CHAPTER 240

WAVE-INDUCED PORE PRESSURE ACTING ON A BURIED SUBMARINE PIPELINE

Waldemar MAGDA *

ABSTRACT

The response of a sandy seabed to surface water waves, with a special emphasis to wave-induced excess pore water pressure oscillations is studied here in relation to the vertical stability of submarine buried pipelines. The main object of the peper is to present a study of the distribution patern of the pore water pressure acting around the pipeline, and to calculate the seepage force, the up-lift force particularly, affecting the pipelne stability, under the assumption of compressible both the pore fluid and soil skeleton, for the case of an arbitrary seabed depth as well as for the infinite thickness of the subsoil.

INTRODUCTION

Generally, the problem associated with buried submarine pipelines depends, on the water and wave conditions. The wave climat plays a very important role and can influence the interaction between the submarine buried pipeline and the surrounding soil significantly. In practice, pipeline located in water depths up to 60 m are buried, whilst the cover must have a thickness ranging from 0.5 to 1.0 m, depending upon the water depth and the covering material.

Submarine pipelines buried in a seabed are an engineering means of transport for crude oil and natural gas from "off-shore" oil fields onto a land. When waves pass over a permeable sandy seabed, pore water pressure is continuously

^{*} Ph.D., Research Engr.

University of Hannover, Sonderforschungsbereich 205, Appelstr. 9A, 3000 Hannover 1, Germany (on leave from the Marine Civil Engineering Department, Hydroengineering Faculty, Technical University of Gdańsk, Poland)

induced within it. Among all environmental loads usually considered in "offshore" pipelines design, the wave-induced pore water pressure plays one of the most important role. The most critical problem in determining the stability of a pipeline buried in permeable soils under wave loading is the prediction of the pore water pressures in the soil in a vicinity of a pipeline (Dursthoff and Mazurkiewicz, 1985). An excess of the pore water pressure can cause instability of a seabed, liquefaction of the upper sand layer and then floatation which can even lead to a failure of a submarine pipeline. The wave-induced excess pore water pressure developed in a vicinity of a buried pipeline is considered as a one of the main parts in a design procedure. The wave-induced uplift force acting on the pipeline is comparable to the displaced water weight if the pipeline is located in the pore water pressure boundary layer and an inadequate design can cause flotation of pipeline and, subsequently, can lead to costly failures. Therefore, it is essential to improve our knowledge on the interaction among waves, seabed and a submarine pipeline.

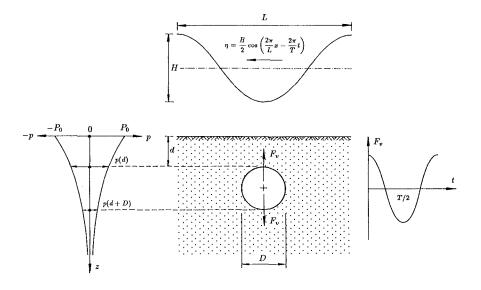


Figure 1 Definition sketch for the uplift force analysis.

It is a very complex and challenging task to define properly the waveinduced excess pore water pressure field around a submarine pipeline buried in a porous seabed. Many researches simplyfied the problem assuming both the porous medium and pore water incompressible. Under this assumption Lai *et al.* (1974), Liu and O'Donnell (1979) and Lennon (1985) investigated this problem using a numerical analysis. Liu and O'Donnell (1979) considered two different types of waves acting on the seabed, namely, monochromatic and solitary, and introduced the integral equation method to solve the resulting integral equation. In a numerical solution procedure developed by Lennon (1985) the pressure distribution on the pipeline was calculated using also the boundary integral equation method (BIEM). Employing conformal mapping techniques, MacPherson (1978) and McDougal *et al.* (1988) presented analytical solutions for the case of an infinite depth of the seabed, whereas Monkmeyer *et al.* (1983) developed a solution using so-called 'image pipe" method which, comparing to the former, can be applicable also to a soil layer of a finite thickness.

