CHAPTER 24

Difference between Waves Acting on Steep and Gentle Beaches

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Abstract

The physical difference of beach erosions between on the steep and on the gentle beaches has been discussed, based on the field data and the theory of generation of the infragravity waves. As a result, it is shown that the incident waves are predominant in the wave run-up phenomena on the steeper beach, while on the gentler beach the infragravity waves are predominant.

Dear Prof. Dalrymple,

Based on the data obtained at the Hazaki Oceanographical Research Facility (**HORF**), I reported at the 23nd ICCE held in Venice, Italy, that the foreshore erodes under the action of infragravity waves of one to several minutes in a period (Katoh and Yanagishima, 1992). This conclusion, however, was distressed with the question made by one of participants, you Prof. Dalrymple, in the conference. Your question was; "There are many examples of beach erosion in experimental flumes with the regular waves, where the infragravity waves cannot exist. How do you explain away the experimental facts of erosion with your conclusion?". Your question was very excellent, because I could not answer on that time.

Since then, I have been studying on the physical difference between the beach erosions in the experimental flume and in the field. Here, I am going to explain the advanced conclusion on the role of the infragravity waves in the process of beach erosion, in the following.

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Critical Level of Foreshore Profile Change

At the HORF which is located on the sandy beach facing to the Pacific Ocean, Katoh and Yanagishima(1990) carried out the field observations on the foreshore berm erosion and formation. By analyzing the small scale sand deposition on the higher elevation when the berm eroded (see Figure 1), Katoh and Yanagishima (1992) presented that the critical levels of foreshore profile change in the processes of both erosion and accumulation are expressed by the significant wave run-up level, R_{max};

$$R_{max} = (\overline{\eta})_o + 0.96(H_L)_o + 0.31 \quad \text{(m)}, \tag{1}$$

where $(\overline{\eta})_o$ is the mean sea level and $(H_L)_o$ is the height of infragravity waves at the shoreline, respectively, and the third constant term is considered to represent the run-up effect of incident wind waves.

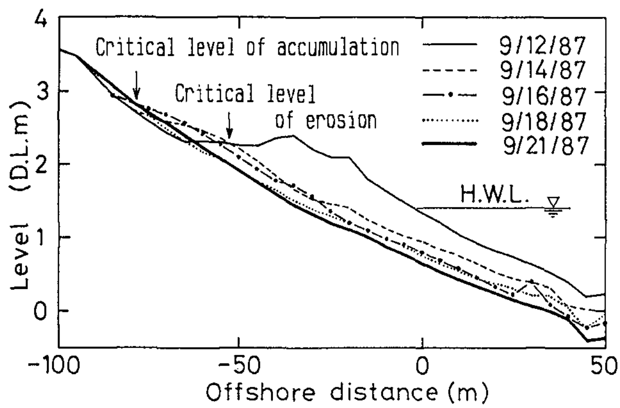

Figure 1 Typical example of berm erosion.

Equation (1) is the empirical relation which has been derived from the field data. The observation spans wide range of incident waves (offshore significant wave heights 0.39 -5.11 m and significant wave periods 4.5-15.6 s), $(H_L)_o$ = 0.15 -1.23 m, and $(\overline{\eta})_o$ = 0.79 -1.88 m. There is, however, a serious problem due to a fixation of observation site, that is to say, the constancy of profile slope. Namely, the effect of beach slope is excluded from equation (1), which restricts its general application.

For making up for this insufficiency of excluding the effect of slope, the empirical equation (1) has been compared with the previous results of experimental and theoretical studies. As the comparison of the results of field observation at the HORF with the Goda's theory (1975) has been already done in regard to the wave set–up at the shorelinc (Yanagishima and Katoh,1990), the run–up heights of incident waves and the heights of infragravity waves at the shoreline have been cxamined in this study.

