CHAPTER 11

WAVE DAMPING BY KELP VEGETATION

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1. INTRODUCTION

Aquatic vegetation, like seagrasses, macroalgae and trees whether submerged or subaerial are an important feature of a coastal ecosystem. In addition to the structural and functional aspects to the environment, they are known to reduce wave and current energies propagating through them. The reduction of energy would then influence sediment motion and thus render an impact on coastal sediment transport. The dissipative character of large stands of kelp has been studied for instance by Jackson and Winant, (1983), Dalrymple et al. (1984) and for artificial seaweed as material for shore protection Price et al. (1968).

Kelp is a macroalga which grows on hard rock and stone and extracts all of its nutrients from the water column. The plant consists of a root-like holdfast organ, a stipe and a frond (Fig.1). The general properties of a fully grown (4-8 years) kelp are summarized as follows:
Length of stipe: 1-2 meters. Fronds have the same length as the stipe.
Specific gravity: 1.18 kg/cu. m; Biomass: 10-30 kg/sq. m
Growth density: 10-15 per sq. meter of horizontal area

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Specimen found at water depths 2-20 metes.

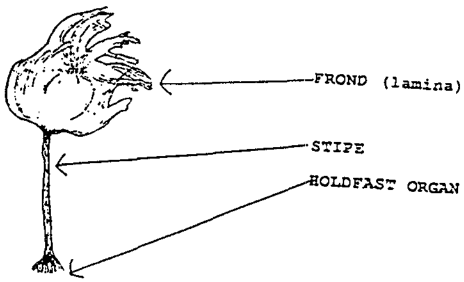

Figure 1: Kelp *Laminaria hyperborea*

Kelp is harvested at several places along the Norwegian coast and is used as a raw material for the manufacturing of various chemicals. In some areas the harvesting of the seaweed has become a controversial issue in which there is suspicion among coastal zone managers that the harvesting results in beach erosion. This study is actually a consequence of the controversy.

Basing on the most recent work by Asano et al. (1992), a new analysis is developed for the flow model and the vegetation motion using field and experimental results carried out on kelp fronds and kelp plant models. The theoretical model is compared with experimental results. The influence of kelp vegetation on beach erosion is not included in this paper because of space limitation.

2. BASIC FORMULATION FOR THE FLOW MODEL

Let us consider small amplitude waves propagating in the x- direction in water of depth h above submerged vegetation of mean height d. We employ cartesian coordinates (x,z) fixed on the mean free surface, z=0, where z is positive upwards (see Fig.2). The surface displacement at the free surface is given by $\eta_1 = a_0 e^{i(kx-\omega t)}$ and displacement at the interface is $\eta_2 = b_0 e^{i(kx-\omega t)}$. Let us assume flat bottom, potential flow in the water layer, frictional flow in the vegetation zone. At the interface the viscous shear stresses and the corresponding layer, δ, are initially neglected. The bottom shear stress is considered to be negligible in comparison with the frictional resistance of the vegetation. Further, let us assume known a priori the wave amplitude a_0, the angular frequency $\omega = 2\pi/T$ both of which are real and positive. k is a wave number and T is the wave period.

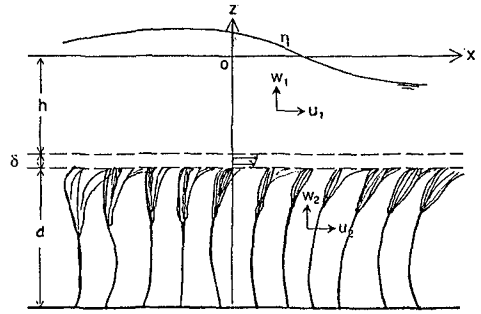

Figure 2: Definition sketch of the flow model

The equations of motion employed are the linearized momentum equations for the water and vegetation zones. For a unit volume

$$\frac{\partial U_1}{\partial t} = -\frac{1}{\rho}\nabla P_1 \tag{1}$$

$$\frac{\partial U_2}{\partial t} = -\frac{1}{\rho}\nabla P_2 - F \tag{2}$$

and the equation of continuity

$$\nabla U = 0 \tag{3}$$

where the subscripts 1 and 2 denote the water and the vegetation zones respectively; t = time; $\mathbf{U} = (\mathbf{u},\mathbf{w})$ = water particle velocity vector; ρ is the fluid density and $\mathbf{P}$ is the dynamic pressure. $\mathbf{F} = (\mathbf{Fx}, \mathbf{Fz})$ = force vector acting on vegetation given as

