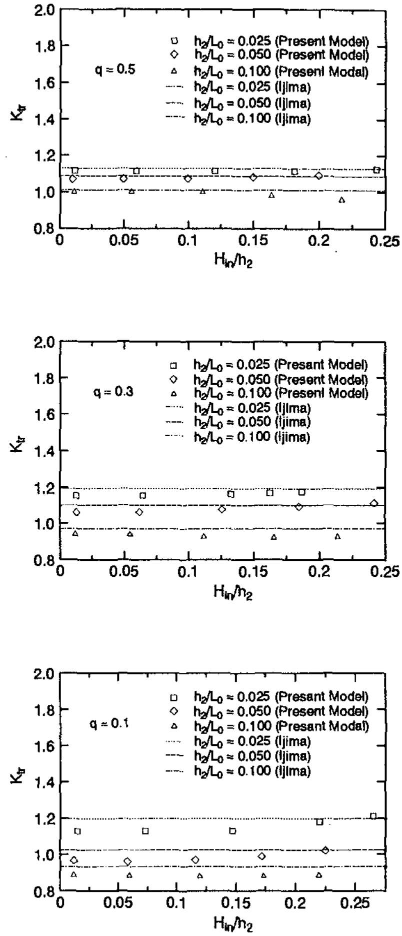

Figure 7: Comparisons of nonlinear and linear transmission coefficients

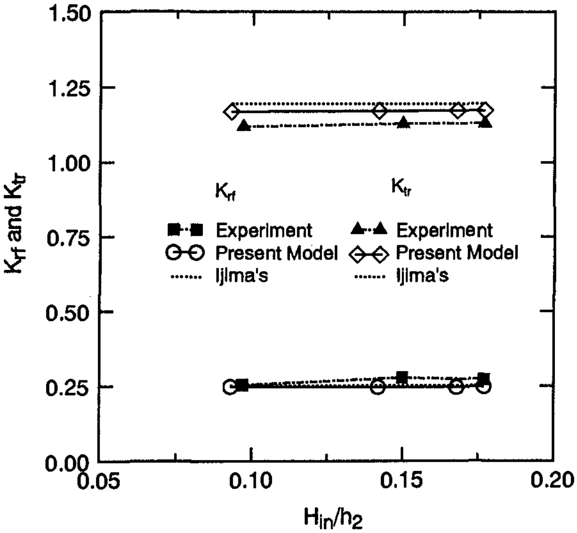

Figure 8: Comparisons of wave reflection and transmission coefficients, K_{rf} and K_{tr} ($h = 0.376$ m, $qh = 0.113$ m, $T = 1.74$ s)

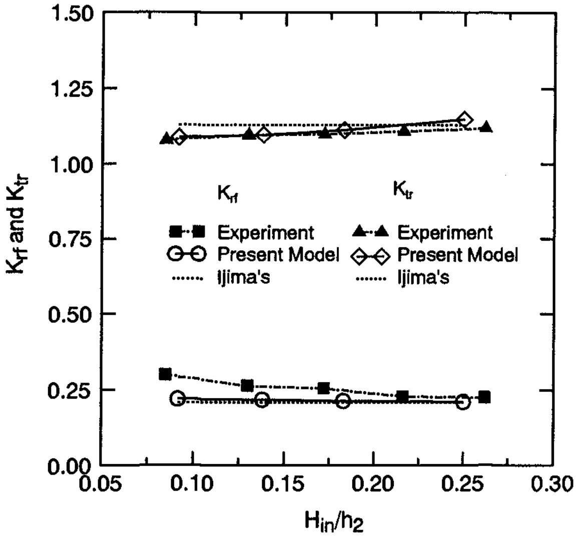

Figure 9: Comparisons of wave reflection and transmission coefficients, K_{rf} and K_{tr} ($h = 0.376$ m, $qh = 0.113$ m, $T = 1.30$ s)

5.2 Comparison of Reflection and Transmission Coefficients with Experimental Data

Laboratory experiments were conducted to verify the reflection and transmission coefficients computed by the present model. The wave flume installed with a rectangular step of 26.3 cm high was used in these experiments. The water depth was set at 0.376 m in front of the step and 0.113 m on the step, the water depth ratio q is equal to 0.3. Comparisons between the reflection and transmission coefficients by the present model, Ijima's solution, and the experiments were made for two different cases of wave period. For the first case, a wave period of 1.74 s was utilized. In the comparison of the coefficients shown in Fig. 8, both Ijima's solution and the present numerical model show an identical prediction of the reflection coefficient K_{rf} which almost agree with the experimental results. For the transmission coefficient K_{tr}, the result from the present model gives a better agreement with the experimental result than that of Ijima's solution.

