CHAPTER 223

MOTION OF MOBILE BEDS AT HIGH SHEAR STRESS

Kenneth C. Wilson[1] and Fidelia N. Nnadi[2]

1. INTRODUCTION

Beds of granular material show various types of behaviour as the dimensionless shear stress or Shields ordinate, Y, is increased. This quantity is defined as $\tau/\rho g(S-1)d$, where τ is boundary shear stress, ρ is fluid density, g is gravitational acceleration, S is the ratio of solids density to fluid density and d is particle diameter. Following the zone of no particle movement at low Y, there is a range at which sand waves are found, and finally, in the high shear-stress region where Y exceeds 0.8, the bed tends to become plane.

This high-stress or upper-plane-bed condition may be encountered in rivers in flood, large flows in estuaries, and closures or breaches of cofferdams or dykes. Because of the very high rates of sediment transport associated with this type of flow, it has a disproportionate effect on both natural topographic features and engineering works. The investigation of such behaviour by traditional flume experiments is not easy, but testing in enclosed pressurised conduits can eliminate many of the difficulties (Wilson, 1966; Nnadi and Wilson, 1992).

Nnadi (1992) has described an experimental program carried out recently using a recirculating system located at Queen's University. The test section was a square conduit with side dimension 98 mm, and series of runs were made with three sizes of sand (S = 2.67), two sizes

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1 Professor, Department of Civil Engineering, Queen's University, Kingston, Ontario, Canada K7L 3N6
2 Post-Doctoral Fellow, Department of Civil Engineering, McMaster University, Hamilton, Ontario, Canada L8S 4K1

of bakelite (S = 1.56) and one size of nylon particles (S = 1.14).

2. ANALYSIS AND EXPERIMENTAL RESULTS

Analysis based on the experimental findings shows that at high shear stress the bed load moves in a shear layer (the sheet-flow layer) with thickness which is proportional to the shear stress applied by the flow to the mobile boundary. Within this layer, the submerged weight of the grains comprising the bed load (also called the contact load) is counteracted by intergranular contacts, which may be either continuous or sporadic. The thickness of the shear layer, δ_s, can be an order of magnitude larger than the particle diameter. For this high-shear-stress configuration, the equivalent roughness of the boundary is not proportional to the size of the individual particles (as had been assumed in the past for the upper-plane-bed regime), but instead varies directly with the shear-layer thickness, δ_s, and hence is proportional to the applied shear stress.

The resulting value of the equivalent roughness, k, is about 0.5 δ_s. It follows that the value of k/d is not constant (as for the rough-boundary relation) but increases with Y in what is essentially a direct proportionality. Wilson and Nnadi (1990) produced a plot of the ratio of equivalent roughness to particle diameter (k/d) _versus_ the dimensionless shear stress. This plot is reproduced here as Figure 1. It clearly shows that k/d is far from constant, but instead increases with Y in accord with the analytical prediction, which can be approximated (Wilson, 1989) by the simple relation

$$\frac{k}{d} \approx 5Y \qquad [1]$$

More detailed analysis is given by Nnadi and Wilson (1992), but the basic points to be noted is that the effective roughness for sheet flow is no longer proportional to particle diameter, but is strongly affected by increasing shear stress. As seen from the figure, a broad range of the variables is represented, the trend is clear, and the scatter band is less than that usually displayed by moveable-boundary data.

Both the experimental work mentioned above and the associated analysis have been concerned with essentially horizontal flows. It was realized that for certain cases of practical importance, such as wave uprush on beaches or bars, the flow is inclined, and the effect of this