$$Fx = \frac{\rho}{2} N\,[C_{Dx} A]_e\, |u_2 - \dot{\xi}|\,(u_2 - \dot{\xi}) + \rho\, N\, |C_{ax} V|_e\,(\dot{u}_2 - \ddot{\xi}) \tag{4}$$

and

$$Fz = \frac{\rho}{2} N\,[C_{Dz} A]_e\, |w_2| w_2 + \rho\, N\,[C_{az} V]_e\, \dot{w}_2 \tag{5}$$

where N is the number of vegetation per unit horizontal area, C_{Dx} and C_{Dz} are drag force coefficients in the x- and z- directions respectively; C_a is the added mass coefficient, u_2 and w_2 are the horizontal and vertical velocities of the fluid particles. ξ is the horizontal displacement of the vegetation stipe with the dot denoting the derivative with respect to time. The subscript e indicates the equivalent value taking into account of both the stipe and the frond. A and V are the total projected area and volume of the plants in this unit volume. More details on the equivalent values are given in the section on the solution for the vegetation motion.

Let us assume that the particle velocities and the dynamic pressure are sinusoidal such that

$$U = U(z)\ e^{i(kx-\omega t)} \tag{6}$$

and

$$P = P(z)\ e^{i(kx-\omega t)} \tag{7}$$

where $i^2 = -1$.

Substituting equations (4), (5), (6) and (7) into equations (1) and (2) we get a new set of equations. For the upper layer we have

$$\frac{\partial u_1}{\partial t} = -\frac{1}{\rho}\frac{\partial P_1}{\partial x} \tag{8}$$

$$\frac{\partial w_1}{\partial t} = -\frac{1}{\rho}\frac{\partial P_1}{\partial z} \tag{9}$$

and for the lower layer we have

$$\frac{\partial P_2}{\partial x} = -\rho\ \omega\ f_x\ u_2 \tag{10}$$

$$\frac{\partial P_2}{\partial z} = -\rho\ \omega\ f_z\ w_2 \tag{11}$$

where fx and fz are the horizontal and vertical force components due to the presence of vegetation and expressed by

$$f_x = f_{Dx} + i\ f_{Ix} \tag{12}$$

and

$$f_z = f_{Dz} + if_{Iz} \tag{13}$$

in which the horizontal and vertical drag force terms are defined by

$$f_{Dx} = \frac{1}{2}\,[C_{Dx}A]_e\,\left|1-\frac{\dot{\xi}}{u_2}\right|\left(1-\frac{\dot{\xi}}{u_2}\right)|u_2|\,/\omega \tag{14}$$

and

$$f_{Dz} = \frac{1}{2}\,[C_{Dz}A]_e\,|w_2|\,/\omega \tag{15}$$

The inertial force terms are expressed by

$$f_{Ix} = [C_{mx}V]_e\,\left|1-\frac{\dot{\xi}}{u_2}\right| \tag{16}$$

Here $C_{mx} = 1 + C_{ax}$

$$f_{Iz} = -\,[C_{mz}V]_e \tag{17}$$

We impose the following linearized boundary conditions at the free surface, interface and bottom boundaries on the momentum equations (8), (9) (10) and (11) :

$$\eta(x,t) = \frac{1}{g}\frac{\partial\phi_1}{\partial t} \quad at \quad z = 0 \tag{18}$$

$$\frac{\partial^2\phi_1}{\partial t^2} + g\,\frac{\partial\phi}{\partial z} = 0 \quad at \quad z = 0 \tag{19}$$

$$\rho\,\frac{\partial\phi_1}{\partial t} = P_1 = P_2 \quad at \quad z = -h \tag{20}$$

$$-\frac{\partial\phi_1}{\partial z} = w_1 = -\frac{1}{\rho\,\omega\,fz}\,\frac{\partial P_2}{\partial z} = w_2 \quad at \quad z = -h \tag{21}$$