For the latter case, the water depths were 0.376 m offshoreward and 0.113 m inshoreward from the tip of the step and the wave period was 1.3 s. In the comparison of the reflection coefficient with Ijima's solution and experiments shown in Fig. 9, the reflection coefficient computed from the present model and Ijima's solution are almost the same. They are in agreement with the experimental data, with the exception of under-prediction for the wave height ratio, H_{in}/h_2, less than 0.2. In the comparison of transmission coefficient with Ijima's solution and experiment, the result of the present model shows a better agreement with the experiment than that of Ijima's solution.

6 Conclusions

Analysis of the coefficients of nonlinear wave reflection and transmission has been made through the present numerical model and experiments, and major conclusions can be described as follows:

1. The numerical model utilizing the boundary element method has been confirmed to accurately predict the profiles of nonlinear waves propagating over a rectangular step, as demonstrated by comparison with experimental data.

2. Waves propagating over a rectangular step are found to exhibit highly nonlinear behavior exemplified by the enhancement of the energy of the second and third harmonic components.

3. A representative wave height based on the assumption of the conservation of energy density is proposed for a situation in which higher harmonic components maintain a significant amount of wave energy. This definition provides a consistent estimate of wave heights on a rectangular step, where the conventional definitions produce a considerable fluctuation of wave heights.

4. The wave reflection and transmission coefficients computed by the model of nonlinear wave propagation over the step are close to the coefficients computed by the solution of Ijima (1971) with the exception of the water depth ratio q equal to 0.1. For this case of water depth ratio, the coefficients computed by the model yield smaller values than those of the analytical solutions.

5. The effect of finite amplitude on the reflection and transmission coefficients of wave propagation over the step is found small as long as the energies of higher harmonic components are taken into account.

Referrences

Bartholomeusz, E. F. (1958): The reflexion of long wave at a step, *Proc. Camb. Phil. Soc. 54*, pp.106-118.

Goda, Y., Suzuki, Y. (1976): Estimation of incident and reflected waves in random wave experiments, *Proc. 15th Int. Conf. Coastal Eng.*, pp.828-845.

Healy, J. J. (1953): Wave damping effect of beaches, *Proc. Minnesota International Hydraulics Convention*, pp.213-220.

Ijima, T. (1971): Solutions of boundary value problems in recent wave theory and their applications, *Summer Seminar on Hydraulic Engineering*, Committee on Hydraulic Engineering, JSCE, Course B, pp.B-1-1-B-1-30 (*in Japanese*).

Kittitanasuan, W., Goda, Y. and Shiobara, T. (1993): Deformation of nonlinear waves on a rectangular step, *Coastal Engineering in Japan*, Vol.36, No.2, pp. 133-153.

Lamb, H., (1932): *Hydrodynamics*, 6th, Cambridge Univ. Press., pp.250-282

Mansard, E. P. D. and Funke, E. R. (1980): The measurement of incident and reflected spectra using a least squares method, *Proc. 17th Int. Conf. Coastal Eng.*, pp.154-172.

Mei, C. C. and Black, J. L. (1969): Scattering of surface waves by the rectangular obstacles in water of finite depth, *Journal of Fluid Mechanics*, Vol.38, pp.499-511.

Newman, J. N. (1965b): Propagation of water waves over infinite step, *Journal of Fluid Mechanics*, Vol. 23, pp.399-415.

Ohyama, T. and Nadaoka, K. (1991): Analysis of wave splitting phenomenon with a submerged breakwater using a numerical wave flume, *Proc. Coastal Eng., JSCE*, Vol.38, pp.16-20 (*in Japanese*).

Thornton, E. B. and Calhoun, R. J. (1972): Spectral resolution of breakwater reflected waves, *Journal of Waterway, Port, Coastal and Ocean Engineering*, vol. 98, pp.443-460.