Run–up Heights of Incident Waves

Mase and Iwagaki (1984) carried out experiments as to the run–up of irregular waves on the uniform slopc (1/5 to 1/30). As a result, they showed the relationship between thc run–up height and the Iribarren number ξ, or a surf similarity parameter, as follow;

$$\frac{R_{1/3}}{H_{1/3}} = 1.378\,\xi^{0.702}, \qquad (2)$$

$$\xi = \tan\beta \,/\, \sqrt{H_{1/3}/L_o} \;,$$

where $R_{1/3}$ is the significant run–up height which is defined by the crest method, $H_{1/3}$ and L_o are a significant wave height and a wavelength in deep water respectively, and $\tan\beta$ is a bottom slope. As Mase *et al.* used a still water level as a reference level for the wave run–up height, the effect of wave set–up is includcd in the run–up height in their analysis. On the other hand, as the referencc level in equation (1) is a mean water level at the shoreline under the action of waves on beach, the wave set–up is included in $(\bar{\eta})_o$, not in thc wave run–up height. In short, a direct comparison of equation (1) with equation (2) is impossible. Then, a magnitude of wave set–up at the shoreline, $\bar{\eta}_{max}$, has been estimated from the data of offshore significant waves by using the Goda's theory (1975). In the calculation, thc bottom slope is 1/60, which is the mean slope in the surf zone at the HORF. By adding $\bar{\eta}_{max}$ to thc third constant term in equation (1), the run–up hcight above the still water level, Rs, is evaluated as follow;

$$Rs = 0.31 + \bar{\eta}_{max}, \quad \text{(m)} \qquad (3)$$

Figurc 2 shows the relation between the non–dimensional run–up height normalized by the offshore significant wave height and ξ, whcre the bottom slopc is fixed to be 1/60. The data plotted by circles and by triangles are obtained in the processes of berm erosion and bcrm formation rcspectively (Katoh and Yanagishima, 1993,1993). From this figure, we have a linear relationship between two parameters as

$$\frac{Rs}{H_{1/3}} = 2.50\,\xi \;, \qquad (4)$$

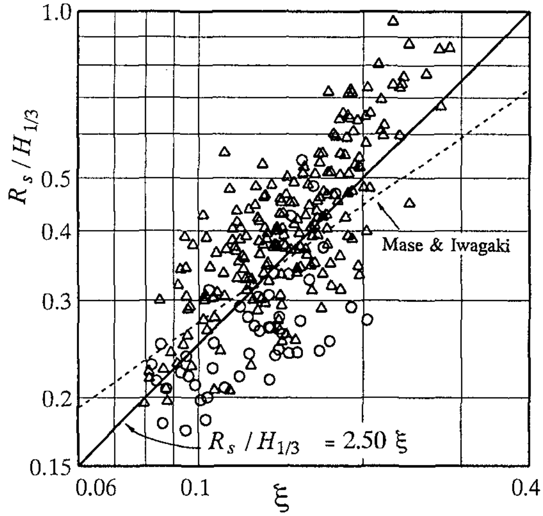

Figure 2 Relation between ξ and run–up heights of incident waves.

which is shown by a solid line closed to the equation (2).

Equation (2) proposed by Mase *et al.* is the significant run–up height by the crest method, while equation (3) is the critical run–up height where the significant profile change occurs. Although the physical meaning are different each other, it can be said that both run–up heights depend on ξ. Here, I have to give a supplementary explanation concerning equation (4). Since the data plotted in Figure 2 do not include the effect of bottom slope which is fixed to be 1/60 in analysis, it must be properly said that the non–dimensional run–up height is inversely proportional to the square root of wave steepness. However, equation (4) is the similar form to equation (2) which is based on the experimental data obtained on the bottom slope from 1/10 to 1/30. By taking this similarity into account, it can be concluded that the non–dimensional run–up height is proportional to ξ.

It is recognized in Figure 2 that the data plotted by the triangles, in the process of berm formation, are scattered mainly above the solid line, while the circles in the process of berm erosion are below the line. The leading cause of these inclined properties is considered to be due to the disregard of the effect of bottom slope in the area from the shallow water depth to the foreshore. If we consider the profile changes in the processes of berm formation, being steeper, and berm erosion, being gentle, the data by the triangles are shifted to the right and the data by circles to the left. As a result, all data will be plotted closer to the solid line.