$$-\frac{1}{\rho\,\omega\,fz}\,\frac{\partial P_2}{\partial z} = 0 \quad at \quad z = -(h+d) \tag{22}$$

$$\frac{\partial P_2}{\partial z} = 0 \quad at \quad z = -(h+d) \tag{23}$$

where $\eta = a_0\, e^{i(kx-\omega t)}$ is the free surface elevation at SWL , a_0 is the wave amplitude at the origin, g is the acceleration due to gravity. k is complex = $k_r + k_i$ in the subscripts r and i denote real and imaginary values. Substituting k into the surface elevation , the local wave amplitude is found to decay exponentially as $a = a_0 \exp(-k_i x)$.

The solutions for the flow model can be shown to be

$$\Phi_1 = i\frac{C}{\omega}[\cosh(\alpha kd)\cosh(k(h+z)) - \frac{i}{\alpha fx}\sinh(\alpha kd)\sinh(k(h+z))]\exp[i(kx-\omega t) \tag{24}$$

$$P_2 = \rho C \cosh(\alpha k(h+d+z))\exp[i(kx-\omega t)] \tag{25}$$

where

$$C = \frac{g a_0}{\cosh(\alpha kd)\cosh(kh)[1 - \frac{i}{\alpha\, fx}\tanh(\alpha kd)\tanh(kh)]} \tag{26}$$

and

$$\alpha = \sqrt{\frac{|fz|}{|fx|}} \leq 1 \tag{27}$$

is the force ratio. In the upper layer a velocity potential Φ_1 exists which satisfy Laplace equation

$$\nabla^2\Phi = 0 \tag{28}$$

where the particle velocities are expressed as

$$u_1 = -\frac{\partial\phi_1}{\partial x}; \quad w_1 = -\frac{\partial\phi_1}{\partial z} \tag{29}$$

In the vegetation zone, however, the particle velocities can be obtained by substituting equation (25) into equations (10) and (11) to give

$$u_2 = -i \frac{Ck}{\omega fx} \cosh(\alpha k(h+d+z)) \exp[i(kx-\omega t)] \tag{30}$$

$$w_2 = -\frac{Ck}{\omega fx} \sinh \alpha k(h+d+z) \exp[i(kx-\omega t)] \tag{31}$$

We remark that the horizontal and vertical wave numbers in the upper layer are the same when the fluid is inviscid and homogeneous. In the vegetation zone they are different due to the different horizontal and vertical resistance forces. At the interface, the horizontal particle velocities are discontinuous, i.e $\mathbf{u}_1 \neq \mathbf{u}_2$, thus a shear stress is present which is accounted for by a boundary- layer type of solution.

Finally, from the combined kinematic and dynamic free surface boundary conditions we derive the dispersion relationship given by

$$\omega^2 = gk \frac{\tanh kh - \frac{i}{\alpha fx} \tanh \alpha kd}{1 - \frac{i}{\alpha fx} \tanh \alpha kd \tanh kh} \tag{32}$$

For given α, ω, h, d and fx the unknown complex wave number k can be found by solving equation (32) by iteration. This is done in the coming section on calculated results.

3. SOLUTION FOR THE VEGETATION MOTION

In order to solve the flow field described above we need the knowledge of the kelp motion. The basic approach is the Morison equation in which the forces resisting the fluid flow are the sum of the drag and inertial forces. Following Asano et al. (1992), the motion of the vegetation is regarded as a forced vibration wih one degree of freedom. Let the horizontal displacement of a single kelp plant be denoted by ξ while the differentiation with respect to time t be denoted by the over dot. For a unit length of kelp, the equation of motion is given by

$$m_0\ddot{\xi} + c_1\dot{\xi} + k_0\xi = \frac{1}{2}\rho C_{Dx}A|u_2-\dot{\xi}|(u_2-\dot{\xi}) + \rho V\dot{u}_2 + \rho VC_{ax}(\dot{u}_2-\ddot{\xi}) \tag{33}$$

where m_0 = mass of kelp per unit length, c_1 = structural damping, k_0 = spring constant, C_{Dx} and C_{ax} are drag and added mass coefficients respectively, A is the projected area and V is the volume per unit length of kelp. Neglecting the structural damping on the assumption that it is small compared to the frictional forces and rearranging we get

$$m\ddot{\xi} + \frac{1}{2}\rho C_{Dx}A|u_2-\dot{\xi}|\dot{\xi} + k_0\xi = \frac{1}{2}\rho C_{Dx}|u_2-\dot{\xi}|u_2 + \rho(1+C_{ax})\dot{u}_2 \tag{34}$$

