CHAPTER 52

Soliton-Mode Wavemaker Theory and System for Coastal Waves

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Abstract

A theory of wavemaker system for coastal waves (nonlinear random waves in very shallow water) having the desired statistics is developed under the assumption that they are treated as random trains of the KdV solitons and further its applicability is examined experimentally. This wavemaker system is shown usefull to generate accurately nonlinear waves having the desired water surface profiles of which nonlinearity is specified by the Ursell number $U_r \geq 15$ for regular waves (cnoidal waves) and $U_r \geq 8$ for random ones (coastal waves).

Introduction

Waves in shallow water come to behave like trains of independent individual waves with the shoaling water. In shallow to very shallow water, particle-like property of each individual wave is more strengthened because of the pronounced nonlinear effect, so that its time sequence becomes increasingly important to coastal engineering problems, such as dynamic response of coastal

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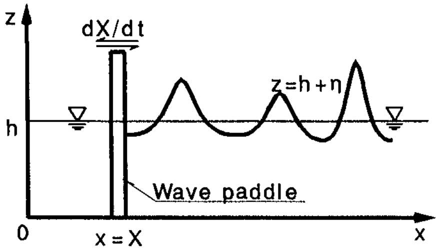


Figure 1. Definition of coordinate system and symbols

structures, nearshore current system and so on. It hence is required in experimental works to generate the wave train having the desired sequence of wave crest. However, usual methods based on the Biesel-Suquet wavemaker theory and the envelope theory assuming a narrow-banded Gaussian process cannot be used to generate strongly nonlinear random waves in very shallow water (called here as coastal waves).

Authors(Yasuda et. al., 1987) already showed that coastal waves can be treated as a random train of solitons and their kinematics can be described using the asymptotic multi-soliton solution of the KdV equation. Hence, treating individual waves defined by the zero-crossing method as solitons and using a stochastic model (Shinoda et al., 1992) of solitons, we can generate straightforwardly coastal waves having the desired sequence of individual waves as a train of solitons.

In this study, we suggest a nonlinear wavemaker theory for the coastal waves and then develop a soliton-mode wavemaker system to generate them in a wave tank. Further, the performance of the system is examined by generating cnoidal waves (treated as uniform trains of solitons) and coastal waves (as random trains of solitons) having the desired sequence.

Wavemaker Theory

The boundary value problem for the wavemaker in a wave tank follows directly from that for two-dimensional waves propagating in an incompressible and irrotational fluid. For the coordinate depicted in **Figure 1**, it is required to solve the following equations,

$$\phi_{xx} + \phi_{zz} = 0, \tag{1}$$

$$\eta_t + \eta_x \phi_x - \phi_z = 0|_{z=h+\eta}, \tag{2}$$

$$\phi_t + (\phi_x^2 + \phi_z^2)/2 + g\eta = 0|_{z=h+\eta}, \tag{3}$$

$$\phi_z = 0|_{z=0}, \tag{4}$$

$$dX/dt = \phi_x|_{x=X}, \tag{5}$$

where ϕ denotes the velocity potencial, η the water surface elevation, g the acceleration of gravity, h the mean water depth, and X the displacement of a piston type paddle.

Integrating eq.(1) from $z = 0$ to $z = h + \eta$ and substituting eqs.(2) and (4) into the resultant equation, we can obtain the following equation,

$$\int_0^{h+\eta} \phi_x dz = -\int \eta_t dx + \text{const.} \tag{6}$$

Since eq.(5) can not be satisfied for the arbitrary z-coordinate as long as piston-type wave paddle is used, the vertically integrated equation instead of eq.(5) is employed as the boundary condition with regard to the depth-averaged velocity by following Goring and Raichlen's approach(1980). Integration of eq.(5) from $z = 0$ to $z = h + \eta$ yields to

$$\int_0^{h+\eta} \frac{dX}{dt} dz = \int_0^{h+\eta} \phi_x dz \Big|_{x=X}. \tag{7}$$

Substituting eq.(6) into eq.(7), we obtain the following relation satisfying eqs.(1), (2), (4) and (5).

$$\int_0^{h+\eta} \frac{dX}{dt} dz = -\int \eta_t dx + \text{const.}\Big|_{x=X}. \tag{8}$$

Since the value of η_t becomes zero in the still water and the wave paddle is also at rest, the integral constant in eq.(8) is regarded as zero. Further, since dX/dt is independent of z in the present piston-type wavemaker system, we can derive the following equation for the paddle motion to generate the wave profile η from eq.(8).

