CHAPTER 230

MIXING BY SHEAR INSTABILITIES OF THE LONGSHORE CURRENT

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ABSTRACT

Shear instabilities of the longshore current are examined as a possible source of non-zero $\overline{u'v'}$ values (mixing) within the surfzone. This term is calculated using model generated stream functions whose amplitudes are calibrated with the observed turbulent kinetic energy spectrum. Data from the DELILAH experiment, conducted at the barred beach at Duck, North Carolina is used. Excellent agreement is found between the predicted range of shear instabilities and observations as seen in frequency-wavenumber plots. Maximum predicted values for $\overline{u'^2}$, $\overline{v'^2}$ and $\overline{u'v'}$ are .04, .20, and .03 m^2/s^2.

INTRODUCTION

During 1986 SUPERDUCK, Oltman-Shay *et al.* (1989) observed low frequency oscillations (<0.01 Hz), with wavelengths less than 300m. Free surface gravity waves below 0.05 Hz consist of two classes of waves: edge waves which are trapped by refraction along the beach face, and those which escape seaward as "leaky" waves; these waves have been observed in great detail and are considered "infragravity" waves because of their low frequencies relative to the sea-swell band. The uniqueness of the oscillations observed lies in the fact that the wavelengths observed were an order of magnitude shorter than the shortest infragravity wave under applicable conditions (a function of frequency and beach slope). These oscillations were considered to be kinematically distinct based upon their frequency / wavenumber range.

Energy density distributions represented by gray shading in wavenumber-frequency space for 10 October during the DELILAH experiment are shown in fig. 1. The theoretical dispersion curves for trapped edge waves, modes 0, 1, and 2 are shown for the appropriate beach slope. Significant energy is seen outside of these edge wave curves; this energy is linear in f-K space (where K, cyclic wavenumber, is equal to $k/2\pi$), indicating that these oscillations (considered to be alongshore progressive waves) are non-dispersive. The relationship between the phase speed of these oscillations, given by the wavenumber-frequency slope, and the magnitude

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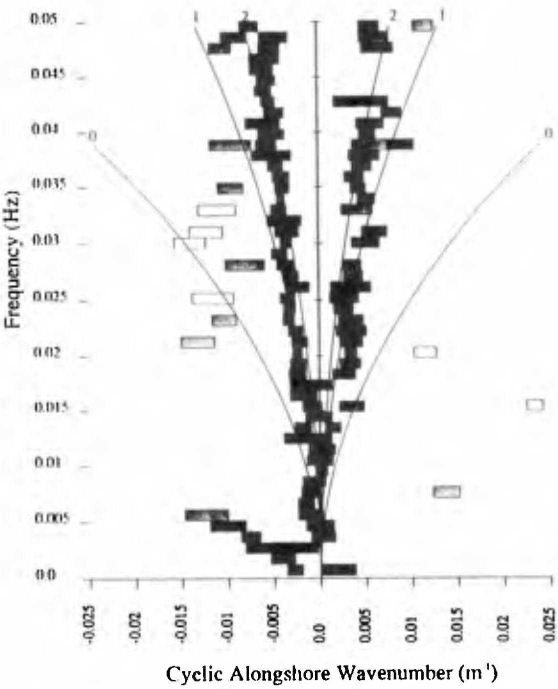

Fig. 1 Frequency-cyclic wavenumber spectrum for 10 Oct. with shading representing log-variance density. Δf=0.00098, ΔK=0.0005. Theoretical edge-wave dispersion curves (modes 0-2) are plotted.

of the mean longshore current, together with the lack of any such oscillations in the absence of mean longshore currents, indicates that the longshore current, and its associated kinetic energy, may be the driving force behind these waves.

Bowan and Holman (1989) formulated a theoretical framework to explain these observations as shear instabilities, deriving a conservation of potential vorticity equation in which the vorticity of the longshore current shear functioned as the restoring force. They also related a phase shift in the stream functions produced by the instabilities to non-zero $\overline{u'v'}$ values which were suggested as possible sources of significant mixing in the nearshore.

Standard longshore current models (based on an alongshore balance between the radiation stress gradient and bottom shear stress) applied to barred topography predict two current maxima in the form of "jets", the first over the bar, and the second at the beach face. Conversely, observations routinely show a single longshore current maximum, found in the vicinity of the trough. Typically, longshore current models employ some sort of horizontal mixing term to try to eliminate this disparity. This mixing term is a parameterization of the turbulent radiation stress gradient and is usually described in terms of eddy viscosity. Turbulence is present in the surf zone over a wide range of frequency and spacial scales; in the present work the turbulence (or perturbations) associated with shear instabilities is studied with the specific objective of calculation of the $\overline{u'v'}$ term, thus avoiding the need for parameterization.