Heights of Infragravity Waves at the Shoreline

Statistics of infragravity waves

In order to observe the waves near the shoreline, an ultra-sonic wave gauge was installed to the pier deck at the location where the mean water depth was about 0.4 meter in M.W.L. The wave measurement was carried out during 20 minutes of every hour with the sampling interval of 0.3 seconds (Katoh and Yanagishima, 1990). By utilizing the wave profile data, the wave heights and periods of infragravity waves were calculated by the following equations based on the result of spectra analysis;

$$H_L = 4.0\sqrt{m_0} \quad , \tag{5}$$

$$T_L = \sqrt{m_0 / m_2} \quad , \tag{6}$$

$$m_n = \int_0^{f_c} f^n S(f) df, \tag{7}$$

where H_L and T_L are the wave height and the period of infragravity waves respectively, f is the frequency, $S(f)$ is the spectral energy density, f_c is the threshold frequency of 0.33 Hz. The wave height of infragravity waves at the shoreline where the water depth was zero, $(H_L)_0$, has been estimated by the following transformation equation (Katoh and Yanagishima, 1990);

$$(H_L)_0 = H_L\sqrt{(1 + h / H_{1/3})} \quad , \tag{8}$$

where h is the water depth at the observation point which can be evaluated from the bottom level and the mean water surface level.

A statistical analysis has been done for the data obtained during 4 years from 1989 to 1990, provided that some data have been excluded according to the following criteria;

(a) The data obtained when the water depth was shallower than 0.5 meter should be excluded, because the sea bottom sometimes emerged when the waves ran down offshoreward.

(b) The data obtained when the water depth was deeper than 1.1 meters is not preferable, because the relative distance from the observation point to the shoreline was large.

Figures 3 and 4 show the frequency distributions of the heights and the periods of infragravity waves at the shoreline, respectively. The highest frequency of wave heights is in the rank of 0.2 to 0.3 meter, while the mean height is 0.38 meter. The number of cases is 45 (4.1%) for the waves higher than 0.8 meter, and 15 (

1.4%) for the waves higher than 1.0 meter. The maximum wave height is 1.34 meters. The wave periods distribute in the range from 40 to 100 seconds, being 62.4 seconds in average.

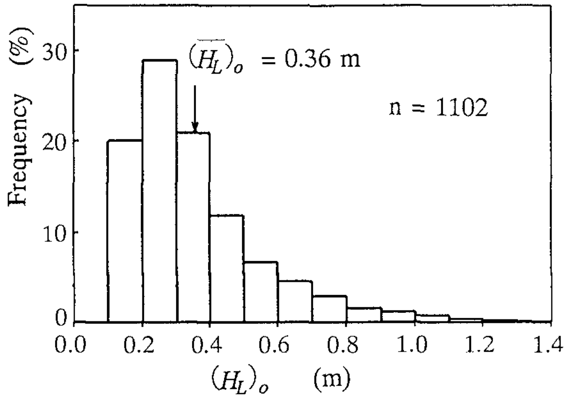

Figure 3 Frequency distribution of heights of infragravity waves.

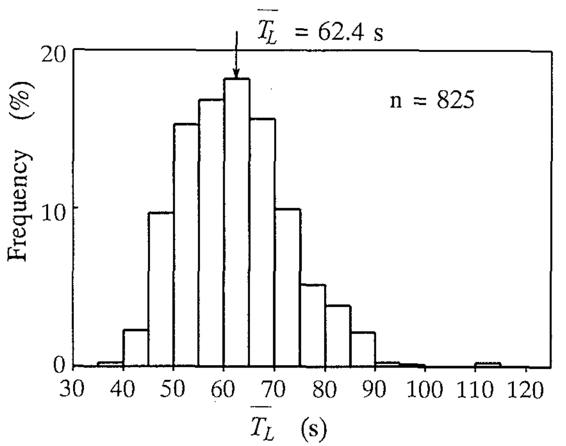

Figure 4 Frequency distribution of periods of infragravity waves.

Figure 5 shows the relation between the heights of infragravity waves at the shoreline and the offshore wave energy flux. On the upper, the significant wave height is shown as an indicator for the case that the wave period is 8.2 seconds. Although there is a little scattering of data, by means of the least square method we have a following relation;

