

CHAPTER 43

EFFECTS FROM DIRECTIONALITY AND SPECTRAL BANDWIDTH ON NON-LINEAR SPATIAL MODULATIONS OF DEEP-WATER SURFACE GRAVITY WAVE TRAINS

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Abstract

The non-linear nature of the largest waves in random wave trains is studied. Experimental wave elevation records from measurements in a large wave basin are analysed. Longcrested as well as shortcrested wave conditions, with various spectral shapes, are included. The spatial development of spectral and statistical properties of the wave trains are analysed. The results indicate that closer than 10-15 wavelengths, observed deviations from linear theory may be explained by 2nd order effects. Further away, higher-order modulational instabilities may lead to a stronger increase of extreme waves. This is especially observed in narrow-banded long-crested waves, while observations in short-crested sea show very little content of these space-dependent modulational effects.

1. Introduction.

Extreme waves observed in random wave trains on deep water have in many cases exceeded predicted levels based on linear wave theory and Rayleigh statistics. Examples on this may be found in laboratory experiments (Stansberg 1991, 1992) as well as in full scale records (Sand et. al. 1990, Kjeldsen 1990, Jonathan et. al. 1994, and others). Second-order random wave theory (Longuet-Higgins 1963) explains a significant part of the deviations from linear theory (Marthinsen and Winterstein 1991, Stansberg 1993, 1994, Vinje and Haver 1994). Some of the observed effects, however, need other explanations. Such results have been published in e.g. Stansberg (1992), where experiments with long-crested waves in a large wave basin were reported. Those experiments seemed to indicate that laboratory generated random wave trains travelling more than 10 - 15 wavelengths

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from the wavemaker are modulated due to higher-order non-linear wave-wave couplings. These may then lead to extreme waves clearly higher than estimates from linear as well as from second-order theory. A possible connection between these observations and the well-known "Benjamin-Feir-modulations" in regular waves (Benjamin and Feir 1967, Lake et. al. 1977, Lo and Mei 1985, and others) was discussed on basis of the experiments. Numerical simulations on related problems have shown a similar connection (Wang et. al. 1992, Yasuda et. al. 1992, Yasuda and Mori 1994).

The significance of such higher-order modulational instabilities in random waves is, however, still not quite clear. Although their existence have been more or less documented, it is a question how stable they are in time and space, and how they depend on various parameters such as the spectral shape and the wave directionality. Previous works (see e.g. the experiments by Su et. al. 1982) have indicated, however, that there is a connection between non-linear modulations and wave directionality, and the simulations by Yasuda and Mori (1994) show some dependence on spectral bandwidth. On this background, new systematic and controlled experiments have been carried out with random waves in MARINTEKs Ocean Basin. Long-crested (unidirectional) as well as short-crested (directional) waves were run, with various spectral bandwidths. The present paper describes a preliminary, empirical analysis study based on the measurements. Thus some main results are shown and interpreted in light of the relationships discussed above. The presentation starts with some simple, but illustrating examples in bichromatic waves, and this is then followed by the main part dealing with random waves.

2. Tests with bi-chromatic waves

2.1. Test facility with set-up.

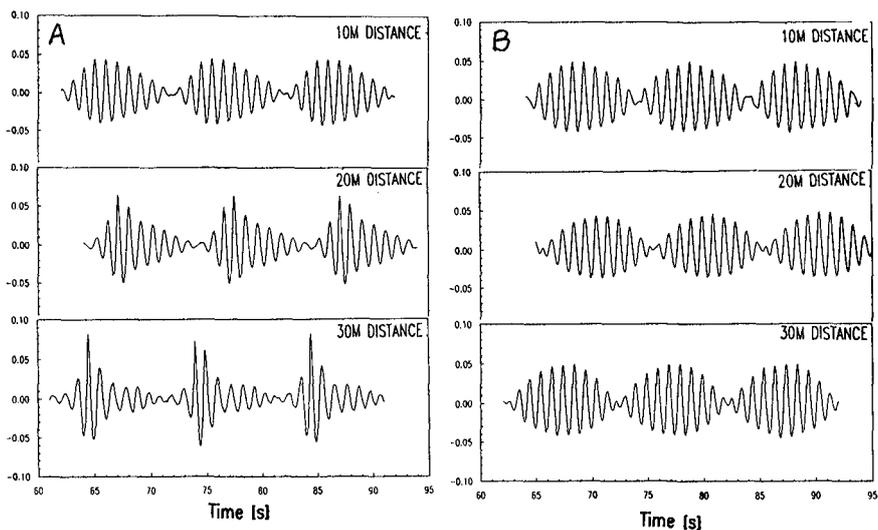
The tests were carried out in MARINTEKs 50mx80m Ocean Basin, which is equipped with a longcrested wavemaker along one full short side, and with a multiflap directional wavemaker along 63m of one long side. Rigid sloped beaches are installed at the opposite sides of the basin. The depth of the adjustable bottom was set to 4m, which corresponds to deep water for the actual waves considered. Wave elevation was measured with wave staffs at a number of locations at different distances from the wavemakers. In the present study, records taken at 10m meters intervals in the actual wave directions are reported.