Putrevu and Svendsen (1992) carried out a numerical study of shear instabilities over various topography and using an order of magnitude analysis, concluded that even a weak shear in the longshore current could be capable of producing significant mixing. To quantify this possibility requires calculation of the $\overline{u'v'}$ associated with the shear instabilities. Unlike non-linear models such as Dodd (1992), which may be used to predict stream function amplitudes, linear models utilize stream functions which are of arbitrary amplitude; thus the predicted velocities, which are based on the gradients of the stream functions, are likewise arbitrary. Calibration of the stream function amplitudes requires one of two approaches. The first, followed in Dodd *et al.* (1992), assumes that the growth rates predicted by the model may be taken as an indication of the ultimate distribution of energy across the wavenumber spectrum. For example, should wavenumber k_1 have a predicted growth rate twice that of wavenumber k_2, it would be assumed that the steady state energy of k_1 will be twice as great also. Linear theory is then used to relate energy to amplitude squared. This method allows for the inter-comparison of different wavenumbers, but still lacks an absolute reference.

The second method is to measure observed energy at the wavenumbers/ frequencies of interest and then scale the stream function amplitudes such that the predicted and observed energies match. This approach does produce an absolute reference and will be used in the current work. The combined u and v energies (i.e. $\overline{u'^2} + \overline{v'^2}$) have been used for calibration, being invariant with orientation. Once the stream function amplitudes have been so calibrated, the result is an alongshore averaged profile of $\overline{u'v'}(x)$ for each wavenumber for which growth is predicted. Integrating these produces a profile which represents the net radiation stress associated with the shear instabilities. Data obtained during the 1990 DELILAH experiment are used to evaluate the magnitude and structure of this term across a barred beach.

SHEAR INSTABILITY THEORY

A. Assumptions

Linear wave theory is utilized, with the x-axis perpendicular to the bathymetry (positive seaward). Both mean and perturbation current velocities are vertically integrated and the mean current is assumed steady state. The longshore current and bathymetry are assumed uniform in the alongshore direction.

B. Shear Instabilities of the Longshore Current

In the companion paper to Oltman-Shay *et al.* (1989), Bowen and Holman (1989) offered a theoretical basis for shear instabilities. Using conservation of potential vorticity as the restoring force, they were able to relate the mean longshore current shear to observed oscillations.

The momentum equations, with the velocity consisting of perturbations (u', v') and a mean longshore current (V) are:

1) $$\frac{\partial u'}{\partial t} + V\frac{\partial u'}{\partial y} = -g\frac{\partial \eta}{\partial x}$$

2) $$\frac{\partial v'}{\partial t} + u'\frac{\partial V}{\partial x} + V\frac{\partial v'}{\partial y} = -g\frac{\partial \eta}{\partial y}$$

where η is surface elevation. These equations are linearized and a non-divergent (rigid lid) approximation is applied allowing the use of a stream function to represent the flow, such that:

3) $$u' = -\frac{1}{h}\frac{\partial \Psi}{\partial y} \qquad v' = \frac{1}{h}\frac{\partial \Psi}{\partial x}$$

Cross differentiating to combine equations and eliminate η, the result is:

4) $$\underset{1}{}\quad\underset{2}{}\quad\underset{3}{}\quad\underset{4}{}$$
$$\left(\overset{1}{\frac{\partial}{\partial t}}+\overset{2}{V\frac{\partial}{\partial y}}\right)\left(\overset{3}{\frac{\Psi_{yy}}{h}+\left(\frac{\Psi_x}{h}\right)_x}\right)=\overset{4}{\Psi_y\left(\frac{V_x}{h}\right)_x}$$

where the subscripts denote differentiation. Term 1 represents the local rate of change. Term 2 is the advection by the mean longshore current. Term 3 is the relative potential vorticity of the perturbations. Term 4 represents the advection by the perturbations of the potential vorticity of the mean longshore current. This potential vorticity equation is comparable to the barotropic Rossby equation used for planetary scale flow with the exception that the background vorticity of the current shear is substituted in place of the Coriolis parameter.