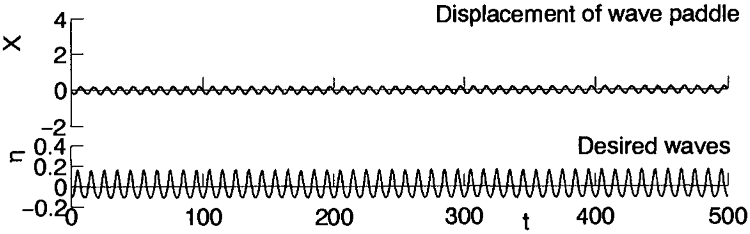
$$\frac{dX}{dt} = \frac{-1}{h+\eta} \int \eta_t dx \Big|_{x=X}. \tag{9}$$

Here, the following dimensionless variables with asterisk are defined for convenience of numerical computations.

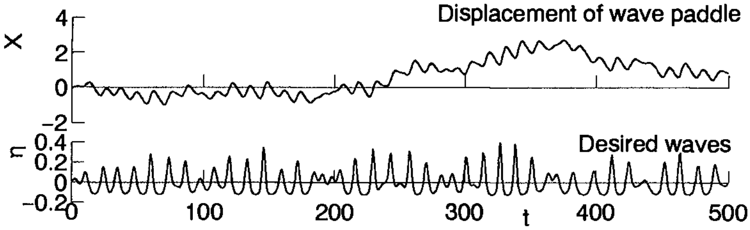
$$\phi^* = \phi/\sqrt{gh^3},\ \eta^* = \eta/h,\ X^* = X/h,\ x^* = x/h,\ z^* = z/h,\ t^* = t\sqrt{g/h} \tag{10}$$

However, the asterisks are omitted for simplicity hereinafter, so that eq.(9) is rewritten as

$$\frac{dX}{dt} = \frac{-1}{1+\eta} \int \eta_t dx \Big|_{x=X}. \tag{11}$$



(a) Cnoidal waves; A uniform train of solitons [$H = 0.3$, $T = 10.0$]



(b) Coastal waves; A random train of solitons [$\bar{H} = 0.218$, $\bar{T} = 9.15$]

Figure 2. Wave profiles of a soliton train having the desired sequence and the displacement of wave paddle to generate it; H is the dimensionless wave height corresponding to the amplitude of solitons, T the dimensionless wave period corresponding to the distance of adjoining solitons, $\bar{H}$ the dimensionless mean wave height, and $\bar{T}$ the dimensionless mean wave period

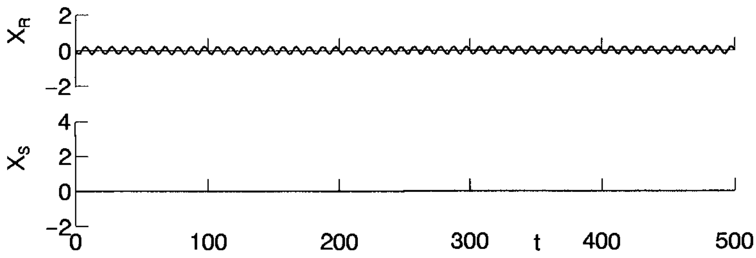
If waves to be generated are trains of solitons and satisfy eqs.(1)~(4) in the order of the KdV equation, η can be represented as