The common feature in the studies mentioned above is that the effect of compressibility of both the pore water and porous medium was neglected. Moreover, some researchers showed that there is a difference between theoretically computed values of pore water pressure and those observed in experiments. In laboratory studies on the stability of buried pipelines, Philips *et al.* (1979) concluded that potential theory did not generally give an accurate representation of the transmission of wave-induced pressures through the sand, when comparing to the test results.

Reported differences between theoretical and experimental results can have three main reasons, namely:

- the theories are based on the Darcy model and therefore they do not contain all important soil/water parameters (incompressible pore water and nondeformable soil skeleton are assumed),
- boundary conditions applied into computation are not realistic, specially when comparing with laboratory tests environment (seabed layer of a finite thickness),
- values of parameters used in calculations are not exactly the same like these 'in-situ' which accompany laboratory investigations.

The proposed method of calculation is based on:

- the pore water pressure theory where the main soil and pore fluid parameters are considered and a finite sand bed layer system is taken into account,
- the 'image pipe' theory which is able to solve both the upper (at the sea bottom) and the lower (at finite depth of the seabed layer) boundary conditions, and also the boundary condition induced by presence of a pipeline (perturbation or scattering effect).

An implementation of certain soil and pore water parameters, e.g. compressibility and permeability, leads not to the Laplace equation, which depends only on a geometry of the problem, but to the storage equation, which is much more complex in form. Using Madsen's (1978) general solution of this equation, an analytical solution for a finite thickness of the seabed layer has been derived and verified qualitatively in numerous large-scale laboratory experiments in a big wave-flume, and quantitatively in small-scale laboratory tests (Magda, 1989, 1991). These tests enabled to study influences of single soil/water parameter changes on the character of the pore water pressure damping within a porous medium. A great attention has been put to modelling and controlling different degrees of saturation which is of a special interest for coastal and tidal areas where, because of a continuous water table movement and wave-breaking zones, the sediment is not and cannot be treated as a saturated medium.

MATHEMATICAL FORMULATION OF THE PROBLEM

Introducing a pipeline-like structure into a soil body, it is not so easy to derive a solution to the governing equation for flow of a compressible pore fluid in a compressible porous medium (e.g. given by: Madsen, 1978; Yamamoto et al., 1978). Therefore, after some mathematical manipulations, and presenting the solution in terms of the pore-water pressure and effective stresses, a new form of the governing equation can be obtained (Okusa, 1985):

$$\nabla^2 \left(\nabla^2 - \frac{1}{c_v} \frac{\partial}{\partial t} \right) p = 0 \tag{1}$$

where p is the wave-induced excess pore water pressure, c_v is the coefficient of consolidation, t is the time, and ∇ is the Laplacian operator. The coefficient of consolidation, c_v , can be defined for the unsaturated soil as

$$\frac{1}{c_v} = \frac{\gamma}{k} \left[\frac{n}{K} + \frac{1 - 2\mu}{2G(1 - \mu)} \right] \tag{2}$$

where γ is the unit weight of the pore fluid, k is the isotropic coefficient of soil permeability, n is the porosity of the porous bed, μ is the Poisson's ratio, K, is the bulk modulus of water, and G is the shear modulus of soil. From this it is easily seen that the solution of Eq. (1) can be formulated as a mixed solution of both the Laplace equation

$$\nabla^2 p = 0 \tag{3}$$

and the consolidation (diffusion) equation

$$\nabla^2 p - \frac{1}{c_v} \frac{\partial p}{\partial t} = 0 \tag{4}$$

in two dimensions. It has to be pointed that sometimes (e.g. Qiu and Sun, 1987) the simplification of the solution to the governing flow equation is going too far and, due to the total elimination of the soil displacements, the problem is reduced only to the consolidation equation. However, the correct solution has to be treated as a sum of the general solutions to the last two differential equations of the second order.