2.2. Directional effects in bichromatic waves.

As shown in previous experiments (e.g. Stansberg 1992), non-linear wave modulations are rather rapidly developing in long-crested bichromatic wave trains with the 2 frequencies quite close to each other. These modulations, also identified through spatially developing side-bands in the frequency domain, give rise to asymmetry in the wave groups. This develops into groups with extreme individual

waves that may become significantly higher than in the initial wave trains. In the new experiments, similar tests were run with longcrested as well as with shortcrested (bi-directional) waves. Examples from the results are shown in fig. 1, including time series samples taken at 3 different distances from the wavemakers. As seen from this, the strong modulations in the longcrested case (which confirms the previous results referred to above) almost disappear completely in the shortcrested case. This is a strong support of the idea that the directionality is an important parameter in the study of non-linear wave modulations.

As a check on the influence from the basin geometry, additional long-crested wave tests were run from the multiflap wavemaker, as well as from the longcrested wavemaker. The characteristic nature of the modulations in fig. 1a was observed in both cases.



*Fig. 1. Wave elevation from bichromatic test records.
A = long-crested. B = short-crested (bi-directional).*

3. Tests with random waves.

All wave records presented below are from measurements in the MARINTEK Ocean Basin. For a brief description of the facility and the set-up, see section 2.1 above.

3.1. An example.

The random wave elevation sample records in fig. 2a illustrate how a long-crested wave group develops throughout space, finally resulting in a very large wave event. Similarly, fig. 2b shows how a group propagates in a directional sea. A basic difference is observed between the 2 cases: The long-crested group continues over a long distance, while the short-crested one exists only over a limited area. It seems reasonable to state that the spatial development of non-linear modulations in such wave groups must be different in the 2 cases. Intuitively, one would expect stronger modulational effects in the longcrested case, where these non-linear effects may grow over many wavelengths, than in the shortcrested case, where the wave groups disappear more quickly. (This can be seen in relation to a comment in Taylor and Haagsma (1994), where a simple argument indicates that these wave-wave interactions are less significant in spread sea than in unidirectional sea). Thus the study of non-linear extreme waves is closely connected to the study of non-linear wave group evolution, as indicated in Wang et. al. (1994).

3.2. Brief description of the tests and procedures.

A systematic test program with a large number of different longcrested and shortcrested wave conditions was run. Shortcrested waves were run from the multiflap wavemaker, while longcrested waves were basically run from the other, longcrested wavemaker. Since they generate waves in different directions in thebasin, some longcrested waves were also run from the multiflap wavemaker, as a check on possible noise effects from the basin geometry etc. Here we report mainly on tests with initial spectral peak periods T_p at 1.0 seconds, which corresponds to relatively short waves in this large basin. This is chosen in order to let the waves propagate a large number of wavelengths across the measuring area, for the study of spatially developing non-linear modulations.

The wave elevation was simultaneously recorded at a number of locations in the basin, as described in 2.1 above. Each random wave record includes about 1500 zero-crossing waves, in non-repeating wave trains. This is considered sufficient for the study of non-linear wave statistics. The following presentation focuses on the spatial development of spectral, grouping and statistical properties of the wave records. The dependence on the travelled distance (in number of wavelengths), directionality, spectral bandwidth and wave steepness is emphasized. Wave groups are analysed by the Hilbert envelope technique, where the square wave envelope represents the continous "group signal" (Stansberg 1983).