$$\eta(x,t) = \sum_{j=1}^{N} \eta_j(x,t) \tag{12}$$

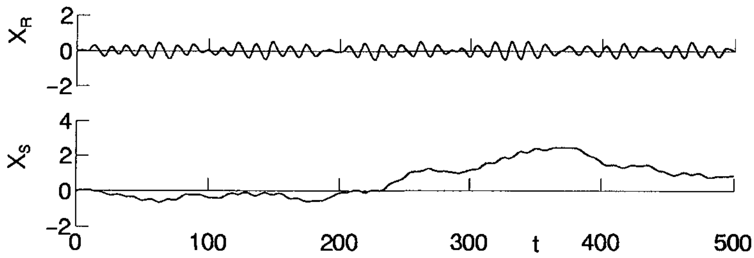
$$\left. \begin{array}{l} \eta_j(x,t) = A_j \operatorname{sech}^2 \frac{\sqrt{3A_j}}{2}\{x - c_j(t-\delta_j)\} - \frac{\eta_0}{N}, \\ c_j = 1 + \frac{A_j}{2} - \frac{3}{2}\eta_0, \quad \eta_0 = 4\sqrt{\frac{A_j}{3}\frac{c_j T_o}{h}} \end{array} \right\} \tag{13}$$

where A_j is the amplitude of the j-th soliton non-dimensionalized with h, N the number of solitons, T_o the observation period and δ_j the phase constant of the j-th soliton.

As a result, the problem to solve eqs.(1)~(5) can be finally reduced to that to solve the following nonlinear ordinary differential equation with regard to the



(a) Cnoidal waves; A uniform train of solitons [$H = 0.3$, $T = 10.0$]



(b) Coastal waves; A random train of solitons [$\bar{H} = 0.218$, $\bar{T} = 9.15$]

Figure 3. Decomposed displacement of the wave paddle; time history of X_R and X_S

displacement X, by combining eqs.(11) and (12),

$$dX/dt = \sum_{j=1}^{N} c_j \eta_j(X,t) \Big/ \left\{ 1 + \sum_{j=1}^{N} \eta_j(X,t) \right\}. \tag{14}$$

Thus, eq.(14) is solved numerically for given values of η_j and c_j ($j = 1, \ldots, N$), so that the displacement X of the wave paddle required to generate the desired waves is uniquely determined. Hence, if the waves to be generated are represented as a train of solitons described by eq.(12) and the ensembles of the soliton parameters A_j and δ_j governing them are substituted into eq.(14) through eqs.(12) and (13), the displacement X required to regenerated the waves in a wave tank can be easily known by solving eq.(14).

Wavemaker System

Wavemaker system to be used here is required to enable the wave paddle to follow accurately the displacement X given by eq.(14). Hence, in order to investigate the displacement X required to generate cnoidal waves (uniform trains of solitons) with $H = 0.3$ and $T = 10.0$ and coastal waves (random

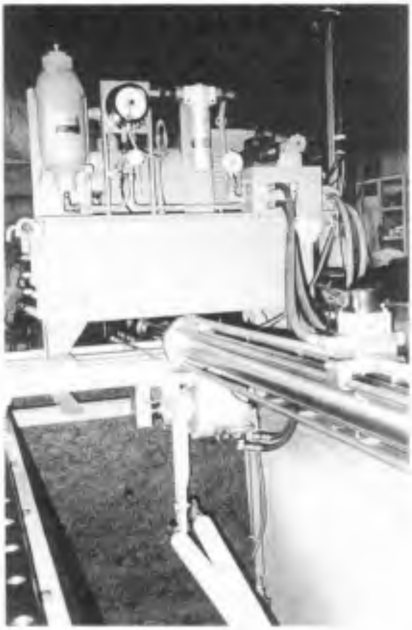


Photo 1. A rotary actuator and cylinder actuator attached to the wavemaker

trains of solitons) with $\bar{H} = 0.218$ and $\bar{T} = 9.15$, time histories of X are shown in **Figure 2** together with their surface profiles. Here, H and T denote the dimensionless wave height and period, respectively, and the over-bar means their mean values. While the displacement for cnoidal waves is periodic and stays within a stroke less than the mean water depth, that for coastal waves is aperiodic and accompanies a slow drift oscillation with a large amplitude exceeding two times of the water depth. Thus, a wavemaker system having a long stroke wave paddle is required to generate coastal waves. However, the motion of wave paddle seems to be a rapid movement with short stroke riding on a slow drift with long stroke, although the stroke of the paddle exceeds two or more times distance of water depth. By aiming at this point, the displacement $X(t)$ was decomposed into a rapid but short displacement $X_R(t)$ and a slow but long one $X_S(t)$ as shown in **Figure 3**, and thus a double mode wavemaker having a high-speed driving part with short stroke and a low-speed driving part with long stroke was newly developed.