$$(H_L)_o = 0.23\ E_f^{0.51} \tag{9}$$

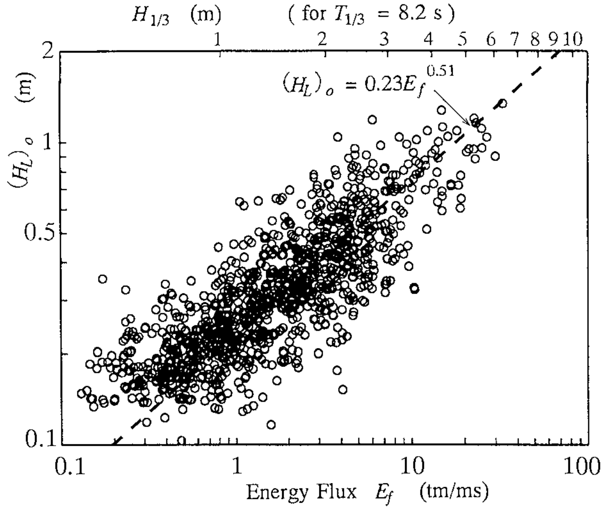

Figure 5 Relation between offshore wave energy flux and the height of infragravity waves at the shoreline.

where E_f is the offshore wave energy flux. Yanagishima and Katoh(1990) reported that the wave set-up at the shoreline, $\bar{\eta}_{max}$, at the HORF can be well explained only by the offshore wave energy flux,

$$\bar{\eta}_{max} = 0.14\, E_f^{0.4} \tag{10}$$

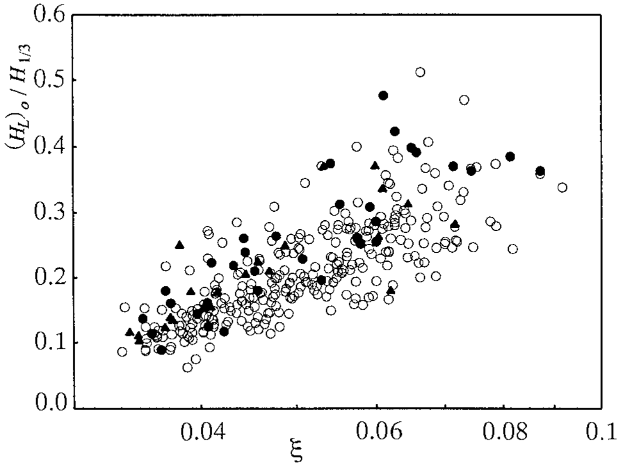

Figure 6 Relation between ξ and height of infragravity waves.

By substituting equations (9) and (10) into the first and second terms of equation (1) respectively, the critical level of foreshore change is expressed only by the wave energy flux. This result corresponds to another result that the daily changes of shoreline position can be well predicted by the offshore wave energy flux (Katoh and Yanagishima, 1988).

Figure 6 shows the relation between the non-dimensional height of infragravity waves at the shoreline, which are normalized by the offshore significant wave heights, and the Iribarren number ξ, provided that the mean slope in the area of 5 to 8 meters in depth where the incident waves break in a storm is utilized, which is 1/140. In this figure, the data obtained when the offshore significant wave height was larger than 2 meters is plotted. The closed triangles and the closed circles are the data which have been analyzed with respect to the berm formation and erosion (Katoh and Yanagishima,1992). Figure 6 shows that the non-dimensional height of infragravity waves increases with ξ. However, since the bottom slope is fixed to be 1/140 in the analysis, it must be properly said that the non-dimensional height increases with a decrease of wave steepness.

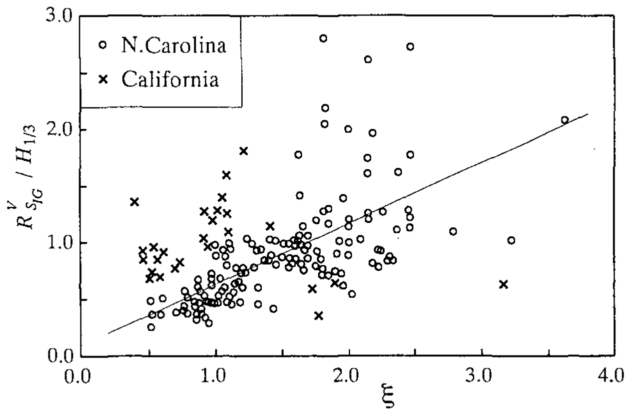

Figure 7 Relation between ξ and significant vertical swash excursion of infragravity waves (ξ is defined with the foreshore slope, from Guza et al., 1984).