A solution is then assumed of the form:

5) $$\Psi = Re\{\phi(x)e^{i(ky-\omega t)}\}$$

where ϕ is the cross-shore structure function. The alongshore wavenumber, k, is taken to be real, but ω, the angular frequency, and ϕ may be complex. The form of the solution which allows growth with time is then:

6) $$\Psi = \exp(\omega_{im}t)Re\{\phi(x)\exp[i(ky-\omega_{re}t)]$$

Inserting this solution in the previous equation yields:

7) $$(V-c)\left(\phi_{xx}-k^2\phi-\frac{\phi_x h_x}{h}\right)-h\phi\left(\frac{V_x}{h}\right)_x=0$$

where c is the phase speed of the shear wave, equal to ω/k. Dodd *et al.* (1992)

include the dissipative effects of bottom friction through a parameterization, $\mu=2c_f U_0/\pi$. The resulting modification of the basic equation produces:

8) $$(V-\frac{i\mu}{kh}-c)(\phi_{xx}-k^2\phi-\frac{\phi_x h_x}{h})-h(\frac{V_x}{h})_x\phi+\frac{i\mu}{kh}(\frac{\phi_x h_x}{h})=0$$

The principle result of the inclusion of dissipation is a dampening effect on instabilities as indicated through the model by the reduced range over which growth is predicted. This equation, with c_f=0.003, is the essence of the shear instability model employed in this study.

After inserting known topography and an *a priori* longshore current profile, this equation takes the form of a quadratic equation in ω. This may be written in matrix form as $[A]\{\phi\}=c[B]\{\phi\}$ which produces the eigenvalues, c, for each wavenumber. Using $c=\omega/k$ the real and imaginary parts (should ω be complex) may be found. It is the cases when ω_{im} is positive that growth is predicted for an instability of that particular wavenumber.

For any instability to grow, (i.e. to have a positive ω_{im}) there must be some source of energy, be it either potential (baroclinic instability) or kinetic (barotropic instability). A mechanism must then exist to transfer this energy from its source, here the longshore current, to the growing perturbation. Dodd and Thornton (1990) derive a set of energy equations to further study this transfer, yielding:

9) $$\frac{\partial}{\partial t}(KE) = -\int_0^\infty \overline{u'v'}V_x dx - g\int_0^\infty \frac{h_x}{h}\overline{u'\eta}dx$$

where KE denotes the kinetic energy of the perturbations and the averaging has been done over the y direction. The first term on the right hand side represents the role of the Reynolds stresses($\overline{u'v'}$) in transferring energy and the second term the work done by the surface pressure gradients. This second term can be expected to be small as a result of the ratio of the depth, h, to the bottom slope in the x direction. Thus simplified, the required condition for a growing instability is that there must be a negative correlation between $\overline{u'v'}$ and the shear of the longshore current.

EXPERIMENT

The 1990 DELILAH experiment, the data from which is the basis of this paper, was conducted at the U.S. Army Corps of Engineers Field Research Facility at Duck, North Carolina, (same site as SUPERDUCK), and was designed specifically to measure shear instabilities. Two alongshore arrays, the first of 5 current meters and located in approximately 1.5 meters of water, monitored conditions in the trough, while the second longshore array, located in approximately 3 meters of water, was positioned on the seaward face of the bar. It is these two arrays which were used in obtaining the normalized, spatially lagged, cross-spectral matrix used with the Iterative Maximum Likelihood Estimator (Pawka 1982) to produce the f-K spectra. An autonomous Coastal Research Amphibious Buggy (CRAB), which was designed to provide a stable platform for operations within the surf zone, was used for daily bathymetric measurements A cross-shore array of 9 current meters and wave sensors extending across the surf zone was deployed to define the longshore current. These three principle arrays, shown in figure 2, were

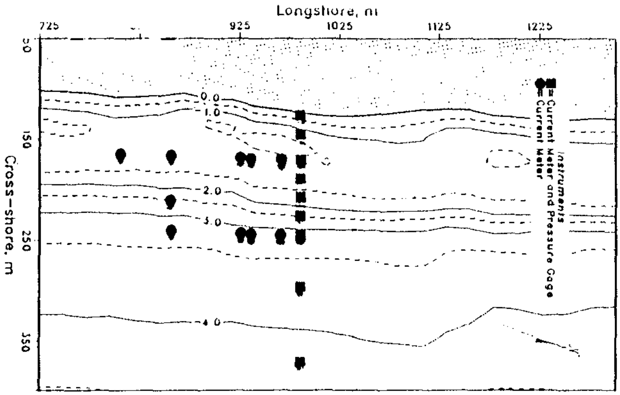

Fig. 2 Meter/gage placement

used to acquire data near-continuously throughout the experiment with a sampling rate of 8 Hz. A wide variety of wave conditions occurred during the one month experiment, including a northeaster which drove broad banded waves and a hurricane which generated narrow banded swell, each resulting in strong longshore currents and concomitant shear instabilities.