The double-mode wavemaker has two types of movement corresponding to X_R and X_S. **Photo 1** shows the high-speed driving part with a rotary

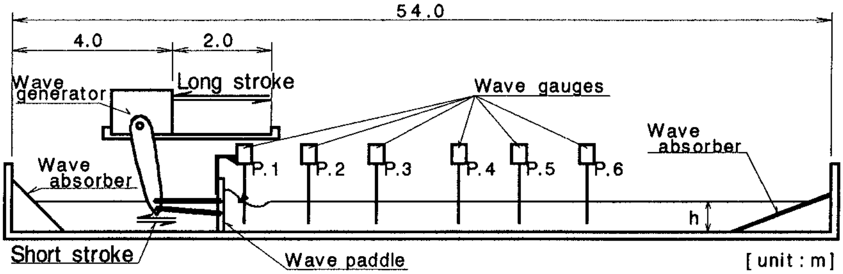


Figure 4. Sketch of the wave tank used here and location of wave gauges installed

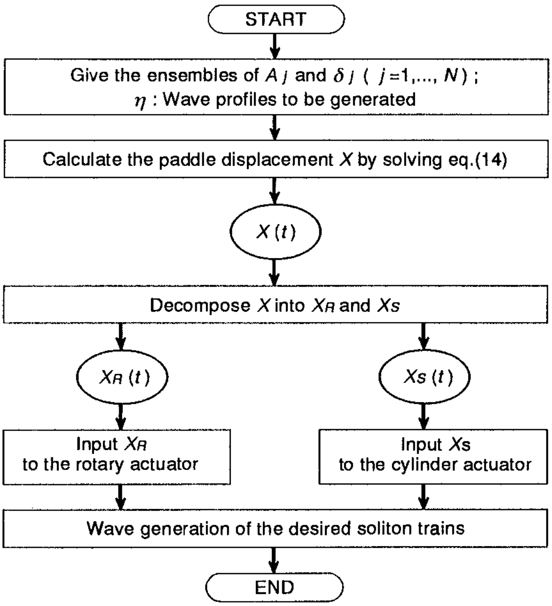
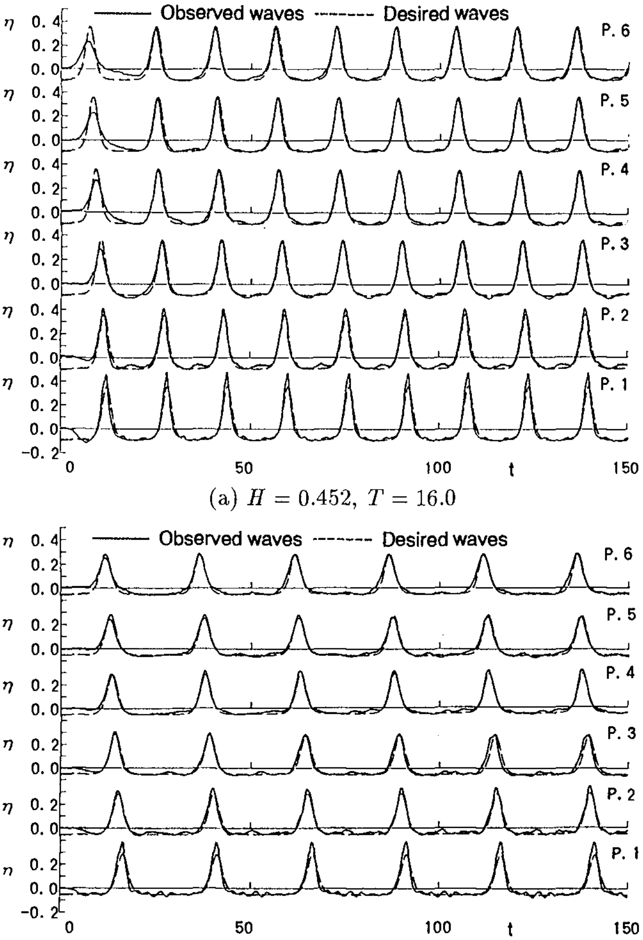