In the same way, Guza *et al.*(1984) analyzed the two sets of wave run-up data, which were obtained on the beaches facing to the Pacific Ocean and the Atlantic Ocean. Figure 7 shows their result, where R^V_{SIG} is the significant vertical swash excursion of infragravity waves. We should remark that the foreshore slope was considered in their calculation of ξ. Figure 7 shows that two data sets are systematically different in this parameter space. While the data obtained in the

Carolina beach show a clear trend as shown by the best fit solid line, the data obtained in the California beach show no significant slope when ξ is large. Guza *et al.*(1984) said that the apparent discrepancy between the data sets lay in the rather arbitrary choice of a cutoff frequency for the infragravity band. However, I think as explained later that it depends on the difference of bottom slope in the wave breaking zone.

Effect of bottom slope on the infragravity waves

As the dependence of infragravity waves at the shoreline on ξ is different from beach to beach, the theory on the generation of infragravity waves by Symonds *et al.*(1982) has been examined. They used the non-dimensional, depth-integrated, and linearized shallow water equations, that is,

$$\chi \frac{\partial U}{\partial t} + \frac{\partial \zeta}{\partial x} = -\frac{\partial (a^2)}{2x\partial x} \tag{11}$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial (xU)}{\partial x} = 0 \tag{12}$$

$$\chi = \frac{\sigma^2 X}{g\tan\beta}, \quad U = \frac{2U'}{3\gamma\sigma X}, \quad t = \sigma t',$$
$$\zeta = \frac{2\zeta'}{3\gamma^2 X\tan\beta}, \quad x = \frac{x'}{X}, \quad a = \frac{a'}{\gamma X\tan\beta}, \tag{13}$$

where $\sigma=2\pi / \overline{T_R}$; $\overline{T_R}$ is a repetition period of wave group, x' is a distance offshore with the origin at the shoreline, X is the mean position of break point, g is the gravitational acceleration, U' is the depth-integrated velocity, γ is the ratio of the incident wave height to the water depth in the surf zone, t' is the time, ζ' is the level of sea surface, and a' is the amplitude of incident waves.

Nakamura and Katoh(1992) pointed out that the Symonds' theory overestimates the height of infragravity waves in comparison with the field data. They modified the Symonds' theory by taking a time delay of small wave breaking due to propagation into consideration, which well predicts the wave height of infragravity waves. According to the modified theory, the non-dimensional wave profile of infragravity waves at the shoreline, ζ_o, is as follow (see, Nakamura and Katoh, 1992);

$$\zeta_o = \sum_{n=1} C_n \sin(n t + \varepsilon_n), \tag{14}$$

where C_n are the coefficients which have been expressed in complicated forms, and ε_n are the phase lags.

In order to examine the characteristic of wave profile expressed by equation (14), relatively simple, but acceptable, assumptions will be introduced, although they make a little sacrifice of quantity. By assuming that the ratio of the wave amplitude to the water depth, γ, is constant, we have,

$$X = \frac{\bar{H}}{2\gamma \tan\beta} \quad , \tag{15}$$

where $\bar{H}$ is the mean wave height which is correlated with $H_{1/3}$ as

$$H_{1/3} = 1.6\,\bar{H} \,. \tag{16}$$

Next, let us assume that the height of incident waves in groups varies sinusoidally (Nakamura and Katoh,1992), that is,

$$H = \bar{H} + \sqrt{2}\; H_{1/3} \cos(\, \sigma t \,) \,/\, 3. \tag{17}$$

By utilizing equations (15),(16), and (17), the ratio of the amplitude of wave break point varying to X is 0.75 ($= \Delta\, a$).

Figure 8 shows the theoretical relation between C_n (; n=1 to 4) and χ , which have been obtained based on the above assumption. In Figure 8, the amplitude of infragravity waves at the shoreline is also shown, which is calculated by means of $\sqrt{2}\;(\zeta_o)_{rms}$ from the wave profile composed by equation (14) for n=1 to 4.

