CHAPTER 213

Theoretical Study of the Wave Attenuation in a Channel with Roughened Sides

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Abstract

An eigenfunction expansion method is used to analyze the propagation of a plane wave train along a rectangular channel, the side walls which are provided with regularly spaced thin vertical strips. The presence of such strips may produce cross-channel seiching and energy dissipation. The method is extended to take into account the dissipation of energy at each pair of strips. Analytical solutions are obtained for the general case. Theoretical results of wave attenuation along the channel are compared to laboratory experiments, Battjes (1965).

1 INTRODUCTION

A harbor is a partially enclosed area connected to the sea by an opening. Sometimes the connection to the sea is through a channel of finite width and length. Entrance channels bounded by rubble-mound jetties are a common way to control wave propagation along the channel and to the harbor. Others, are channels with roughened sides, Battjes (1965) or with corrugated boundaries, Liu (1987). Further references can be found in Liu (1987).

Battjes (1965), did a semiempirical study of the attenuation of water waves in a rectangular channel, the side walls of which had been provided with

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regularly spaced roughness strips. Such strips were found to be highly effective wave dampers. However, the damping effectiveness of the strip sharply decreases when resonance occurs in the space between the strips.

In this paper the eigenfunction expansion method (hereafter called EFEM) is used to analyze the propagation of a plane wave train down a channel provided with thin vertical strips, regularly spaced. The wave solutions are expanded in eigenfunctions over the channel width. Dalrymple and Martin (1991), examined periodically spaced offshore breakwaters, matching eigenfunction expansions at the downwave and upwave side to show that a single incident wave train can generate directional wave trains downwave of the openings. Losada et al. (1992), used a two-dimensional (over the depth and across the channel cross-section) EFEM to analyze the generation and propagation of linear water waves down a wave flume, consisting of a wavemaker, an abrupt expansion, a breakwater and a fully absorbing ending wall. The EFEM method allowed the proper description of the wave motion, including cross-channel seiching as well as the prediction of the reflection and transmission which occurs at the channel junction and the porous structure. In order to evaluate the wave attenuation along the channel under seiching regime, (frictionless case), and energy loss regime, (friction case), an EFEM is used, providing matching conditions at each pair of strips which take into account a loss of kinetic energy.

FORMULATION OF THE PROBLEM 2

Figure 1 shows a rectangular channel.



Fig.1.- Definition Sketch

The channel is symmetric about its centerline with constant depth h, the side walls of which are provided with regularly spaced strips attached vertically to them and extended as flat, thin (theoretically infinitesimal) plates into the channel.

The cross sectional width of the channel is 2b. The gap between aligned pairs of strips is 2l and the distance between two consecutive pairs of strips is denoted by s. The channel has N pairs of strips at the centerline of the channel at the still water level, with z directed upwards and x pointing downwave. Region 1 is defined by $x \leq 0$. Between two consecutive strips a new region is defined. A total of N-1 regions are defined. Region N+1 is extended over $x \geq (N-1)s$.

The waves propagate down the channel in the positive direction. Then, as they encounter the first row of strips they get scattered by the roughness elements, partially reflected, and partially transmitted into Region 2. The transmitted waves in Region 2 diffract into the wider channel, dissipating energy through the formation of vortices, jets and eddy zones and reflecting from the side walls. They are then partially reflected and partially transmitted and dissipated after encountering the next row of strips. The transmitted and reflected waves are subsequently scattered and dissipated back and forth between the succesive rows of strips until they encounter Region N+1 where they are partially reflected, dissipated and transmitted into the leeward semiinfinite fluid region. As the waves propagate down the channel, they decrease in magnitude.

For an incompressible fluid and irrotational motion, the wave field outside the strip regions can be specified by the velocity potentials: ϕ_1 in Region 1 and ϕ_{N+1} in the leaward region. The linear boundary value problem for water of constant depth, h in a channel of width 2b, is well known. In the regions enclosed by strips it is assumed that the wave field is also describable by a velocity potential, ϕ_n , with $2 \leq n \leq N$. Since the solution in the adjacent regions must be continuous at each interface, continuity of mass flux and pressure must be required over the water column and across the channel width. Then, following Mei et al. (1974), it is possible to incorporate to the pressure matching condition a head drop consisting of a loss of kinetic energy due to flow separation and an apparent inertia.