Figure 5. Flow chart of the procedure to generate the soliton trains as desired

actuator and the low-speed driving part with a cylinder actuator attached to the wavemaker, and **Figure 4** illustrates its schematic sketch. The rotary actuator enables short stroke but high speed motion corresponding to X_R and has a performance of the maximum speed of 60cm/s and stroke of 30cm. The cylinder actuator enables slow but long stroke motion corresponding to X_S and has a performance of the maximum speed of 7.3cm/s and stroke of 2m.

As a result, the procedure using the double-mode wavemaker to generate the desired waves is summarized as follows: The ensemble of A_j and δ_j for the



(a) $H = 0.452$, $T = 16.0$

(b) $H = 0.334$, $T = 25.0$

Figure 6. Comparisons for a train of uniform solitons between the desired wave profiles and the observed ones

soliton trains to be generated are given to eq.(14) and then the displacement X required to generate them is determined by solving eq.(14). Further, the displacement X is decomposed into X_R and X_S and the values of X_R and X_S at each time step are given to the rotary actuator and cylinder one, respectively, as input signal. Thus, the wave paddle follows the composed motion of X_R and

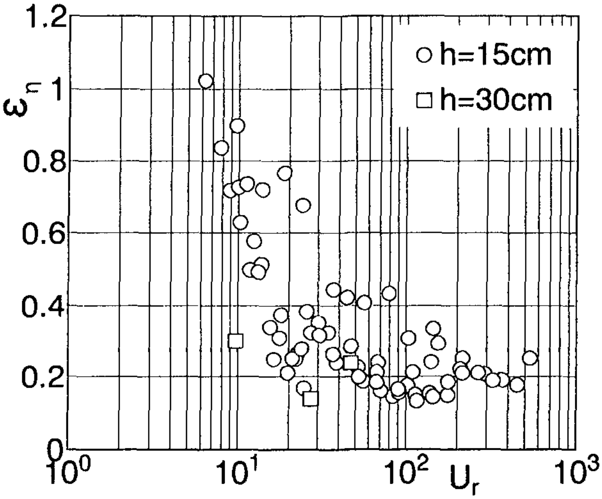


Figure 7. Relationships for cnoidal waves between the Ursell number U_r and the error index ε_η

X_S and produce the soliton trains as desired. Flow chart of the procedure explained above is illustrated in **Figure 5**.

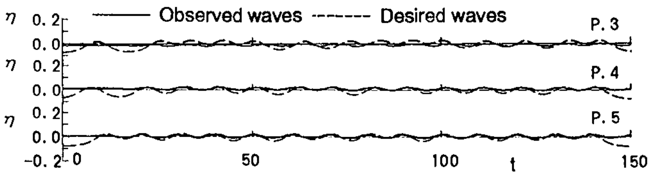
Performance Test

Experiments using the wave tank and gauges shown in **Figure 4** were undertaken to examine the performance of the double-mode wavemaker system. Six wave gauges were installed at six locations from P.1 to P.6 in the tank. Distance between each wave gauges is evenly about 2.5m.