3.3. Presentation of results.

Figs. 3 & 4 show examples on the spatial development of the elevation power spectra. 3 different long-crested wave conditions are presented in fig. 3, while 2 short-crested conditions are presented in fig. 4. The short-crested conditions are

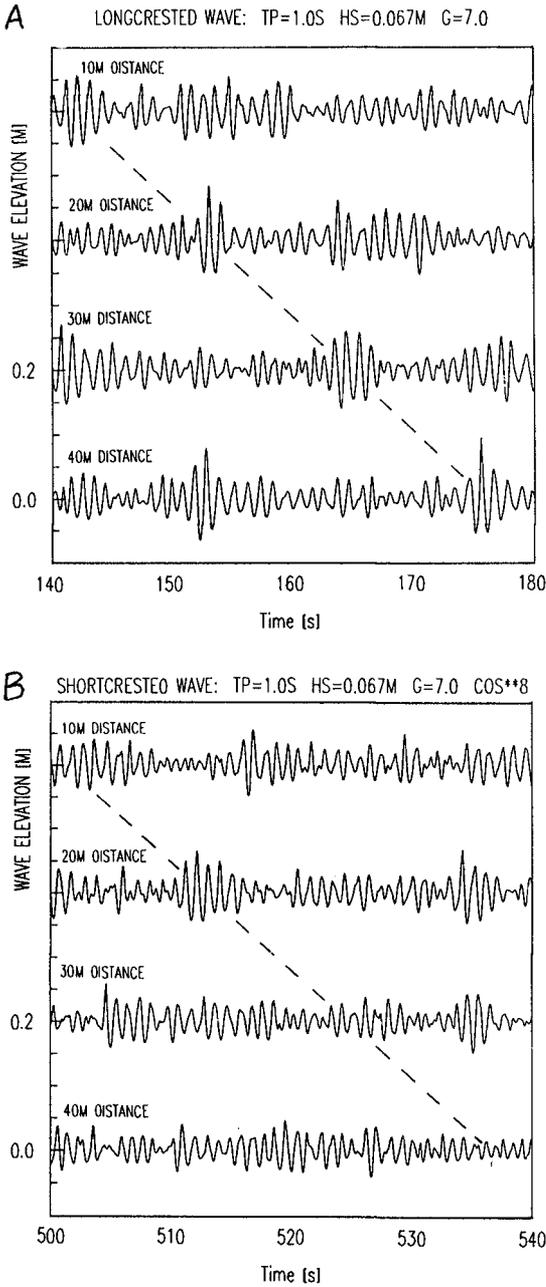


Fig. 2. Sample records illustrating wave group development.