SOLUTION METHOD

Assuming that the wave-induced hydrodynamic pressure at the seabed is described by the periodic function

$$p = P_0 \exp[i(ax - \omega t)] \tag{5}$$

where $a = 2\pi/L$ is the wave number, L is the wave length, $\omega = 2\pi/T$ is the angular velocity, T is the wave period, and P_0 is the pressure amplitude at the seabed, and due to linearity of the above mentioned component equations, all the unknowns in the problem considered (among others: the wave-induced pore pressure) are periodic with a and ω . Then, the wave-induced pore pressure p is represented by

$$p = f(z) \exp[i(ax - \omega t)] \tag{6}$$

where f(z) is a function of z only. Introducing this into Eqs. (3) and (4), the general solution is represented by the sum of the solutions from the two following differential equations

$$\frac{d^2f}{dz^2} - a^2f = 0 \tag{7}$$

$$\frac{d^2f}{dz^2} - \left(a^2 - \frac{\omega}{c_v}i\right)f = 0 \tag{8}$$

Because the governing equations are linear, the wave-induced stresses can be obtained by superposing, as previously indicated by Yamamoto (1981) and Okusa (1985a). The general solutions f_1 of Eq. (7) and f_2 of Eq. (8) are

$$f_1 = C_L \exp(az) + D_L \exp(-az) \tag{9}$$

$$f_2 = C_C \exp(\kappa z) + D_C \exp(-\kappa z) \tag{10}$$

where C_L, C_C, D_L, D_C are integral constants depending on the boundary conditions and

$$\kappa = \sqrt{a^2 - \frac{\omega}{c_v}i} \tag{11}$$

For the case of infinitely thick homogeneous sediment, the wave-induced pore pressures, stresses, and displacements must tend to zero as $z \to \infty$. Therefore (Okusa, 1985):

$$C_L + C_C = 1 \tag{12}$$

$$C_L = \frac{2(1+\mu)B}{3+2\mu B-B} \qquad C_C = \frac{3(1-B)}{3+2\mu B-B}$$
(13)

where the Skempton's pore pressure coefficient B is defined as

$$\frac{1}{B} = 1 + \frac{n\beta}{\alpha} \tag{14}$$

where α is the volume compressibility of the sediment, β is the volume compressibility of the pore fluid.

Now, using a complementary wave loading method, *i.e.* two waves having the same phase and different amplitudes $(C_L \text{ and } C_C)$ are assumed for solving the Laplace equation and consolidation equation separately, one can write

$$p = C_L \times \mathbf{L} + C_C \times \mathbf{C} \tag{15}$$

where L and C denote values obtained from the solutions of Laplace equation and consolidation (diffusion) equation, respectively, assuming for both of them a unit amplitude of the inducing hydrodynamic pressure wave at the seabed.

A contribution of the particular components, supplied by the solutions of Laplace equation and consolidation equation, to the total solution of the problem is illustrated in Tab. 1.

Degree of Saturation	C_L	C_{C}
S = 1.00	0.998	0.002
S=0.99	0.855	0.145
S = 0.98	0.748	0.252
S = 0.97	0.665	0.335
S = 0.96	0.598	0.402
S = 0.95	0.544	0.456

Table 1Contribution of single components (from the Laplace equation
and consolidation equation) in the total solution, with regard
to different saturation conditions.

It has to be stressed once more that the influence of partly saturated seabed conditions is predominant for the investigated case of the uplift force acting on a buried submarine pipeline. Therefore, the both solutions to the Laplace equation and consolidation equation have to be always taken into account simultaneously. To confirm an existence of partly saturated conditions in a natural environment, a measuring campaign was conducted on Norderney Island (Germany). After sampling and statistical analysis of the measured and calculated

3140

results, the mean value of the degree of saturation was found to be 0.975 (Magda and Davidov, 1990).

Fig. 2 shows a set of pore pressure profiles with depth calculated for the singular solutions, *e.g.* the Laplace and consolidation problems, considered separately, comparing them with a solution for the compounded problem of the compressible fluid flow through compressible media.

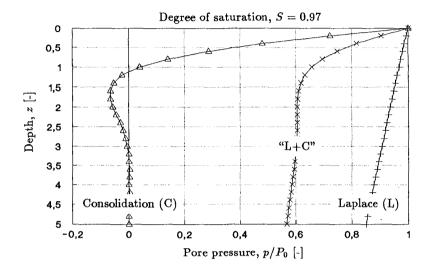


Figure 2 Comparison of different solutions for the pore pressure distribution with depth.