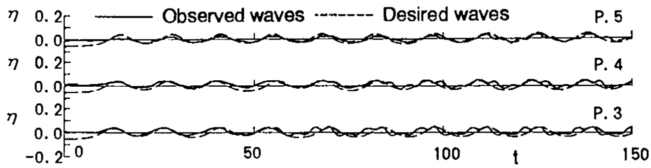
Cnoidal waves (Uniform trains of solitons)

Since cnoidal waves can be represented as a uniform train of solitons having constant amplitude and period, they can be generated in a wave tank by producing a uniform soliton train having a given constant amplitude and period in order. Water surface elevations of generated trains of solitons can be regarded as cnoidal waves as shown in **Figure 6**, where the comparisons between the measured temporal water surface elevations of the waves generated as a train of solitons and theoretical ones of the cnoidal waves to be generated are made This result further demonstrates that the soliton is the elementary excitation of waves in very shallow water and therefore the desired cnoidal waves can be generated as train of solitons.

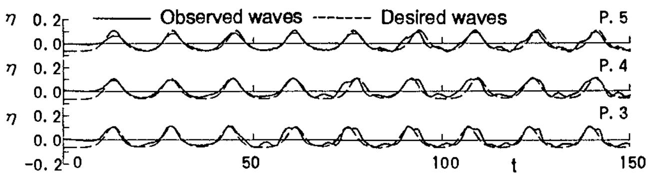
However, the soliton can not exist in shallow to deep water and becomes



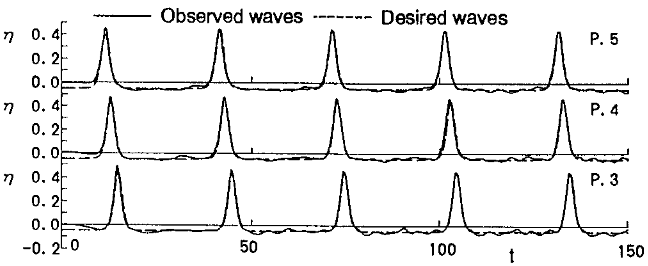
(a) $H = 0.097,\ T = 10.0,\ \varepsilon_\eta = 0.897$



(b) $H = 0.097,\ T = 14.0,\ \varepsilon_\eta = 0.765$



(c) $H = 0.174,\ T = 16.0,\ \varepsilon_\eta = 0.422$



(d) $H = 0.502,\ T = 30.0,\ \varepsilon_\eta = 0.177$

Figure 8. Comparisons for a train of uniform solitons between the desired wave profiles and the observed ones having a typical value of ε_η

physically meaningless wave there. The present soliton-mode wavemaker can not generate waves in shallow to deep water and therefore its applicable region should be made clear since there is the applicable limit to the wavemaker. In

Table 1. Experimental code numbers and conditions for coastal waves

Code No.	$\bar{H}$	$\bar{T}$	U_r	Skewness
R01	0.218	9.15	23.95	1.136
R02	0.321	10.25	40.44	0.760
R03	0.102	10.12	13.06	1.308
R04	0.207	12.21	38.21	1.305
R05	0.297	11.80	55.16	1.612

order to clarify quantitatively the applicable limit, an error index ε_η is defined as

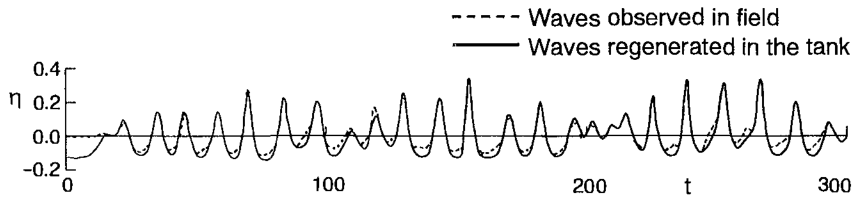
$$\varepsilon_\eta = \sqrt{\int_0^{T_o} (\eta_{obs} - \eta_{des})^2 dt \Big/ \int_0^{T_o} (\eta_{des})^2 dt}, \tag{15}$$

where η_{obs} denotes the water surface elevation of the observed waves in the wave tank and η_{des} that of the desired waves given to eq.(12) as input data. Relationships between the Ursell number U_r of generated waves and the error index ε_η are shown in **Figure 7**. The values of ε_η increase with the decreasing of the value of U_r and particularly is amplified rapidly from the region where the value of U_r begins to be less than about 15. Moreover, the difference of surface profile between η_{obs} and η_{des} can be approximately ignored as shown in **Figure 8** if the value of ε_η is less than 0.8. It hence could be stated that the present wavemaker system can generate cnoidal waves with $U_r \geq 15$ as desired.