The general solution of equation (34) requires iterations involving volumetric integration of unknown variables. For our particular case of kelp we shall simplify the equation of motion before attempting to solve the coupled system. The kelp plant consists of a stipe and a frond which together make a total height d. The stipe can be represented as a slender vertical cylinder of height $d-l_k$ with uniformly distributed mass and the frond is taken as a concentrated mass at the top of the stipe. Here l_k is the half length of the frond. Integrating over depth gives the equation that represents the integrated effect over the water column by the motion at the top of the stipe. Now the equation of motion (34) becomes

$$m_e\ddot{\lambda} + \frac{1}{2}\rho[C_{Dx}A]_e|u_\lambda-\dot{\lambda}|\dot{\lambda} + k_0 = \frac{1}{2}[C_{Dx}A]_e|u_\lambda-\dot{\lambda}|u_\lambda + \rho[C_{mx}V]_e\dot{u}_\lambda \tag{35}$$

where we have assumed that the velocities of both the stipe and fluid can be treated as varying linearly from the bottom to the top of the stipe, that is

$$\dot{\xi}=\frac{h+d+z}{d-l_k}\dot{\lambda}\ ; \qquad u_2=\frac{h+d+z}{d-l_k}u_\lambda \tag{36}$$

where λ refers to the level at the top of the stipe and the subscript *e* stands for equivalent values for the stipe and frond such that

$$m_e = \frac{1}{2}m_0 + m_f + \rho(\frac{1}{2}C_{as}V_s+C_{af}V_f) \tag{37}$$

Here subscripts s and f denote the stipe and frond properties respectively. The mass per unit length, m_0 , of the stipe and its equivalent added mass is weighted by ½ to imply the conversion of the entire kelp motion to the top. This is valid also for the drag and inertial force coefficients which are given by

$$[C_{Dx}A]_e = \frac{1}{2}C_{Ds}A_s + C_{Df}A_f \tag{38}$$

$$[C_m V]_e = \frac{1}{2}(1+C_{as})V_s + (1+C_{af})V_f \tag{39}$$

Prior to linearization of equation (35) we need to establish the relationship between the drag forces and the flow velocity. One complication with kelp is that the projected area of the frond varies in a flow field. When the velocity (or the relative velocity as the case may be) is zero, the kelp will assume an upright position and the projected area is largest. As the velocity increases, the plant tends to bend over and the fronds tend to streamline in the direction of the flow thereby reducing the projected area. A field experiment has been carried out by the authors in collaboration with prof. Martin Mork and dr. scient. student Kjersti Sjøtun of the University of Bergen (UiB) using a research vessel " *Hans Brattstrøm*" belonging to UiB. Drag forces were measured on 7 fronds of different sizes of the Norwegian kelp by ship towing. In the laboratory, the forces have been measured using a shear plate on which 95 model kelp plants were fixed. Details of the procedure follow in the section on experimental set-up. Results from the two experiments as shown in Fig.3 show that the drag force does not follow the normal quadratic relationship with velocity, instead, the force is linearly proportional to the velocity.

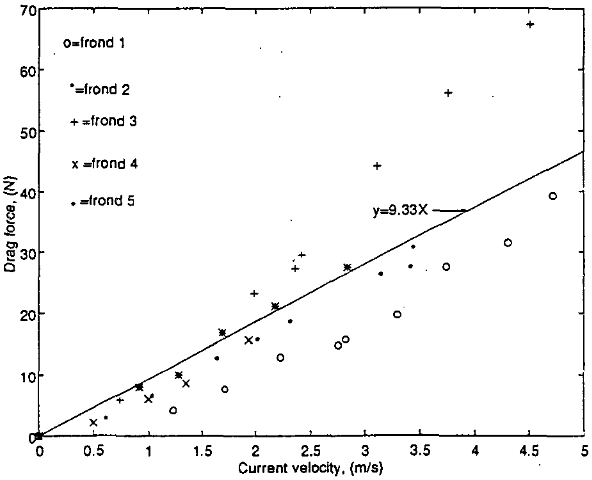

Figure 3: Variation of the drag force on kelp fronds with current velocity

Basing on the two experiments, the following general relationship is proposed between the projected area and the flow velocity:

$$\frac{\rho}{2}[C_{Dx}A]_e = F_\lambda |u_\lambda - \dot{\lambda}|^{-m} = constant \tag{40}$$

where F_λ = equivalent drag force coefficient evaluated at elevation λ , or at

$z = -(h + l_k)$. Substituting equation (40) into equation (35) gives

$$m_e\ddot{\lambda} + N_D\dot{\lambda} + k_0\lambda = N_D\, u_\lambda + N_I\, \dot{u}_\lambda \tag{41}$$

where

$$N_D = F_\lambda\ |u_\lambda(1-\frac{\dot{\lambda}}{u_\lambda})|^{1-m} \tag{42}$$

Both F_λ and m are empirically determined constants. The experimental results shown in figure 3, suggest that m=1 and F_λ=9.3 and 30.9 for currents and waves (fig. 7) respectively. With this information we can now proceed with the linearization of equation (35) to get an analytical solution.

Assuming small amplitudes for the vegetation and fluid particle motion and that the particle velocity amplitude is much greater than the maximum velocity amplitude of vegetation, equation (35) now becomes

$$m_e\ddot{\lambda} + \frac{1}{2}\rho[C_{Dx}A]_e|u_\lambda|\dot{\lambda} + k_0\lambda = \frac{1}{2}\rho[C_{Dx}A]_e|u_\lambda|u_\lambda + \rho[C_{mx}V]_e\dot{u}_\lambda \tag{43}$$

which represents a linear system provided k_0 is also constant. The solution to this equation gives a ratio known as the velocity amplification factor

$$A_m = \frac{\dot{\lambda}}{u_2|_{z=-(h+l_k)}} = \frac{1-i\omega\frac{N_I}{N_D}}{1+i\omega(\frac{\omega_n^2}{\omega^2}-1)\frac{m_e}{N_D}} \tag{44}$$

where $\omega_n = \surd(k_0/m_e)$ in which k_0 is the spring constant which was determined experimentally to be 20 N/m for deflections up to 55 cm at the top of the plant. From equation (44) we can derive the quantity

$$|1-A_m| = \frac{\omega[(\frac{\omega_n^2}{\omega^2}-1)+N_I]}{\sqrt{N_D^2+\omega^2(\frac{\omega_n^2}{\omega^2}-1)^2}} \tag{45}$$

The linearized damping force coefficient used in the solution for the flow model can be established by applying the principle of equivalent work which the energy dissipation of the actual system to that of a linear system (Wang and Tørum,1994). The time averaged work done by the actual system per unit surface area is given by

$$W_a = \alpha_k\, N\, F_\lambda (1 - \frac{\dot{\lambda}}{u_\lambda}) \overline{u_\lambda^2} \tag{46}$$

where N is the number of plants per unit surface area, α_k is a force reduction factor due to group effect and the over bar denotes the time averaging. For the linearized flow system

$$W_L = \rho f_{Lx} \overline{\int_{-(h+d)}^{-(h+l_k)} u_2^2\, dz} \tag{47}$$

Equating equation (46) and (47) gives

$$f_{Lx} = \frac{\alpha_k\, N\, F_\lambda (1 - A_m) \overline{u_\lambda^2}}{\rho \overline{\int_{-(h+d)}^{-(h+l_k)} u_2^2\, dz}} \tag{48}$$