MODEL/DATA COMPARISON

Required as input to the shear instability model is the longshore current profile. In the present work this is obtained through application of a cubic spline to the nine observations. The resulting profile and the measured bathymetry are shown in figure 3 together with the calculated background vorticity (term 4 in eq. (4)). This background vorticity exhibits a relative minima approximately 75 meters offshore. This relative minima corresponds to an elimination of the restoring force and so is the location of the predicted instabilities, an example of which (cyclic wavenumber = .0065) is shown in figure 4. The longshore current profile is overlaid on figure 4 and demonstrates the transfer of energy described by equation (9), (i.e. that the axis of the instabilities are generally opposite to the local current shear).

Predicted growth rate, ω_i, and frequency, ω_r, (which is linear and therefore non-dispersive) versus cyclic wavenumber are shown in figure 5. Overlaying this predicted dispersion relation on the observed f-K spectra shown in figure 6. demonstrates excellent agreement.

To calibrate the amplitude of the model produced stream functions, the energy density spectra ($u'^2 + v'^2$) were calculated for each of the nine cross-shore current meters. Two hour time series were used with sampling at 8 Hz. The total record length was broken up into 8 sub-records based on the required time interval necessary to produce frequency resolution comparable to that used in the model. An unfortunate result is that the degrees of freedom available remain fairly small and the confidence intervals are therefore excessively large. As an alternative test

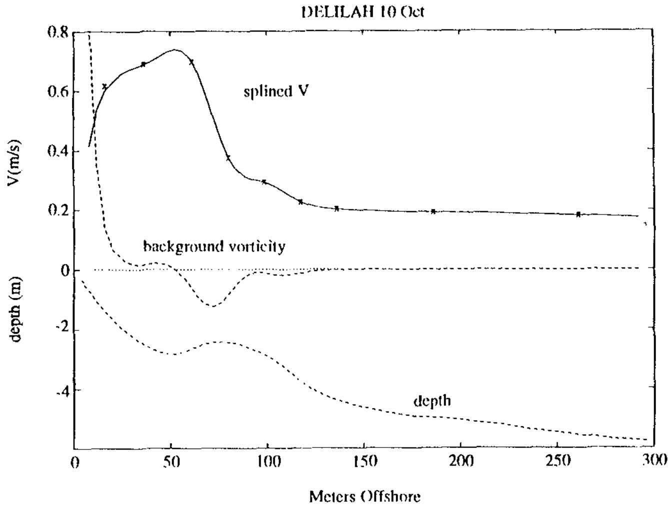

Fig. 3 Splined longshore current velocity profile, measured depth, and calculated background vorticity.

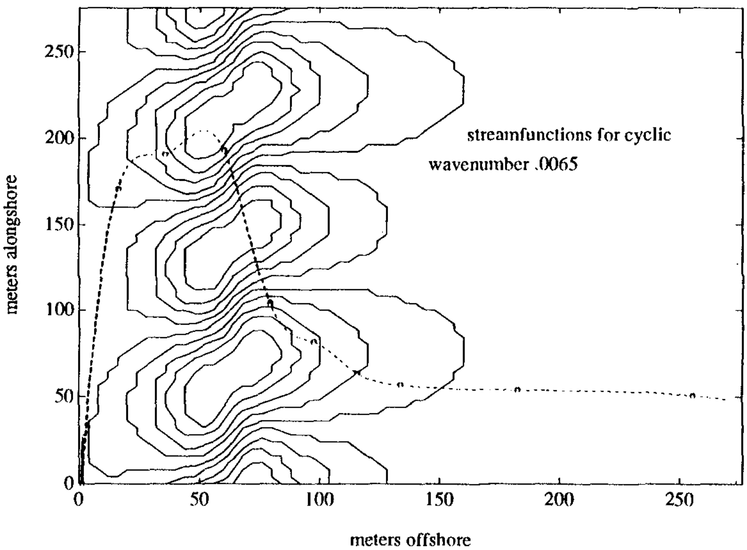

Fig. 4 Model predicted stream functions and superimposed longshore current profile showing opposing tilt of system as required for energy transfer.