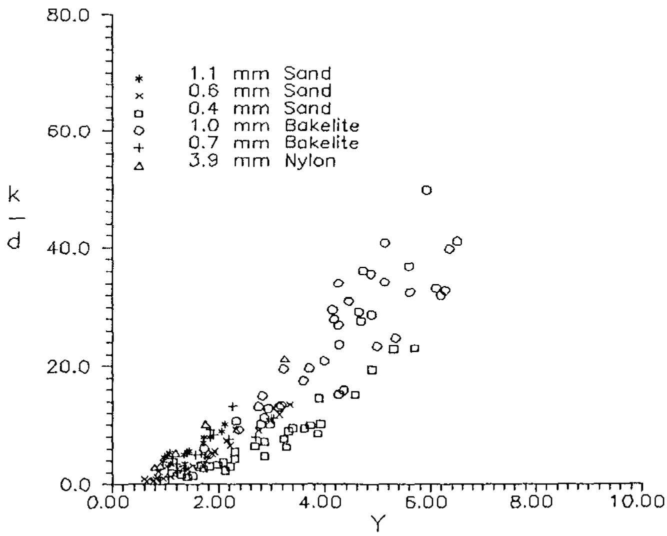

Figure 1 Effective Roughness Ratio for Horizontal Flow

inclination can be important. In order to investigate this effect, it was arranged that the closed-conduit test section could be inclined to the horizontal. Several series of runs in the upper-plane-bed regime were made at inclination angle θ (elevation increasing in the direction of flow) of 10°, 20° and 30°. The values of manometric gradient, mean flow velocity and bed-associated hydraulic radius were obtained for each run, and used to calculate k/d and Y in the same fashion employed previously for the horizontal tests. The results for 10°, 20° and 30° inclination are shown on Figures 2, 3 and 4, respectively. The general trends are very similar to that of the plot for horizontal flows but it is worth noting that for the inclined flows the range of the variables is larger than for the horizontal case, and k/d has a somewhat larger rate of increase with Y. The data for the various inclinations indicate that, in the range investigated, the rate of increase of k/d with Y is monotonically related to the inclination angle.

Analysis of particle motion in inclined flows showed that a positive inclination introduces a submerged-weight force component which acts opposite to the direction of motion. The resulting behaviour is equivalent to a horizontal flow with a larger effective value of

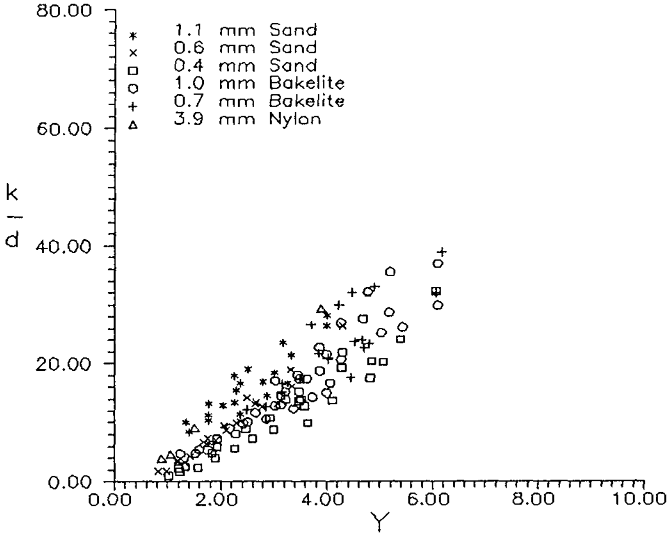

Figure 2 Effective Roughness Ratio for $\theta = 10°$

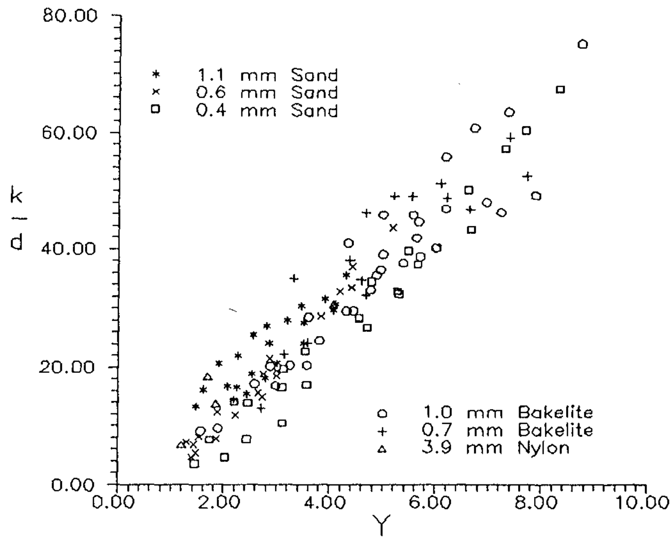

Figure 3 Effective Roughness Ratio for $\theta = 20°$

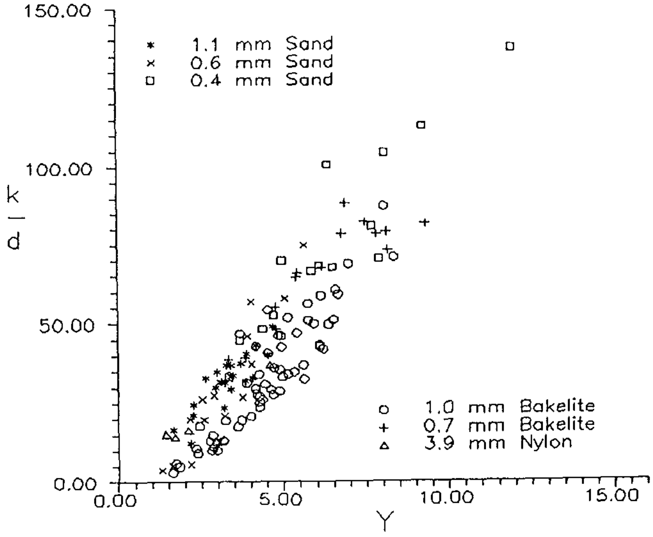

Figure 4 Effective Roughness Ratio for $\theta = 30°$

submerged relative density, equivalent to multiplying (S-1) by the factor $\cos\theta(1+\tan\theta/\tan\phi')$, where ϕ' is the dynamic friction angle of the solids. On this basis it is predicted that the solids transport rate will diminish with increasing inclination θ (for given values of hydraulic gradient and bed-associated hydraulic radius).