In regard to χ , an interesting rewriting will be possible. By referring the empirical relation between $\bar{T_R}$ and $T_{1/3}$ ($\bar{T_R}$ = 9.24$T_{1/3}$, Nakamura and Katoh, 1992), we have

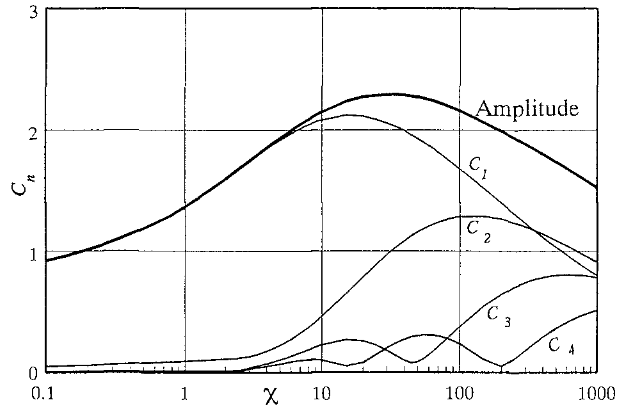

Figure 8 Relation between χ and C_n or amplitude of infragravity waves at the shoreline.

$$\sigma = \frac{2\pi}{9.24\, T_{1/3}} = \frac{1}{9.24}\sqrt{\frac{2\pi g}{L_o}} . \tag{18}$$

Then, by substituting equations (15),(16), and (18) into equation (13), we have

$$\chi = \frac{0.023}{\gamma}\,\frac{H_{1/3}\,/\,L_o}{\tan^2\beta} = \frac{0.023}{\gamma\xi^2} , \tag{19}$$

that is to say, χ is the function of ξ. Furthermore, another relation in equation (13) can be rewritten for the condition at the shoreline as follow;

$$\sqrt{2}\ (\zeta_o)_{rms} = \frac{(H_L)_o}{3\gamma^2 X\tan\beta} . \tag{20}$$

According to the parameter definitions, we have

$$\overline{H} = 2\gamma X \tan\beta . \tag{21}$$

By substituting equation (21) into equation (20) and taking equation (16) into account, we finally have

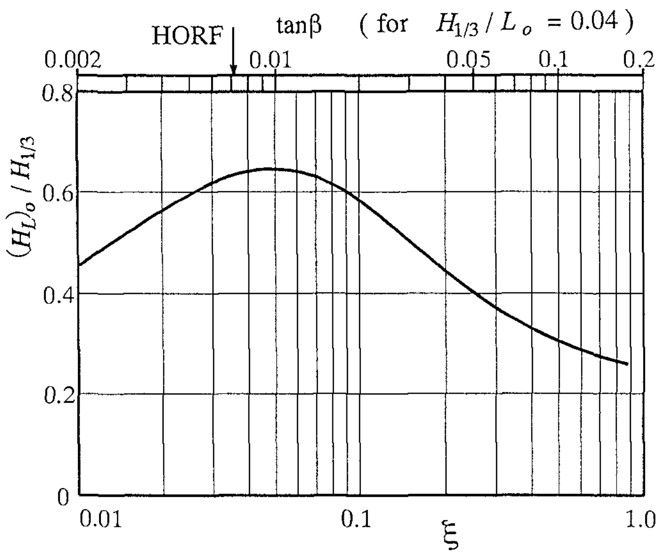

Figure 9 Relation between ξ and height of infragravity waves at the shoreline (theory).

$$\frac{(H_L)_o}{H_{1/3}} = \frac{3\gamma}{3.2}\sqrt{2}\,(\zeta_o)_{rms}\,. \tag{22}$$

By utilizing equations (19) and (22), Figure 8 can be represented by using new parameters.

Figure 9 is the transformed relation between new parameters, by assuming γ =0.3. On the upper side, a scale is marked for the bottom slope when the mean value of offshore wave steepness at the HORF, 0.04, is adopted. The mean bottom slope in the area of wave breaking in a storm is 1/140 at the HORF, which is indicated by an arrow on the upper side. In a range of bottom slope up to 0.01 (=1/100), the non-dimensional wave height increases with ξ, of which tendency is the similar as that shown in Figure 6. On the other hand, it decreases with ξ in a range of bottom slope steeper than 0.01.