The boundary value problem can be completely solved if the potential, $\phi_j(x, y, z, t)$, in the *jth* region, is known for j = 1, 2, ..., N+1. For each constant depth region, the Laplace's equation and the non-flow boundary conditions at the bottom and at the vertical channel boundaries are assumed to hold. The potentials in each region can be separated as:

$$\Phi_j(x,y,z,t) = \Re[\phi(x,y)I_j(z)e^{i\sigma t}]$$
(2.1)

where

$$I_{j} = \frac{ig\cosh k(z+h)}{\sigma \cosh kh}$$
(2.2)

where g is the gravitational constant, T is the wave period, t stands for time and $\sigma = 2\pi/T$, is the wave frequency. The wave number in each region, k_j , satisfies, for any j = 1, 2, ..., N + 1, the linear dispersion relationship

$$\Gamma_j = \frac{\sigma^2 h}{g} = k_j h \tanh k_j h \tag{2.3}$$

Because of the constant depth in all regions, eq. (2.3) has real roots k_{j1} , where $k_{j1} \ge 0$ and has an infinite number of purely imaginary roots, k_{jm} , with $m \ge 1$.

The potentials ϕ_j must solve the following problem

$$rac{\partial^2 \phi_j}{\partial x^2} + rac{\partial^2 \phi_j}{\partial y^2} + k_j^2 \phi_j = 0 \qquad \qquad -h \le z \le 0 \qquad (2.4)$$

$$\frac{\partial \phi_j}{\partial z} = 0 \qquad \qquad z = -h \qquad (2.5)$$

$$\frac{\partial \phi_j}{\partial z} - \frac{\Gamma_j}{h} \phi_j = 0 \qquad \qquad z = 0 \qquad (2.6)$$

Every potential has to satisfy a non-flow condition across the boundaries given by

$$rac{\partial \phi_j}{\partial y} = 0$$
 $y = \pm b$ (2.7)

Finally, it is assumed that the downwave end of the channel, Region N+1, is fully absorbent. To take into account this absorbing character, it is enough to specify a radiation condition, requiring that the potential in that region is a downstream progressive wave.

2.1 **Matching Conditions**

Since the solution in adjacent regions must be continuous at each interface, continuity of mass flux and pressure must be required over the water column and across the channel width.

Frictionless Case

At the strips, the matching conditions for the frictionless case are

$(\phi_j)_x = (\phi_{j+1})_x = 0$	$at_{-}x = (j-1)s_{-}$	and $ l \leq y \leq b$	(2.8)
$(\phi_j)_x = (\phi_{j+1})_x$	$at \ x = (j-1)s$	and $\mid y \mid \leq l$	(2.9)
$\phi_i - \phi_{i+1} = 0$	at $x = (j-1)s$	and $ y \leq l$	(2.10)

for
$$j = 1, 2, ..., N + 1$$

which guarantee the non-flow condition through the strips and the continuity of mass flux and pressure in the gap.

Friction Case

Based on a model of quadratic loss and scattering of long waves Mei, Liu and Ippen, (1974), Losada (1991), proposed a similar model to apply for the case of wave scattering by thin vertical barriers. In this model the continuity of pressure at the interface is obtained based on the Bernouilli equation. Thus, the analytical solution satisfies the following matching conditions (2.8) and (2.9), and a new condition given by,

$$rac{i\sigma}{g}(\phi_j-\phi_{j+1}) = rac{f}{2g}(\phi_{j+1})_x \mid (\phi_{j+1})_x \mid + rac{L}{g}(\phi_{j+1})_{xt}, x = (j-1)s \ and \ \mid y \mid \leq l \ (2.11)$$

for j = 1, 2, ..., N + 1

where, f and L are two empirical coefficients related to the loss of kinetic energy and apparent inertia respectively at each pair of strips. Following Mei (1974), we will use the following expression for f: $f = [(2b/c.2l) - 1]^2$, with $c = 0.62 + 0.38(l/b)^3$.

The good agreement between the analytical and experimental solutions for the case of thin vertical plates, suggests the extension of the model to the multistrips case. Losada (1991), showed that the proposed approximation gives the best results for the intermediate and shallow water waves. Furthermore, he showed that for relatively long waves, the apparent inertia term is not too important in comparison to the friction term. In this paper only the friction term will be considered. Moreover, eq. (2.11) will be linearized, therefore

$$\phi_j - \phi_{j+1} = \frac{-ig}{\sigma} C_{ej}(\phi_{j+1})_x$$
 (2.12)

where C_{ei} is an equivalent friction term. Following Mei (1974),

$$C_{ej} = \frac{f}{2g} \frac{8}{3\pi} \left| \frac{\partial \phi_j}{\partial x} \right|$$
(2.13)

which depends on the potential value at each gap and it is not known before the solution is completed. To solve the problem an iterative procedure has to be used.