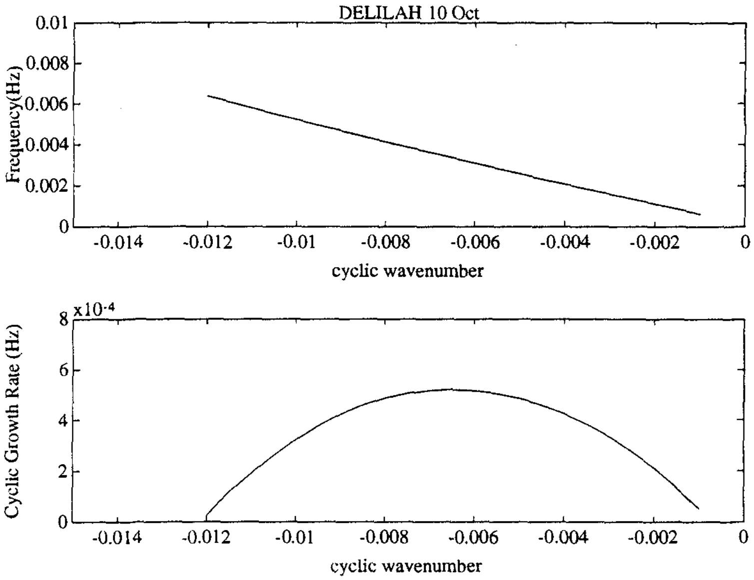

Fig 5. Model predicted frequency (ω_{re}) and growth rate (ω_{im}) versus cyclic wavenumber.

of the statistical strength of the data, the consistency over the nine current meters is offered, shown in figure 7. The observed $u'^2 + v'^2$ as a function of frequency is thus obtained for each of the nine current meters of the cross shore array.

To calculate $u'^2 + v'^2$ for the model, an analytical solution was obtained by substituting an amplitude variable, A, into eq.(6). The modeled $\overline{u'^2} + \overline{v'^2}$, averaged over one wavelength in the alongshore direction can be written then as:

10) $$\overline{u'^2} = \frac{A^2k^2}{2h^2}(\phi_r^2 + \phi_i^2) \qquad \overline{v'^2} = \frac{A^2}{2h^2}\left[\left(\frac{\partial\phi_r(x)}{\partial x}\right)^2 + \left(\frac{\partial\phi_i(x)}{\partial x}\right)^2\right]$$

These are combined, providing the modeled $u'^2 + v'^2$ as a function of cross-shore distance for each of the wavenumbers for which growth was predicted (in this particular case 22). Since the predictions are in wavenumber space, while the observations are in frequency space, the model predicted dispersion relation is used to translate the output into frequency space for comparison.

The observed and predicted $u'^2 + v'^2$ spectra may each be thought of as an energy surface in two dimensional x-f space. In the case of the observations there are 9 lines of E(f) spread across x, and in the case of the model output, there are 22 lines of E(x) spread across k. The surface given by the model output may be adjusted based upon the values of A(f) and so the two surfaces are matched in a best fit manner. These values of A(f) are then applied to the calculation of $u'v'$ based on the analytical form (again averaged over the alongshore wavelength) given by:

11)
$$\overline{u'v'} = -\frac{A^2 k}{2h^2}\left(\phi_r \frac{\partial \phi_i}{\partial x} - \phi_i \frac{\partial \phi_r}{\partial x}\right)$$

This produces a $\overline{u'v'}(x)$ profile for each of the modeled wavenumbers. Finally, these are integrated to provide a single $\overline{u'v'}(x)$ profile representing the net effect of the predicted range of shear instabilities (shown together with $\overline{u'^2}$ and $\overline{v'^2}$ in figure 8).