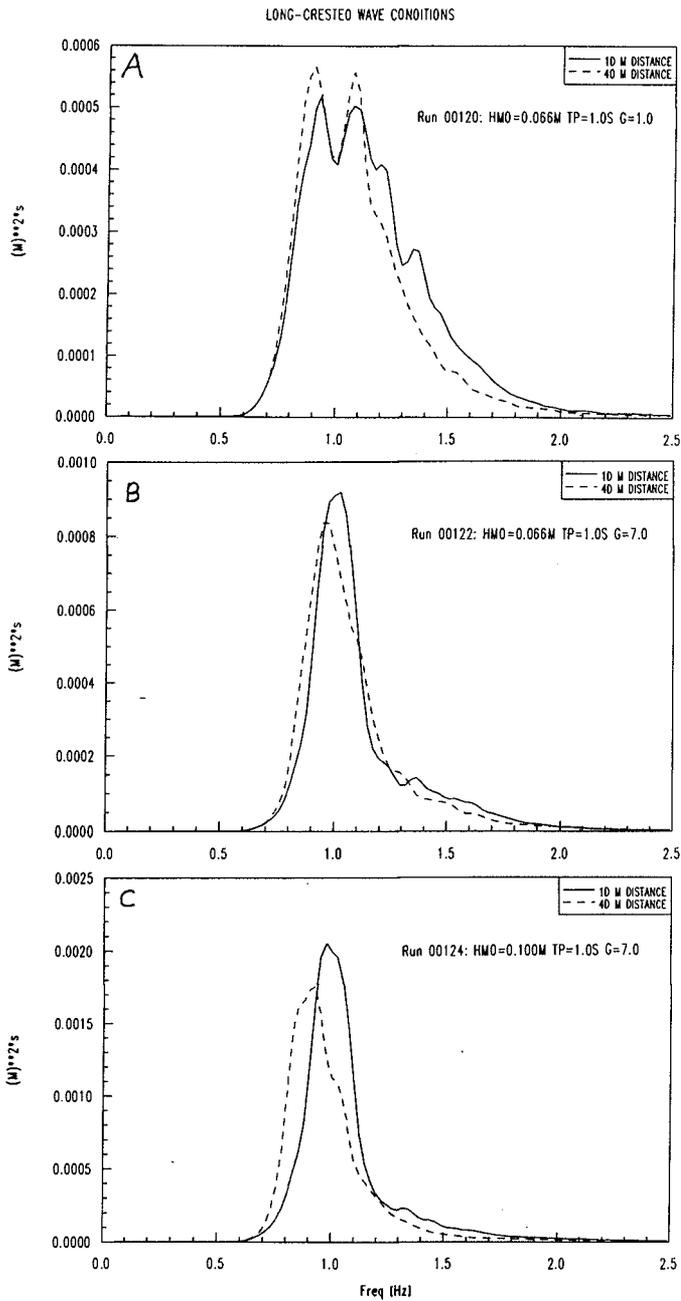


Fig. 3. Power spectra, 3 longcrested wave conditions, each at 2 locations (10 m and 40 m).

comparable, in scalar spectral parameters, to the 2 narrow-banded long-crested spectra. Each plot includes spectra measured both at 6 and 25 wavelengths distances (corresponding to 10 and 40 meters).

With location 10 m, nearest to the wavemaker, defined as the reference signal, amplitude amplification factors at location (40 m) are calculated for the same 5 sea states as in figs. 3 & 4. The results are presented in fig. 5, with 1 plot for longcrested, and 1 plot for short-crested waves. This factor is simply estimated as the square root of the ration between the corresponding spectra.

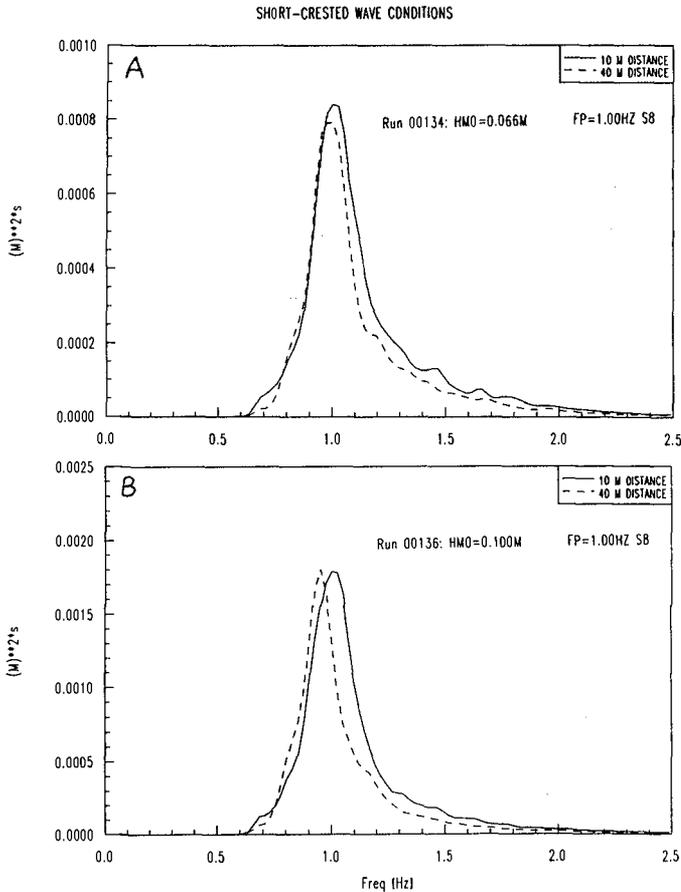


Fig. 4. As fig. 3, but for 2 shortcrested conditions. (\cos^8 - distributions).