In the analyses by Guza *et al.*(1984), they defined ξ by the foreshore slope, not by the bottom slope in the wave breaking area. Then, I have inspected the bottom slopes of two beaches in literature. The mean bottom slope in the area from −2 to −6 meters is 1/127 in the North Carolina beach (Holman and Shallenger,1985), which is too much gentler than the foreshore slope. The bottom slope in the Torrey Pines Beach, California, is almost constant up to the water depth of 7 meters, being 1/45 (Guza and Thornton,1985). By utilizing these slopes, the values of ξ have been calculated, provided that the foreshore slopes are 6 degrees

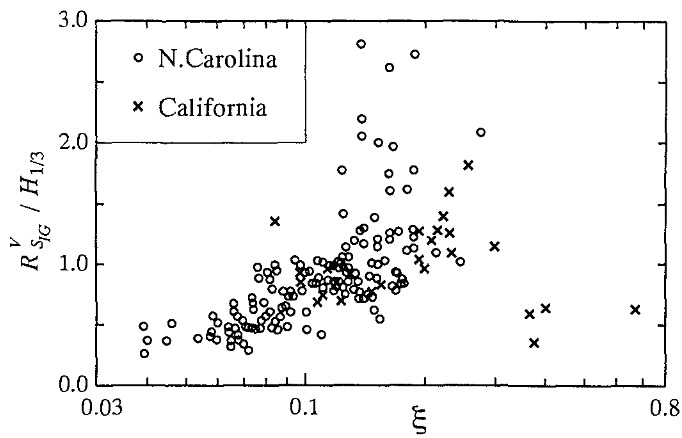

Figure 10 Relation between ξ and significant vertical swash excursion of infragravity waves (ξ is defined with the mean bottom slope in the wave breaking area).

in both the beaches. Figure 10 is the result of calculation, in which the data are rearranged to give the qualitative agreement with the theoretical relation in Figure 9.

Discussion and Conclusions

Figure 11 shows the result of present study. A thick line is the theoretical relation between the non-dimensional height of infragravity waves at the shoreline and ξ, which has been qualitatively verified with three sets of field data. The linear relationship between the non-dimensional run-up height of incident waves and ξ, equation (4), is superimposed on this figure by a thin line, which is curved in a semi-log space. There is an intersection of two lines, which is roughly corresponding to the bottom slope of 1/25 when the wave steepness is 0.04. In the right-hand side from the intersection, on the steeper beach, the incident waves are predominant in the wave run-up phenomena, while on the gentler beach the infragravity waves are predominant.

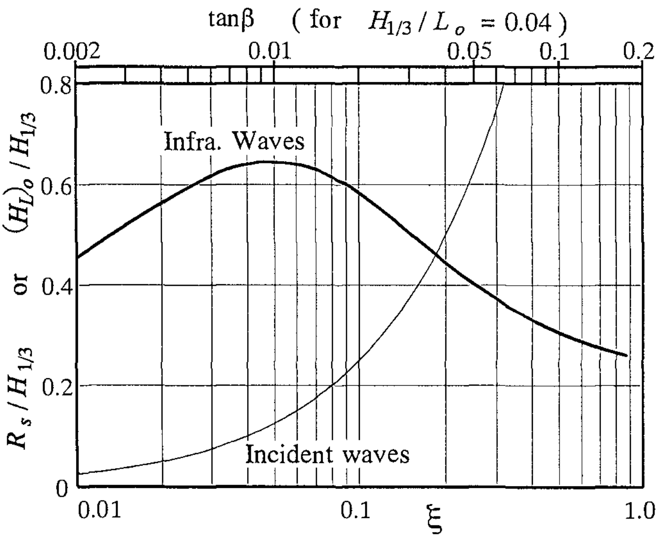

Figure 11 Dependences of infragravity waves and incident waves on ξ.

Almost all of the model experiments were conducted on the model beach steeper than 1/30, probably due to the limited length of flume. Under this condition, the dominating external force for profile changes is the incident waves. Then, the foreshore erodes even though only the incident waves are reproduced in the experiment. On the other hand, the infragravity waves exist on the gentle beach in the field. However, the evidences which show the cross relation between the beach erosion and the infragravity waves are scarce. One of reasons is that many field observations in connection with the wave run-up were carried out on the steep beach (according to literature survey by Kubota ;1991). Another reason is that many field observations were done in the relatively calm wave conditions. The smaller waves break closer to the beach where the bottom slope is steeper because the profile of nearshore topography is usually concave upward. In short, the development of infragravity waves is weak when the incident waves break on the steep bottom. Therefore, in order to have quantitative information on the relation between the infragravity waves and the beach erosion, the further field observations are required concerning the phenomena on the gentle beaches in storms.

Finally, the author is grateful to Mr. Satoshi Nakamura, a member of the littoral drift laboratory, for his courtesy in using the computer program which he developed to predict the height of infragravity waves in the surf zone.

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