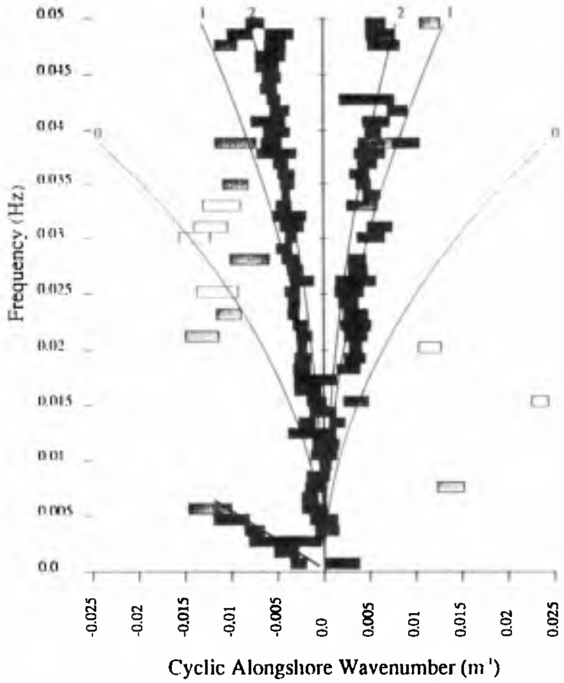

Fig. 6 Same frequency-cyclic wavenumber spectrum as shown in figure 1 with model predicted dispersion relation superimposed.

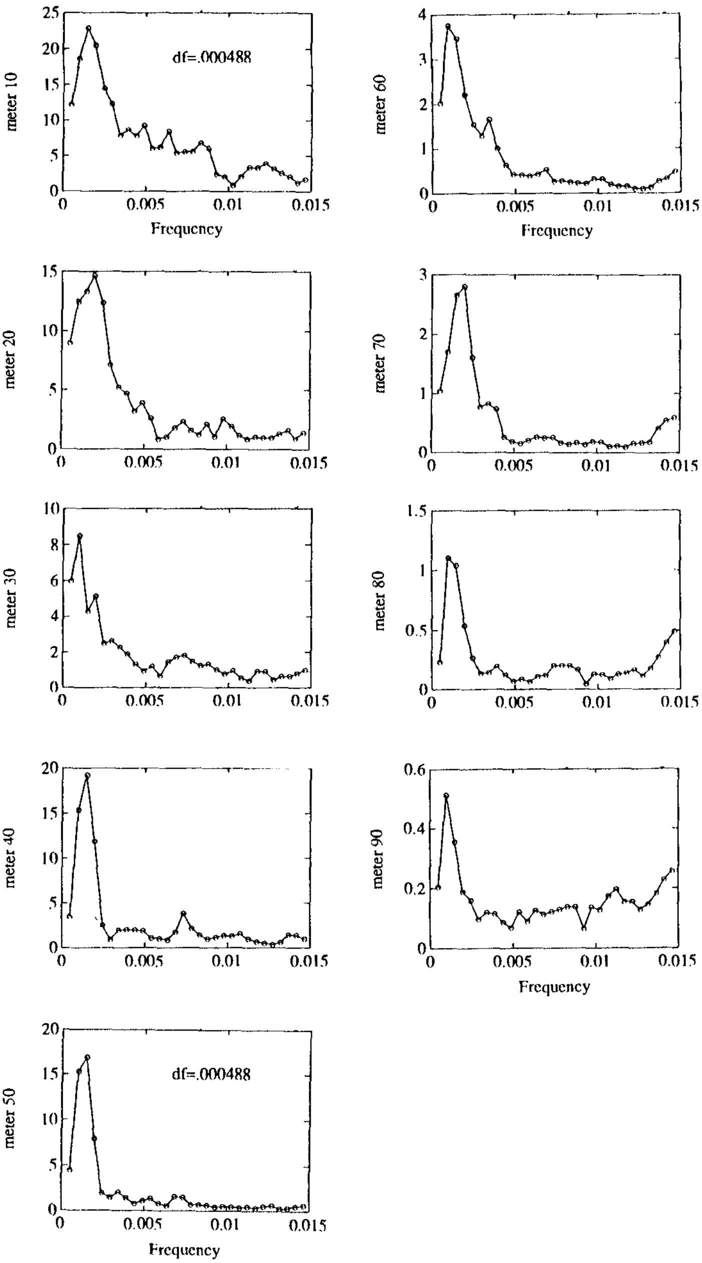

Fig. 7 $(u'^2 + v'^2)$ **energy density spectra over frequency range of interest for each of 9 current meters (cross-shore positioning shown on Figs. 2 and 3).**