This effect can be expressed by applying a suitable adjustment to the dimensionless bed-load transport coefficient G_s, defined by the equation

$$q_s = \frac{G_s}{(S-1)g}\left(\frac{\tau}{\rho}\right)^{1.5} \qquad [2]$$

Here q_s is the volumetric discharge of bed-load solids per unit breadth, and the other symbols have been defined previously. In the high-shear-stress range (where the critical shear stress for the beginning of particle motion is a negligibly small fraction of the applied shear stress) Equation 2 is compatible in form to the equation of Meyer-Peter and Müller. That equation produces a G_s value of 8.0 for near-horizontal flow. However, as proposed earlier (Wilson, 1966 and 1987), $G_s \simeq 12$ gives a better fit both to sand data at high shear stress and to the theoretical expectations for horizontal flow in the upper-plane-bed region. The present analysis indicates that G_s will vary with both i/(S-1) and the inclination angle θ. Figure 5 shows calculated curves for various inclinations up to 30°. Each curve displays an essentially flat initial section, followed by a downturn at higher values of i/(S-1). The series of curves shows the expected decrease of G_s with increasing θ.

The experimental data for inclination angles of 0° to 30° were found to be in good accord with the trends shown by the predicted curves. Figures 6, 7, 8 and 9 show the data for θ values of 0°, 10°, 20° and 30°, respectively.

Although there is a fair degree of scatter in the experimental results for G_s, each data set clusters about the predicted line. The droop in G_s at high i/(S-1), which is particularly clear from the nylon data, occurs for each of the inclination angles. Comparison of the four data sets clearly shows the downward trend of G_s with increasing θ that follows the general trend of the predicted curves.