A similar analysis can also be performed for the case of a finite thickness of the seabed layer. The formulas describing coefficients C_L, C_D, D_L, D_D are, however, much more complicated.

The solution to the Laplace equation, Eq. (3), for the boundary conditions problem created by a finite thickness of the seabed layer and a pipe-like structure embedded in the soil sediment, is not trivial but does not bring any troubles. As documented in the introduction, it is possible to obtain this solution using, for example, one of the reported conformal mapping techniques.

It is not an easy task to solve the consolidation partial differential equation, Eq. (4), in the Cartesian coordinate system for the identical to the above mentioned boundary conditions problem. Therefore, to overcome the difficulties, the solution method presented below is based on the cylindrical (circular-cylinder) coordinate system.

The consolidation equation, also known as diffusion or heat conduction equation, is considered. It can be presented in general form as

$$\nabla^2 \varphi = \frac{1}{h^2} \frac{\partial \varphi}{\partial t} \tag{16}$$

The solution of any of the scalar equations like: the Laplace equation, the Poisson equation, the diffusion equation, the wave equation, the damped wave equation, transmission line equation, and the vecotr wave equation may be reduced to a solution of the scalar Helmholtz equation, or its special case – the Laplace equation (Moon and Spencer, 1971). For the consolidation equation (16), let

$$U(u_i)T(t) \tag{17}$$

where U is a function of the space coordinates and T is a function of time only. Substitution into the consolidation equation allows the separation of the time part, giving

$$\nabla^2 U + \kappa U = 0 \tag{18a}$$

$$\frac{dT}{dt} + \kappa^2 h^2 T = 0 \tag{18b}$$

where κ is the separation constant.

The solution of the Helmholtz equation (18a) depends on the space variables and the boundary conditions, and will be different for each problem. The equation in time (18b), however, is independent of the coordinate system. Thus the solution of the consolidation equation is always

$$\varphi = U(u_1, u_2, u_3) e^{-\kappa^2 h^2 t} \tag{19}$$

Geometry of the problem, *i.e.* circular pipe buried in a seabed (see Fig. 1), advices to use the circular-cylinder coordinates

$$\begin{cases} u_1 = r & 0 \le r < \infty \\ u_2 = \theta & 0 \le \theta < 2\pi \\ u_3 = z & -\infty < z < +\infty \end{cases} \qquad \begin{cases} x = r \cos \theta \\ y = r \sin \theta \\ z = z \end{cases}$$
(20)

Rewriting the Helmholtz equation (18a), U must satisfy, in polar coordinates,

$$\nabla^2 U + \kappa^2 U = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial U}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 U}{\partial \theta^2} + \kappa^2 U = 0$$
(21)

Separation of the Helmholtz equation (21), in two-dimensional polar coordinate system (in a plane problem θ is independent of z and the circular-cylinder coordinate system is simplified and becomes the polar coordinate system), leads to (Moon and Spencer, 1971):

$$rac{d^2 R}{dr^2} + rac{1}{r} rac{dR}{dr} + (\kappa^2 - rac{\lambda^2}{r^2})R = 0$$
 (22a)

$$\frac{d^2\Theta}{d\theta^2} + \lambda^2\Theta = 0 \tag{22b}$$

where R, θ are functions of r, θ , respectively, and λ and c are separation constants. These equations are solved for R and Θ , and the solution of the Helmholtz equation has a following form

$$U(r,\theta) = R(r) \Theta(\theta)$$
(23)

Differential equation (22b) has a following general solution

$$\Theta(\theta) = \alpha \cos \lambda \theta + \beta \sin \lambda \theta \tag{24}$$