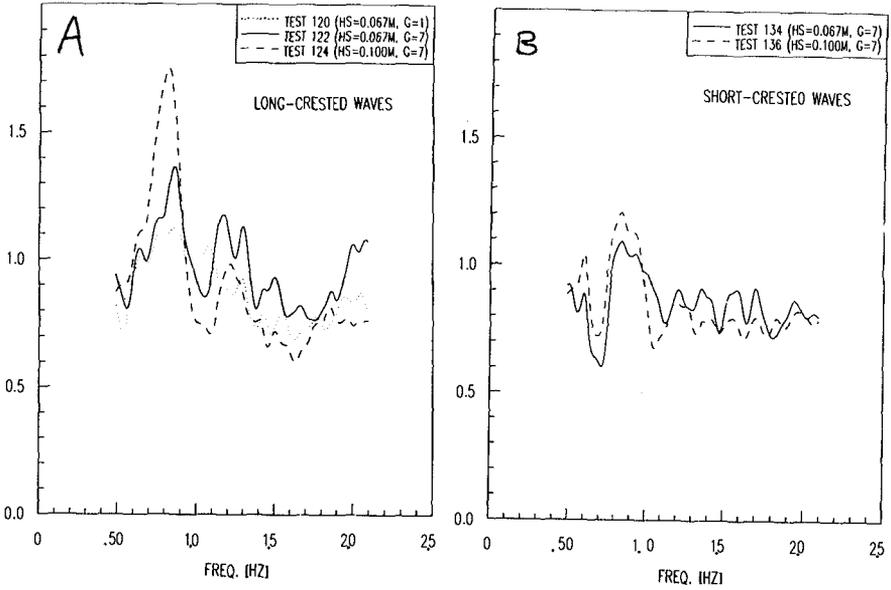


Fig. 5. Amplitude amplification from 10 m to 40 m distance.

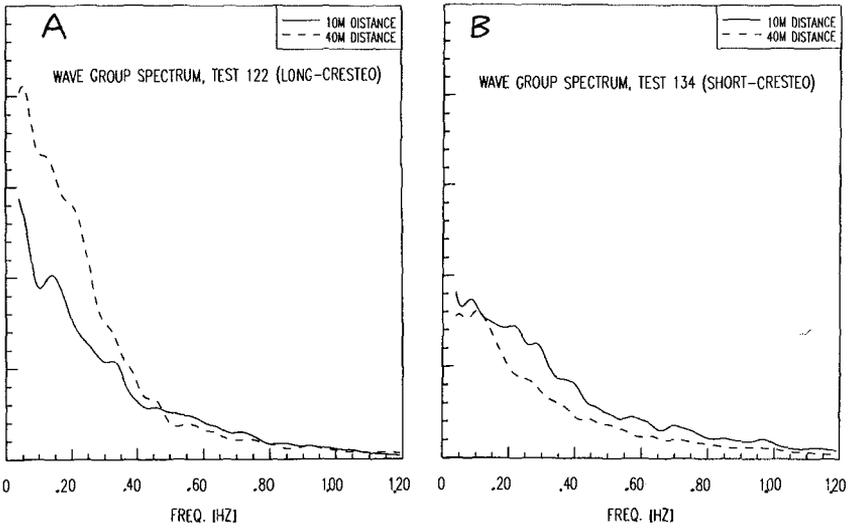


Fig. 6. Wave group spectra, for conditions in figs. 3B (long-crested) and 4A (short-crested).

The most pronounced modulations and non-Gaussian statistics are observed in the narrow-banded long-crested test case 122 (figure 3 B). For this case, wave group spectra and crest and waveheight distributions are presented in figs. 6A and 7, respectively. For comparison, the same type of results for the corresponding short-crested case are plotted in figs. 6B and 8.

For a concentrated illustration of the basic non-linear statistical properties of the recorded data, and their relations to other relevant parameters, the statistical skewness and kurtosis of some of the records are shown in fig. 9. The skewness is based on 3rd order statistical moment of the time series, and is calculated as:

$$\gamma_1 = \frac{1}{N\sigma^3} \sum_{i=1}^N (x_i - \bar{x})^3 \quad (1)$$

where

- x_i = sample value of time history
- \bar{x} = mean value of x_i
- σ = standard deviation of x_i
- N = number of time history samples.