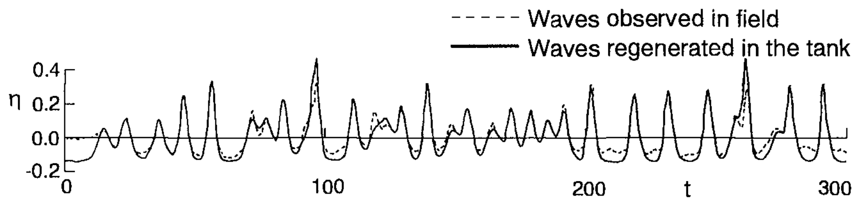
Coastal waves (Random trains of solitons)

Intensive experiments generating various trains of solitons were undertaken to examine how to present wavemaker can accurately regenerate coastal waves having various statistics as desired. Representative cases of them are shown in **Table 1**, in which their code numbers and wave statistics are indicated. R01 and R02 denote the coastal waves observed on Ogata coast facing the Japan sea, and R03, R04 and R05 indicate the coastal waves simulated as random trains of solitons by using a digital simulation method (Shinoda et al., 1992). $\bar{H}$ and $\bar{T}$ are their mean wave heights and periods respectively. The values of U_r and Skewness show that all the waves to be regenerated have pronounced nonlinearity and can not be generated with usual methods based on the Biesel-Suquet wavemaker theory.

Figure 9 shows the comparisons of wave profiles of R01 and R02 between the observed waves in field and the regenerated waves in the wave tank. Both the wave profiles are in almost complete agreement and demonstrate that the



(a) Code No. R01



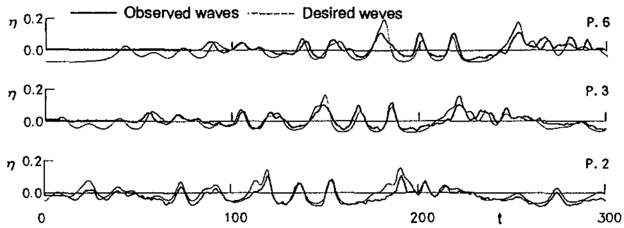
(b) Code No. R02

Figure 9. Comparison of wave profiles between waves observed in field and regenerated ones in the wave tank

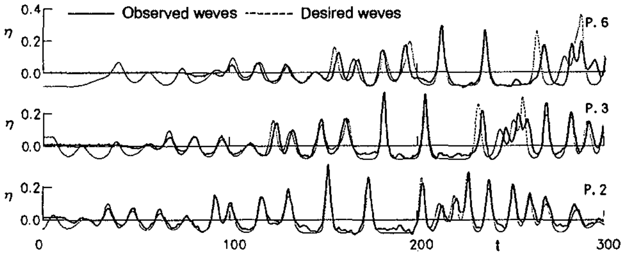
present wavemaker can regenerate coastal waves having the given statistics including time sequence of wave crest as desired. It should be particularly noted that coastal waves in field can be almost completely regenerated in the tank even if they partially break.

Further, **Figure 10** shows the comparisons of wave profiles of R03, R04 and R05 between the digitally simulated waves using a stochastic mode of solitons and the regenerated waves in the tank. Although the surface profiles of the regenerated waves get to deviate slightly from those of the simulated waves as the waves propagate from P.2 toward P.3 and further P.6, both the profiles keep a good agreement and shows that the present wavemaker can generate the coastal waves having arbitrary wave statistics by making random trains of solitons in order only if the value of the statistics are given.