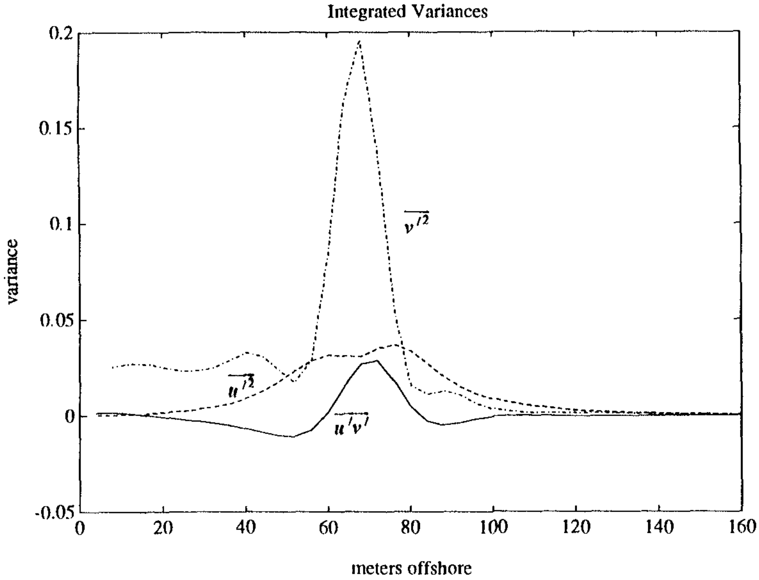

Fig. 8 Integrated $\overline{u'^2}$, $\overline{v'^2}$ and $\overline{u'v'}$ profiles based on calibration of model predicted stream functions

Summary

The magnitude and cross-shore structure of the turbulent radiation stress associated with shear instabilities of the longshore current has been examined. Through calibration of the model-generated stream function amplitudes, an absolute reference has been incorporated such that dimensional values of $\overline{u'v'}(x)$ have been obtained. The cross-shore gradients of the radiation stresses due to both waves (calculated through a Thornton and Guza (1983) model) and shear instability turbulence are compared in fig. 9. The model suggests that the turbulence term may well exceed the wave forcing term for this data. Further study will examine additional data from DELILAH and attempt to link the longshore current model predicted "two-jet base state" to the single maximum observed profile, through a series of progressive steps.

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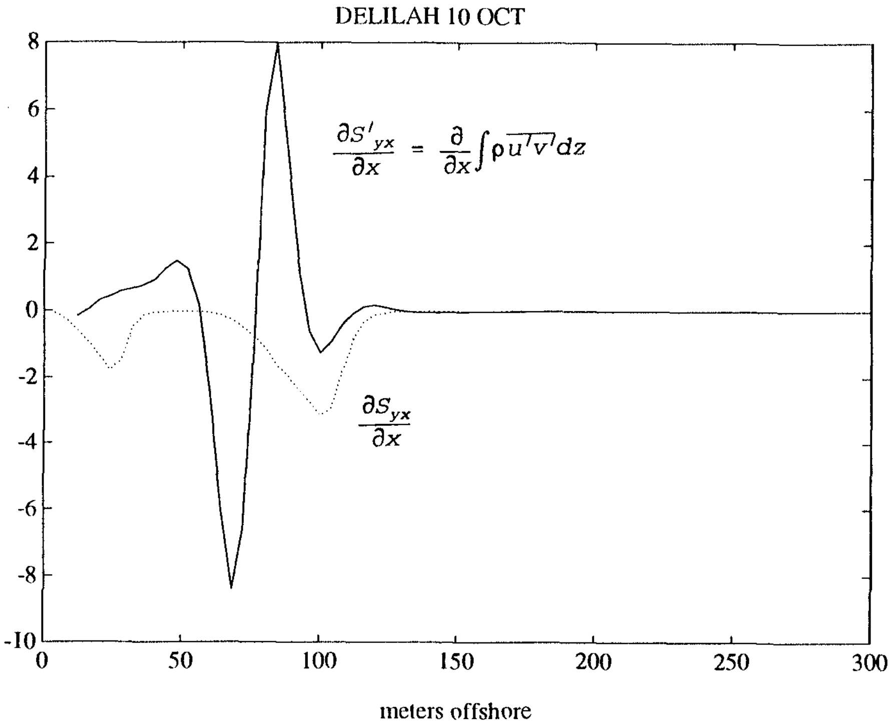

Fig. 9 **Comparison of the model predicted gradients of the shear instability-turbulence radiation stress and the wave radiation stress obtained through a Thornton and Guza (1983) model.**

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