3 FULL SOLUTION

The potentials ϕ_j for the frictionless and friction cases in each region satisfy the same boundary value problem, and therefore have the same analytical expression. They differ only in the matching conditions, that is, in the numerical value of the coefficients. Because of the constant water depth along the channel the potential $\phi_j(x, y)$ at each region is

$$\phi_1(x,y) = \sum_{n=0} [A_n^{(1)} e^{-iq_n x} + B_n^{(1)} e^{iq_n x}] \cos(n\lambda y)$$
(3.1)

$$\phi_j(x,y) = \sum_{n=0}^{\infty} [A_n^{(j)} e^{-iq_n(x-(j-2)s)} + B_n^{(j)} e^{iq_n(x-(j-1)s)}] \cos(n\lambda y) \quad (3.2)$$

$$\phi_{N+1}(x,y) = \sum_{n=0}^{\infty} [A_n^{(N+1)} e^{-iq_n(x-(N-1)s)}] \cos(n\lambda y)$$
(3.3)

where $q_n = \sqrt{k^2 - (n\lambda)^2}$, $\lambda = \pi/b$.

The inclusion of the Fourier terms, $cos(n\lambda y)$, $n = 1, 2, ..., \infty$ for all velocity potentials assures no flow through the channel walls. There is an infinite number of eigenvalues, $\lambda_n = n\lambda$. The corresponding eigenfunctions form a complete orthogonal set in the domain $(-b \leq b)$.

The wave field at each region consists of the incident plane wave train propagating down the channel and the reflected plane wave trains, which are independent of the y coordinate (n = 0) plus progressive and evanescent standing waves travelling in the negative x direction. The evanescent modes occur when $\lambda_n \geq k$, leading to a dampened motion in the x direction. The progressive standing wave modes consist of two intersecting wave trains travelling at

$$\theta = \cos^{-1}(\frac{\sqrt{k^2 - (n\lambda)^2}}{k^2})$$
 (3.4)

to the x axis.

Note that eqs. (3.1) to (3.3) do not include a family of vertical evanescent modes, which has to be included to satisfy the matching conditions at each interface. However, for intermediate and shallow water waves, the relative error between the plane wave approximation and the full solution is small.

The unknowns of the problem are $A_m^{(j)}$ and $B_m^{(j)}$, with m = 0, 1, 2, ..., n and j = 1, 2, ..., N + 1. The incident plane wave train is defined by $A_o^{(1)} = 1$ and $A_m^{(1)} = 0$, with $m \ge 1$. In order to satisfy the radiation condition in the leeward region, $B_m^{(N+1)} = 0$, with $m \ge 0$.

Substituting the expression of the potentials into the mass flux condition, eq. (2.9), an expression from $B_m^{(j)}$ as a function of $A_m^{(j)}$ can be found.

Next, the two remaining matching conditions are to be prescribed on the velocity eq. (2.8) (non-flow through the strips) and on the pressure eq.(2.10) or on the momentum eq. (2.11). Here, a mixed boundary condition must be prescribed (Dalrymple and Martin, 1991, and Losada et al., 1991).

3.1 Dual Series

The two remaining matching conditions to be satisfied at the gap are known as dual series relations, (Sneddon, 1966). They have to be solved for the values of the coefficients $A_n^{(j)}$. The two conditions can be combined to make one mixed boundary condition. This condition is

$$G(y) = 0 \qquad at \quad 0 \leq |y| \leq b \qquad (3.5)$$

To determine the $A_n^{(j)}$ several techniques can be used, e.g. least squares method, which requires the value of

$$\int_{-b}^{b} |G(y)|^2 dy$$
 (3.6)

to be a minimum.

Minimizing this integral with respect to each of the $A_n^{(j)}$ leads to the following system of equations

$$\int_{-l}^{l} G^{*(2)}(y) \frac{\partial G^{(2)}(y)}{\partial A_{n}^{(j+1)}} + 2 \int_{l}^{b} G^{*(1)}(y) \frac{\partial G^{(1)}(y)}{\partial A_{n}^{(j+1)}} dy = 0 \qquad n = 0, 1, 2, ..., \infty$$

$$j = 1, 2, 3, ..., N \qquad (3.7)$$

where G^* is the complex conjugate of G and G^1 and G^2 are the matching conditions at the gap and at the strip respectively. Truncating eq. (3.7) to M terms and solving for the M * N values of $A_m^{(j)}$ simultaneously, a complex system of M * N * N matrix equations is obtained which can be solved with the IMSL routine, LEQT1C.