The kurtosis is based on the 4th order statistical moment, and is calculated as:

$$\gamma_2 = \frac{1}{N\sigma^4} \sum_{i=1}^N (x_i - \bar{x})^4 - 3 \quad (2)$$

(with this definition also called "the excess of kurtosis")

Both these parameters are expected to be zero for Gaussian records (linear waves), but they are always subject to some statistical scatter. With the long records available here, the scatter is believed not to be a problem. The skewness is an indicator of 2nd order contents in the wave signal (Vinje and Haver 1994), while the kurtosis indicates the presence of 3rd and higher order contents. Thus the skewness gives information on crest heights, while the kurtosis indicates possible non-linear wave height statistics. The skewness values are plotted for all records of the total experiment, as a function of the typical steepness of the record. ($S = H_s / L_p$, with $L_p = (g / 2 \pi) \cdot T_p^2$, T_p = spectral peak period). The kurtosis values, however, are here plotted only for one set of H_s and T_p ($H_s=0.067\text{m}$, $T_p=1.0\text{s}$), as a function of the normalized travelled distance D_r (number of propagated wavelengths). ($D_r = D / L_p$). This is done in order to highlight observed relationships.

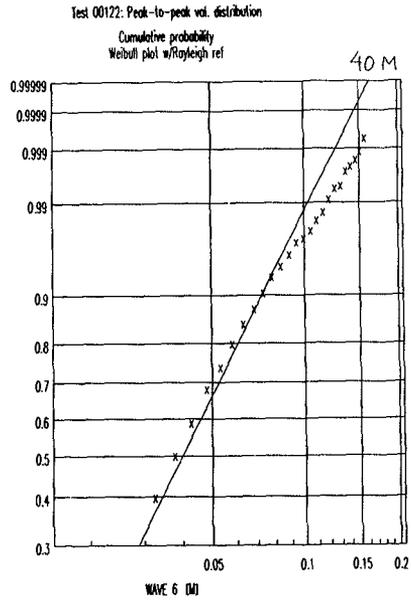
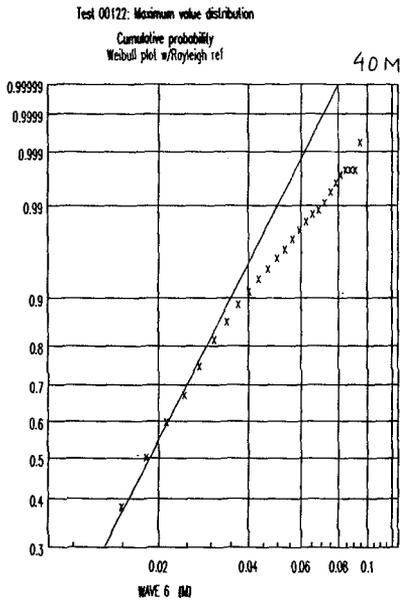
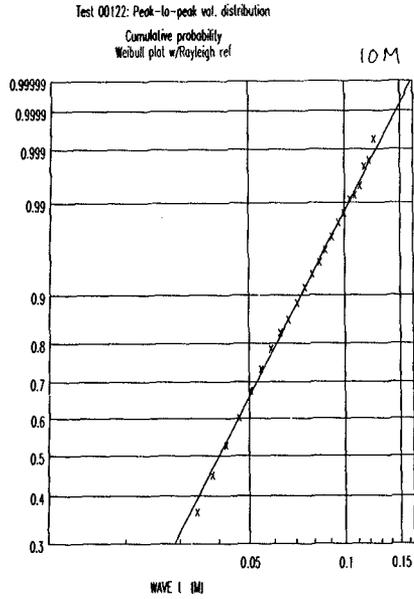
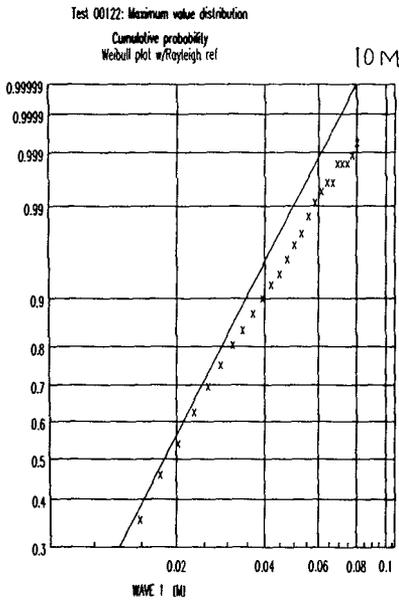


Fig. 7. Crest and wave height distributions, same conditions as in fig. 6A (long-crested).