Substituting equation (30) into equation (48) gives

$$f_{Lx} = \frac{\alpha_k\, N\, F_\lambda (1 - A_m) \cosh^2 k_s (d - l_k)}{\rho (\frac{1}{2} + \frac{\sinh 2k_s d}{4k_s d})} \tag{49}$$

where $k_s = \alpha k$. Then the linearized damping force coefficient becomes

$$fx = \frac{f_{Lx}}{\omega} + [C_m V]_e\, (1 - A_m) \tag{50}$$

4. EXPERIMENTAL RESULTS AND DISCUSSION

4.1 Experimental setup

The experiment was carried out in a 33 m long, 1 m wide and 1.6 m high wave tank as shown in Fig.4. Five thousand models (scale 1:10) of typical Norwegian kelp plants were fixed in the wave flume bottom over a span of 9.3 meters. This represented a density of about 12 plants per horizontal square meter in the field. Eight capacitance wave gauges were used to measure surface elevations, one shear plate to measure the horizontal force and a mini current meter was inserted in the plants 4 centimeters above the shear plate. One of the wave gauges was fixed above the shear plate.The shear plate was fixed flush with the bottom. The location of the first wave gauge, taken to be x=0 and the last taken to be x = 7m were fixed about 1.2 meters inward from the outer boundaries. In total, 50 tests were carried out for different wave periods (6-14 s, full scale) and wave heights in water depth of 60 cm. Analysis of the results is done within $0 \le x \le 7$ m.

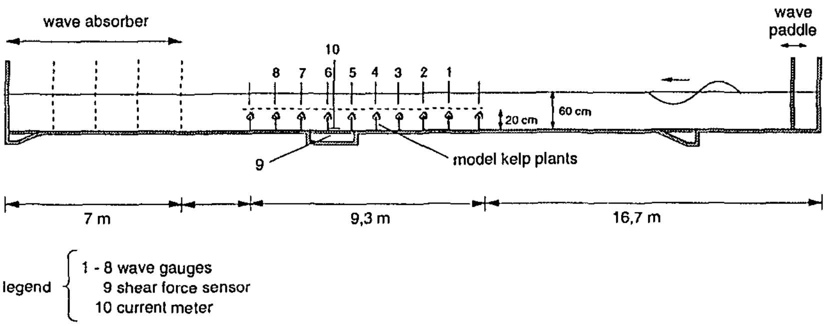

Figure 4: Test setup

4.2 Comparison of theoretical and experimental results

The wave heights measured at the eight locations along the wave channel are fitted to an exponential decay curve H/Ho = exp(-0.00327X) as shown in Fig. (5) whereby H is the local wave height and Ho is the incident wave height measured at x=0, ki =0.00327/m was found by regression based on the least squares in the MATLAB environment.

Basing on the relationship proposed in equations (40) and (42), the force can then be generalized as

$$F = F_\lambda \ |1-A_m| \ u_\lambda \tag{51}$$

where $|1\text{-}A_m|$ is given by equation (45). Fig. 6 shows this function fitted to the measured force for given wave heights. Fig. 7 and Fig. 8 show the linear variation of the measured force with the horizontal particle velocity and wave height respectively.

Finally, inserting equation (50) into the dispersion relationship given by equation (32) for a given water depth ,(h+d) wave period, T, and number of vegetation per unit surface area, the damping coefficient, ki, is found by iteration in the MATLAB environment . This solution, however, does not include the contribution of the shear stress to damping.