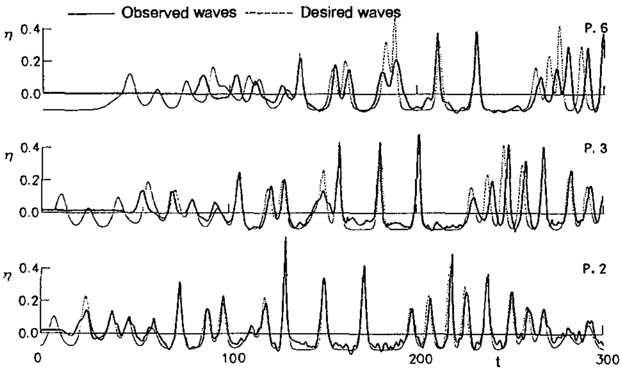
In order to make clear the applicable limit of the present wavemaker system to coastal waves, the relations between the Ursell number U_r of coastal waves to be generated and the error index ε_η for the regenerated waves in the tank are examined and shown in **Figure 11**. The value of ε_η is less than 0.8 almost independently of the Ursell number. Since the values of ε_η in all the cases shown in **Figures 9** and **10** are at most 0.644, the generated waves in the tank could



(a) Code No. R03
$[\bar{H} = 0.102,\ \bar{T} = 10.12,\ \rho_{HH} = 0.319,\ \rho_{HT} = 0.0,\ \varepsilon_\eta|_{P.2} = 0.644]$



(b) Code No. R04
$[\bar{H} = 0.207,\ \bar{T} = 12.21,\ \rho_{HH} = 0.534,\ \rho_{HT} = 0.216,\ \varepsilon_\eta|_{P.2} = 0.405]$



(c) Code No. R05
$[\bar{H} = 0.297,\ \bar{T} = 11.80,\ \rho_{HH} = 0.532,\ \rho_{HT} = 0.368,\ \varepsilon_\eta|_{P.2} = 0.484]$

Figure 10. Comparisons between the desired wave profiles for a random train of solitons and those measured in the wave tank

be regarded as almost desired if the value of ε_η is less than 0.8, that is, the value of U_r is almost more than 8. It hence could be stated that the present wavemaker can generate the coastal waves having given statistics as desired if their Ursell numbers are more than 8.

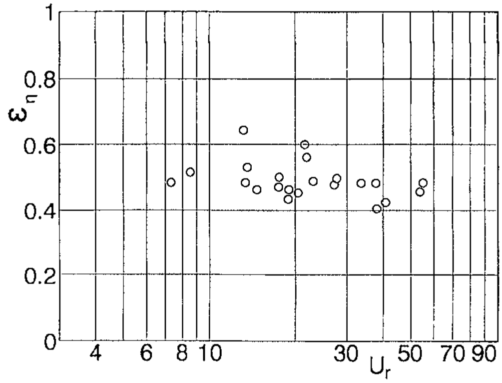


Figure 11. Relationships for coastal waves between the Ursell number U_r and the error index ε_η at the measurement point P.2

Conclusions

A wavemaker theory to generate nonlinear waves in very shallow water as trains of solitons was developed by solving mathematically wavemaker problem in the order of the KdV equation. The paddle motion given by the theory showed that the displacement required to generate them as desired exceeds two times or more of the water depth and a wavemaker having long stroke paddle is necessary. Hence, a new type wavemaker was thought out and developed under the cooperation of ISEYA Manufacturing Co. in order to solve this problem, by noticing that the displacement can be separated into two parts; the one is rapid but short stroke part and the other is slow but long stroke part. This wavemaker can generate the desired soliton trains in order by equipping a double-mode movement having a rotary actuator enabling short but rapid motion and a cylinder actuator enabling slow but long stroke motion was thought out and newly developed. Experimental examination verifies that the system is useful to generate strongly nonlinear waves having the desired water surface profiles of which nonlinearity is specified by the Ursell number $U_r \geq 15$ for cnoidal waves and $U_r \geq 8$ for coastal waves. Thus, the double-mode movement is essential to the soliton-mode wavemaker to generate the desired coastal waves.

Hence, it could be stated that the present wavemaker system can almost completely generate both the cnoidal and coastal waves having not only the desired statistics including sequence of wave crest but also the desired wave profiles, if the input signal to the wave paddle is given by the present wavemaker theory and given to the soliton-mode wavemaker as input signal and further the waves to be generated are within the aforementioned applicable region of this system.

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