For the governing problem, U is a harmonic function of θ with a period 2π , therefore, Θ must have the same feature. It is possible only when λ is represented by an integer number. By limiting the range of values of λ only to positive ones ($\lambda = 0, 1, 2, ..., n, ...$) both functions $\Theta(\theta)$ and R(r) can be written accordingly as

$$\Theta_{0}(\theta), \Theta_{1}(\theta), \Theta_{2}(\theta), ..., \Theta_{n}(\theta), ...; R_{0}(r), R_{1}(r), R_{2}(r), ..., R_{n}(r), ...$$
 (25)

In this way, an infinite system of solutions for Eq. (23) is obtained which now can be written as

$$U(r,\theta) = \sum_{n=0}^{\infty} [\alpha_n \cos n\theta + \beta_n \sin n\theta] R_n(r)$$
(26)

Eq. (22a) can be considered as the Bessel equation which in general form can be written

$$\frac{d^2W}{dw^2} + \frac{1}{w}\frac{dW}{dw} + (\mu^2 w^2 + q^2 - s^2/w^2)W = 0$$
(27)

The general series solution of Eq. (27) may be written, for $s \neq$ integer,

$$W = A\mathcal{J}_{\boldsymbol{s}}(\mu, q, w) + B\mathcal{J}_{-\boldsymbol{s}}(\mu, q, w)$$
(28)

These series are valid everywhere in the finite complex plane. If s = n, an integer, \mathcal{J}_{-s} is no longer independent of \mathcal{J}_s and the general solution of Eq. (27) is

$$W = A\mathcal{J}_n(\mu, q, w) + B\mathcal{Y}_n(\mu, q, w)$$
⁽²⁹⁾

where \mathcal{J}_n and \mathcal{Y}_n are the Bessel wave functions of the first and second kind (also called the Weber function), respectively. If $\mu = 0$ (this is the case) it can be concluded that the Bessel functions degenerate and Eq. (29) becomes

$$W = A\mathcal{J}_n(qw) + B\mathcal{Y}_n(qw) \tag{30}$$

Introducing the Hankel functions (*i.e.* the Bessel functions of the third kind, which are linear combinations of the Bessel functions of the first and second kinds)

$$\mathcal{H}_n^{(1)}(qw) = \mathcal{J}_n(qw) + i\mathcal{Y}_n(qw) \tag{31}$$

$$\mathcal{H}_n^{(2)}(qw) = \mathcal{J}_n(qw) - i\mathcal{Y}_n(qw)$$
(32)

where: $\mathcal{H}_n^{(1)}$, $\mathcal{H}_n^{(2)}$ are the Hankel functions of the first and second kind, respectively, and of order *n*, the general solution of Eq. (27) may be also written (Moon and Spencer, 1971):

$$W = A\mathcal{H}_n^{(1)}(qw) + B\mathcal{H}_n^{(2)}(qw) \tag{33}$$

Comparing now Eq. (22a) and Eq. (27), and replacing W by R and qw by κr , one has

$$R = A\mathcal{H}_n^{(1)}(\kappa r) + B\mathcal{H}_n^{(2)}(\kappa r)$$
(34)

Two-dimensional Helmholtz equation (18a), describing diffraction, after transformation into the polar coordinates system gets a form which is known as the Bessel equation, the solution of which, in two-dimensional scattering by localized objects in a sea of constant depth can be constructed by superposition of the following terms (Mei, 1989):

$$\begin{cases} \mathcal{H}_n^{(1)}(\kappa r) \\ \mathcal{H}_n^{(2)}(\kappa r) \end{cases} \begin{cases} \sin n\theta \\ \cos n\theta \end{cases}$$
 (35)

Because of the asymptotic behaviour of the Hankel functions

$$\begin{cases} \mathcal{H}_n^{(1)}(\kappa r) \\ \mathcal{H}_n^{(2)}(\kappa r) \end{cases} \simeq \left(\frac{2}{\pi \kappa r}\right)^{1/2} \exp\left[\pm i\left(\kappa r - \frac{\pi}{4} - \frac{n\pi}{2}\right)\right]$$
(36)

 $\mathcal{H}_n^{(2)}$ must be discared when κ is complex with a positive real part (Mei, 1989).