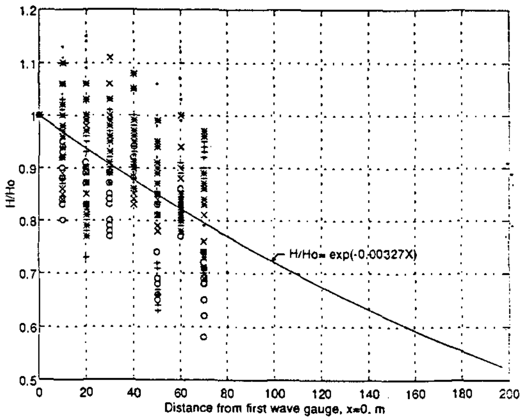

Figure 5. Exponential decay model fitted to data

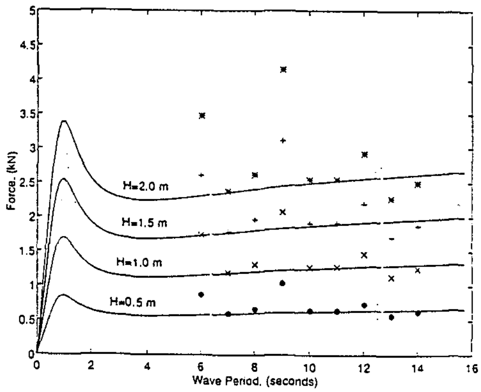

Figure 6. Theoretical force model (Eq. (51)) fitted to measured force

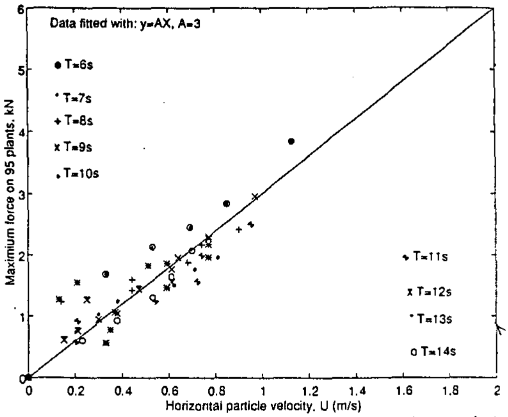

Figure 7: Variation of force with horizontal particle velocity amplitude

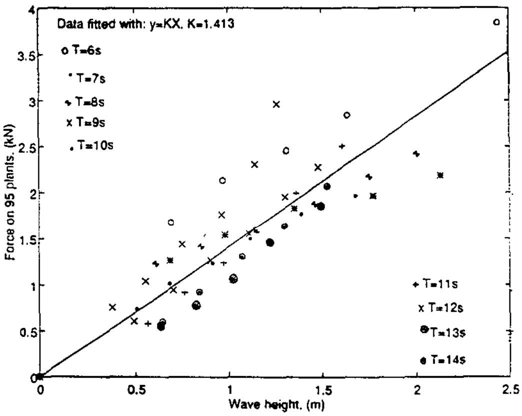

Figure 8: Variation of force with wave height

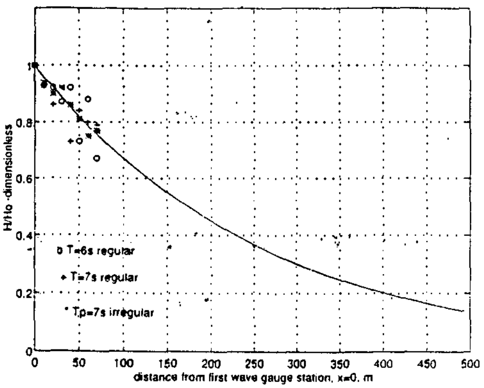

Figure 9: Comparison of scatter for regular and irregular waves

5. CONCLUSION

The present study has given an analytical solution for water waves propagating over submerged vegetation taking account of the vegetation motion using the field experimental results. The linear relationship of the damping force with velocity has been applied effectively to obtain an analytical solution for the otherwise iterative equation of the vegetation motion. The average damping coefficient has been found to be ki=0.00327/m for the type of kelp we considered. However there has been a large scatter of about 30% which may have been due to reflection of the waves from the wave absorber (Fig. 4). A trial run with irregular waves has revealed a smaller scatter (Fig.9). In this study only regular waves were used. It is our intention to use irregular waves to investigate further on the damping force and the damping coefficient.

6. ACKNOWLEDGEMENT

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7. REFERENCES

1. Asano, T., Deguchi, H., and Kobayashi, N. (1992): Interaction between water waves and vegetation. *Proc. 23rd International Conference on Coastal Engineering, Venice, Italy*

2. Dalrymple R.A., Kirby, J.T., and Hwang, P.A. (1984): Wave diffraction due to areas of energy dissipation. *Journal of Waterway, Port, Coastal and Ocean Engineering.* ASCE, Vol.110(1)

3. Jackson, G.A., and Winant, C.D. (1983): NOTE Effect of a kelp forest on coastal currents. *Continental Shelf Research, **2(1)**, pp75-80*

4. Price, W.A., Tomlinson, K.W., and Hunt, J.N., (1968): The effect of artificial seaweed in promoting the build up of beaches. Proc. 11th International Conference on Coastal Engineering, London, pp 570-578

5. Wang, H and Tørum, A. (1994): A numerical model on beach response behind coastal kelp. SINTEF-NHL Report No. STF60 A 94092