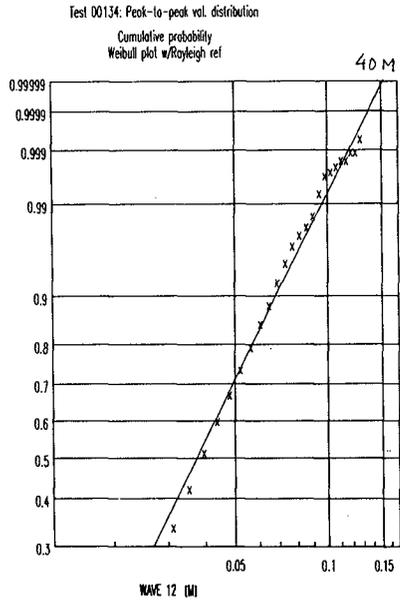
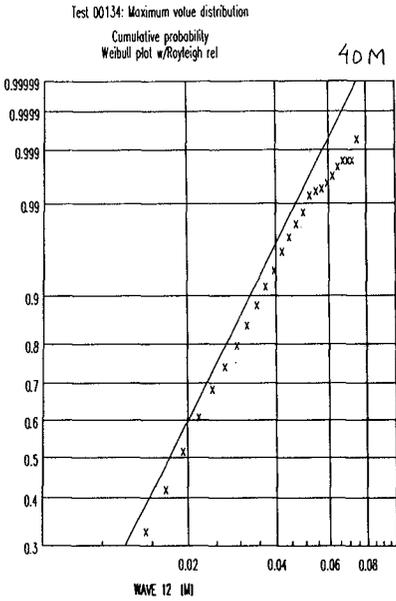
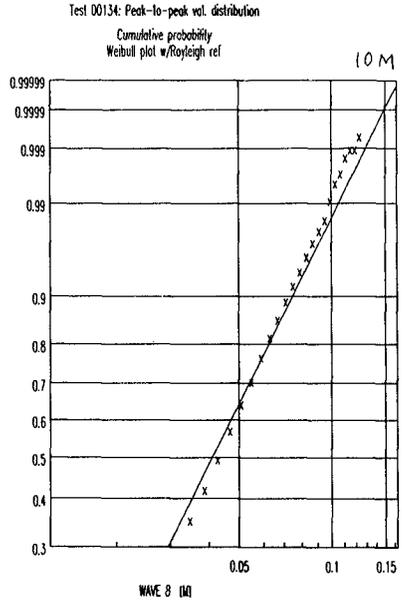
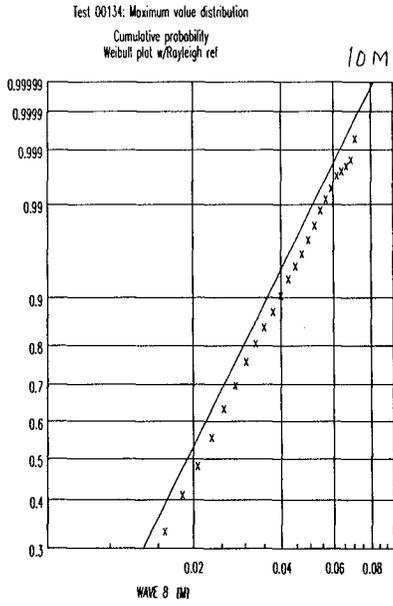


Fig. 8. As fig. 7, but same conditions as in fig. 6B (short-crested).