Assuming the hydrodynamic bottom pressure oscillations of a unit amplitude, expressed by harmonic solution

$$p = e^{-i\omega t} \tag{37}$$

where i denotes the imaginary unit, and comparing it with Eq. (6), the separation constant can be expressed by

$$\kappa = \sqrt{\frac{i\omega}{c_v}}$$
 (38)

In fact, κ is a complex number and can be presented in a general form as

$$\kappa = \sqrt{a + ib}$$
 where $a = 0$ and $b \equiv \frac{\omega}{c_v} > 0$ (39)

This can also be written

$$\mathfrak{c} = a' + ib'$$
 (40)

Comparison of the last two expresions shows that

 $a',b' \ge 1$ when $b \ge 1$ and $0 \le a',b' < 1$ when $0 \le b < 1$ (41)

So, b' is always positive without any respect to the positive value of b ($b \ge 0$).

And thus, the general solution for the scattered (radiated) waves, also pore water pressure waves, may be written as

$$U = \sum_{n=0}^{\infty} (\alpha_n \cos n\theta + \beta_n \sin n\theta) H_n^{(1)}(kr)$$
(42)

An implementation of the "image pipe" theory (Monkmeyer *et al.*, 1983) allowed to fulfil all requirements concerning the boundary conditions of the problem.

RESULTS OF CALCULATION

An application of the method is illustrated by some calculation examples where the meaning of the seabed saturation problem is specially emphased. For a certain geometry (depth of burial, pipeline outside diameter) the pipeline uplift force is computed, taking into account values of the degree of saturation from the range between 0.85 and 1.00, for which the soil can be considered as a saturated soil but with the pore fluid having some degree of compressibility (higher than a pure water) due to the presence of the air bubbles (Esrig and Kirby, 1977).

The solution to Eq. (1) enables to investigate the problem of pore pressure gradient, especially in a vicinity of the seabed surface. Performing series of calculation, the most unfavourable (from the pressure gradient point of view) phase of the pore pressure oscillations can be detected and then, it becomes feasible to calculate the most critical value of the pipeline uplift force. And

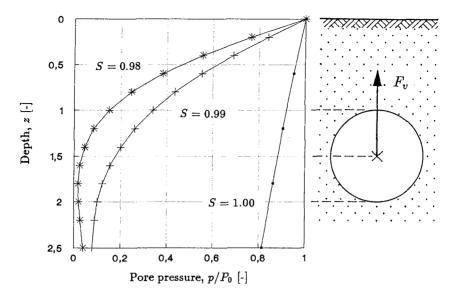


Figure 3 Definition sketch for the uplift force analysis (influence of different saturation conditions).

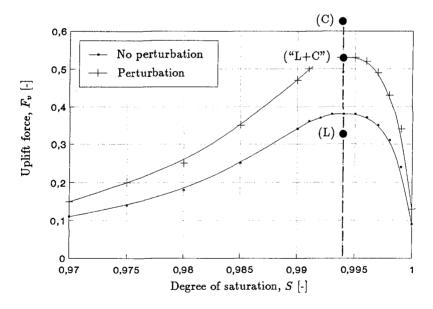


Figure 4 Pipeline uplift force versus different saturation conditions of seabed sediments.

thus, Fig. 3 shows the governing problem and Fig. 4 illustrates the results of calculations for a certain set of data where the uplift force is influenced by different values of the degree of saturation.

It can be easily recognized that the pipeline uplift force depends very strongly on the degree of saturation and has a maximum value for the degree of saturation very close to 1.00.

Changing a value of the degree of saturation with a step of 0.01 (*i.e.*, 1%), for example, the most critical situation can be easily omitted. Therefore, looking for an absolute maximum value of the pipeline uplift force, it is required to apply even smaller increment of the degree of saturation when performing a parameter study by means of numerical calculations to obtain a precise picture of possible variations in the pipeline uplift force.

The elaborated method seems to be very useful in a optimalization design procedure and gives the result which reflects, among others, the most inconvenient case for the pipeline stability with respect to saturation conditions of the seabed which are, on the other hand, extremely difficult and almost impossible to determine 'in-situ', using engineering methods of testing, with the exactness which is comparable to the necessary step of calculation.