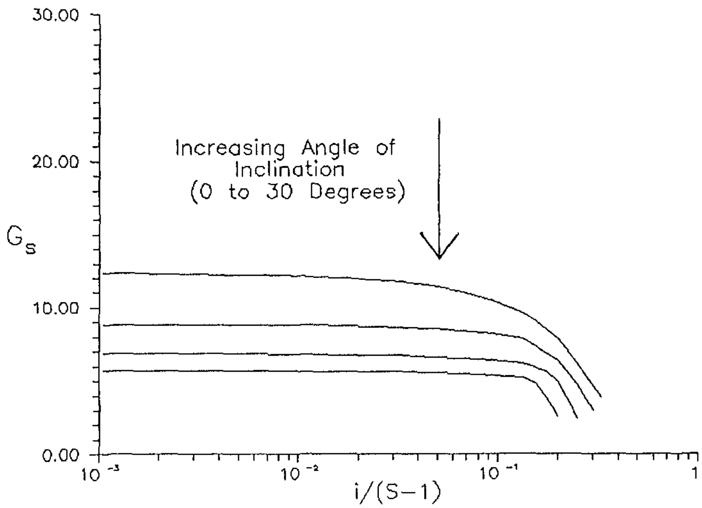

Figure 5 Predicted Transport-Rate Parameter for Various Inclinations

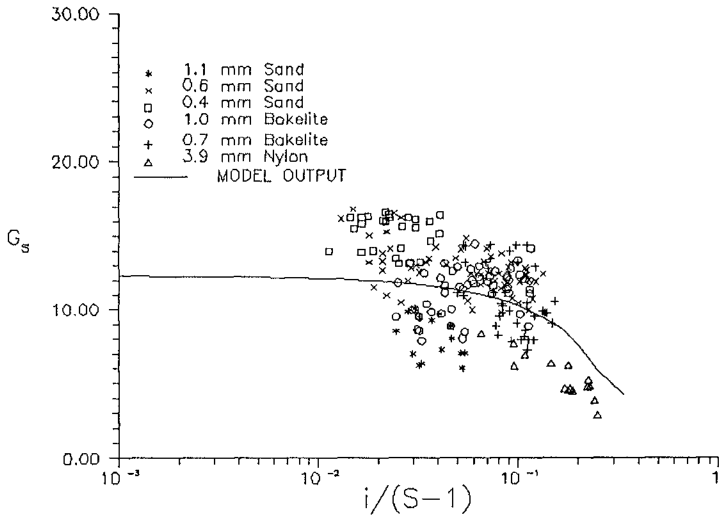

Figure 6 Observed and Predicted Transport-Rate Parameter for Horizontal flow

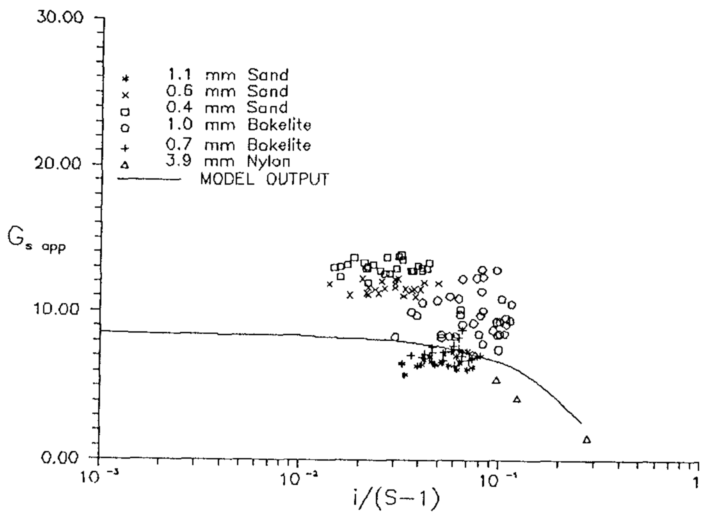

Figure 7 Observed and Predicted Transport-Rate Parameter for $\theta = 10°$

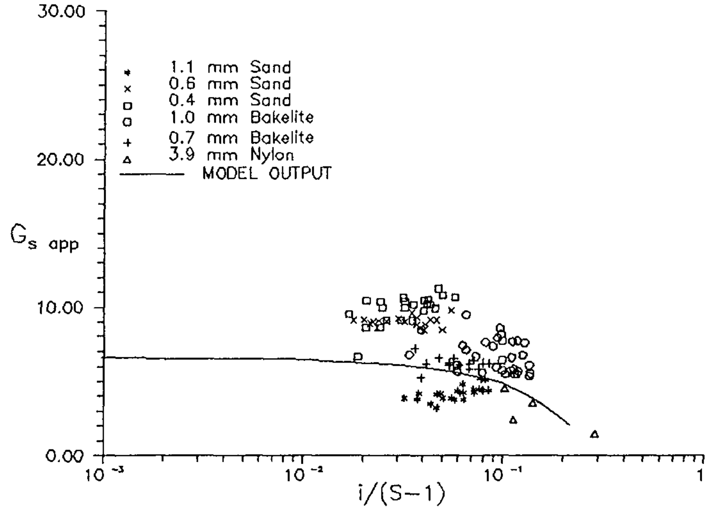

Figure 8 Observed and Predicted Transport-Rate Parameter for $\theta = 20°$

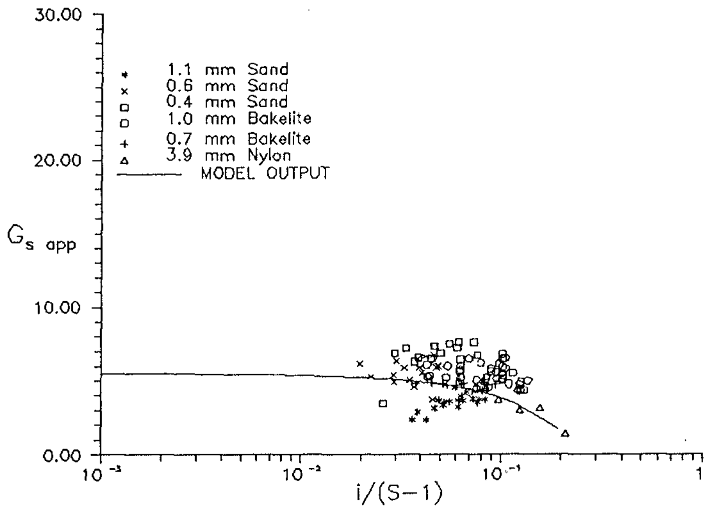

Figure 9 Observed and Predicted Transport-Rate Parameter for $\theta = 30°$

3. CONCLUSION

At high shear stress, beds of granular materials tend to become plane, with bed-load particles forming a sheet-flow layer of thickness proportional to the applied shear stress. Results from both experiments and analysis have shown that this region is characterised by large friction factors and high rates of bed-load transport.

The study of horizontal sheet flow has now been extended to inclined flows, which are significant for beach morphology. The effect of an inclination angle (with bed elevation increasing in the direction of flow) is to increase friction and to diminish bed-load transport rate. Once again, experimental and analytical results are found to be in good accord.

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