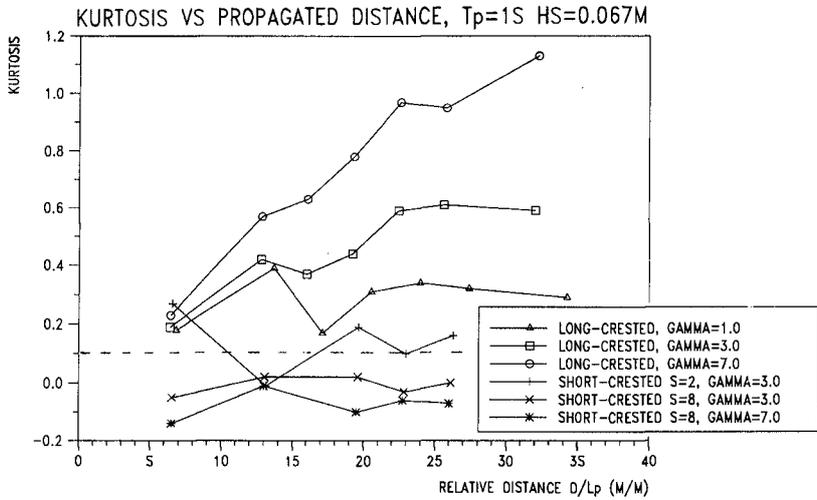
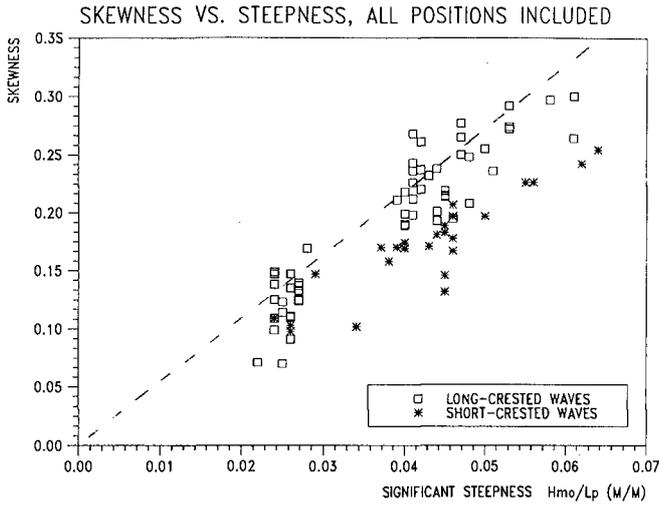


Fig. 9. Skewness vs. steepness, and excess of kurtosis vs. normalized travelling distance. Dotted line: estimated from 2nd order theory (Vinje & Haver 1994).

4. Discussion of results.

The power spectrum plots in figs. 3 & 4 show some changes in the spectra with increasing travelling distance. This is especially seen when the wave steepness is very high. In general, the energy in the high-frequency part of the spectrum is somewhat reduced due to dissipation. (At very high frequencies, however, the energy content seems to be more stable, probably due to bound harmonics). Energy shift is also observed from the peak frequency region to the surrounding upper and lower frequencies, particularly in longcrested waves. This is especially pronounced in the very steep sea condition in which case a net transfer to lower frequencies is also taking place together with some loss of energy (to some extent connected with dissipation due to breaking).

The energy transfer is further illustrated by the amplification plots in fig. 5. Of particular interest here is the apparent growth of side bands around the initial peak frequency, mainly observed in narrow-banded longcrested waves. This observation indicates a similarity with the side bands due to non-linear modulational effects in monochromatic and bichromatic waves discussed earlier. Therefore it fits well together with the observed increase in the far-distance wave group spectrum in fig. 6, and also with the crest height statistics deviating systematically from the Rayleigh model (fig. 7). While the distribution in fig. 7, at 10 m, (with extreme crests exceeding the Rayleigh estimate by 15-20 %) can be explained by 2nd order effects, the distributions at 40 m must be explained by higher-order effects. Crest levels up to 40% higher than the Rayleigh prediction are observed in the latter case.

The non-Gaussian statistics observed in the amplitude distribution plots are also recognized in the skewness and kurtosis plots. The plot of skewness shows that it depends more or less only on the steepness of the sea state, and less on other parameters such as directionality, propagation etc. The theoretical curve from Vinje and Haver (1994), based on 2nd order theory, fits reasonably well. (A slight skewness reduction is observed in directional sea). We interpret the result in the way that the 2nd order contribution is quite stable, regardless of other circumstances. The kurtosis, however, varies significantly with travelling distance, wave directionality and with the spectral width.

A clear increase in non-linear modulation effects with narrower spectral width is observed from the measurements. We interpret this, qualitatively, as a consequence of modulation periods possibly around 4 - 6 times the wave periods. It is reasonable to assume that this will affect long wave groups (narrow spectra) stronger than it will affect short wave groups (broader spectra). (Side-band instabilities may more easily become "smeared out" by disturbing neighbouring frequencies).

As already commented, the above non-linear effects are only to a limited extent

observed in short-crested waves in the present measurements. This is an important observation, which confirms the conclusion from the bichromatic test examples in fig. 1. Thus it seems reasonable to conclude from this that extreme wave events due to non-linear modulations are most pronounced in longcrested waves. It appears to be a possible connection between these effects and the direction of the local phase gradient relative to that of the energy gradient. For more firm conclusions to be drawn on this, more analysis work is to be made, possibly including comparisons to theoretical/numerical models.

5. Acknowledgement

This work has been financially supported by the Royal Norwegian Research Council.

6. References.

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