The calculation procedure, presented in the paper and based on the advanced pore water pressure theory, make it feasible to incorporate important soil/water parameters into the pipeline uplift force analysis. Obtained values of the uplift force appear to be greater than these computed from the potential theory; this finding is in accordance with some observations from laboratory tests reported in the literature.

REFERENCES

DURSTHOFF, W., MAZURKIEWICZ, B. (1985).

"Problems related to submarine pipelines," Mitteilungen des Franzius-Instituts für Wasserbau und Küsteningenieurwesen der Universität Hannover, Heft 61, pp. 176-238.

```
LAI, N.W., DOMINGUEZ, R.F., DUNLAP, W.A. (1974).
```

"Numerical solutions for determining wave-induced pressure distributions around buried pipelines," Texas A & M University, Sea Grant Pub. No. TAMU-SG-75-205.

LENNON, G.P. (1985).

"Wave-induced forces on buried pipelines," Journal of Waterway, Port, Coastal and Ocean Engineering, Vol. 111, No. 3, pp. 511-524.

MACPHERSON, H. (1978).

"Wave forces on pipeline buried in permeable seabed," Journal of Waterway, Port, Coastal and Ocean Division, Vol. 104, No. WW4, pp. 407-419. MADSEN, O.S. (1978).

"Wave-induced pore pressures and effective stresses in a porous bed," Géotechnique, Vol. 28, No. 4, pp. 377-393.

MAGDA, W. (1989).

"Wave-induced pore water pressure generation in a sand bed," Internal Report No. 1, Sonderforschungsbereich-205 "Küsteningenieurwesen," TP A13, Universität Hannover, June 1989, pp. 1-94.

MAGDA, W. (1990).

"Pore water pressure generation in a highly saturated sea bed for shallow water conditions," Proceedings of the 1st International Symposium "LIT-TORAL 1990," Marseille, July 1990, pp. 110-114.

MAGDA, W., DAVIDOV, N. (1990).

"On determining of the degree of saturation from 'in-situ' measurements of the seabed performed on Norderney Island," Internal Report No. 2, Sonderforschungsbereich-205 "Küsteningenieurwesen," TP A13, Universität Hannover, July 1989, pp. 1-57.

McDougal, W.G., Davidson, S.H., Monkmeyer, P.L., Sollitt, C.K. (1988).

"Wave-induced forces on buried pipelines," Journal of Waterway, Port, Coastal, and Ocean Engineering, Vol. 114, No. 2, pp. 220-236.

MEI, C.C. (1989).

"The Applied Dynamics of Ocean Surface Waves," Advanced Series on Ocean Engineering - Vol. I, World Scientific Publishing Co. Pte. Ltd, pp. 1-740.

Monkmeyer, P.L., Mantovani, P., Vincent, H. (1983).

"Wave-induced seepage effects on a buried pipeline," Proceedings of the Coastal Structures '83 Conference, pp. 519-531.

MOON, P., SPENCER, D.E. (1971).

"Field Theory Handbook; Including Coordinate Systems, Differential Equations and Their Solutions," Springer-Verlag, pp. 1-236.

OKUSA, S. (1985a).

"Wave-induced stresses in unsaturated submarine sediments," Géotechnique, 35, No. 4, pp. 517-532.

PHILLIPS, B.A., GHAZZALY, O.I., KALAJIAN, E.H. (1979).

"Stability of pipeline in sand under wave pressure," Proceedings of the Speciality Conference "Civil Engineering in the Ocean IV," San Francisco, pp. 122-136.

QIU, H.H., SUN, Z.C. (1987).

"Wave induced pressures on a buried pipeline," Proceedings of the 6th International Symposium on Offshore Mechanics and Arctic Engineering (OMAE), Houston, Texas, March 1-6, 1987.

YAMAMOTO, T., KONING, H.L., SELLMEIJER, H., HIJUM, E. (1978).
"On the response of a poro-elastic bed to water waves," Journal of Fluid Mechanics, Vol. 87, part 1, pp. 193-206.