Dual Series for the Frictionless Case

Taking into account the non-flow condition at each pair of strips, eq. (2.8), and matching the pressure using eq. (2.10), we get the mixed matching condition $G(y) = G_1(y) + G_2(y)$, which for each j = 1, 2, 3, ..., N, is

$$\sum_{n=0}^{\infty} \{-4(e^{-\alpha i q_n s})^* I_{mn} A_n^{*(j)} + \{4I_{mn} + 2[iq_n(\delta e^{-2iq_n s} - 1)]^* [iq_m(\delta e^{-2iq_m s} - 1)] J_{mn}\} A_n^{*(j+1)} + 2\sum_{p=j+2}^{N_p} \delta(iq_n)^* (e^{-2iq_n s} - 1)^* (e^{-iq_n(p-(j+1))s})^* [iq_m(\delta e^{-2iq_m s} - 1)] J_{mn}\} A_n^{*(p)} - 2\delta(iq_n)^* (e^{-iq_n(N-j)s})^* [iq_m(\delta e^{-2iq_m s} - 1)] J_{mn}\} A_n^{*(N+1)} = 0$$
(3.8)

where $\delta = 0$, if j = N and $\delta = 1$ otherwise, and $I_{mn} = \int_{-l}^{l} \cos(n\lambda y) \cos(m\lambda y) dy$, and $J_{mn} = \int_{l}^{b} \cos(n\lambda y) \cos(m\lambda y) dy$.

Dual Series for the Frictional Case

The non-flow condition at each strip is the same as for the frictionless case. The continuity of pressure, eq. (2.10) can now be written as:

$$G^{(2)}(y) \equiv \sum_{n=0}^{\infty} \{2e^{-lpha i q_n s} A_n^{(j)} - [2 - C'_{ej} i q_n (\delta e^{-2i q_n s} - 1)] A_n^{(j+1)}$$

$$+\sum_{\substack{p=j+2\\e_j iq_n}}^{N} C'_{e_j} iq_n \delta(e^{-2iq_n s} - 1) e^{-iq_n (p-(j+1))s} A_n^{(p)} \\ -C'_{e_j} iq_n \delta e^{-iq_n (N-j)s} A_n^{(N+1)} \cos(n\lambda y) = 0$$
(3.9)

where $C'_{ej} = -(ig/\sigma)/C_{ej}$

Substituting eqs. (3.9) and the corresponding $G^{(1)}(y)$ into eq. (3.7) and truncating the series to M terms, we get a set of M*N*N matrix equations that can be easily solved.

4 THEORETICAL RESULTS

The reflection coefficient, C_r , is defined as the absolute value of the most progressive coefficient $B_o^{(1)}$ of the reflected potential in Region 1. Similarly, the transmission coefficient, C_t , is defined as the absolute value of the most progressive coefficient, $A_o^{(N+1)}$, of the transmitted potential in Region N + 1.

For the frictionless case, the propagation of a wave train down a channel in two separate cases, one using two pairs of strips and the second using ten pairs of strips, is examined for the following characteristics: T = 1.373s, h = 0.6m, 2b = 3.0m and 2l = 2.6m.

Figure 2 shows C_r versus ks where,k is the wavenumber. Resonant conditions occur at $ks = p\pi$, p = 1, 2, ... Further results have shown, that increasing the gap to 2l = 2.8m, the amplitude of the resonance is reduced.



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Note, that the resonant conditions are stronger for the channel with a larger number of strips.

Figure 3 shows the dependency on C_r on the relative gap width l/b. This time the separation of the strips is s = 0.4. A sharp increase in C_r occurs, by decreasing the relative gap from 70% to a 60%.



Fig.3.- Reflection Coefficient versus l/b (without friction) (T=1.373 s, 2.b=3.0 m, s=0.4 m, h=0.6 m, M=10)

Battjes, (1965) reported experimental values of the wave attenuation in a channel with roughened sides. The wave height was measured in eight equidistant points in each cross-section and the average of these heights was taken to represent the wave attenuation at each respective section.

Results without friction have shown that the analytical model underpredicts the wave attenuation measured in the experiments. Further, the amplitude attenuation depends linearly on x. A similar behavior is obtained if the friction coefficient is kept constant along the channel, as Battjes, (1965) suggested.

Figure 4 shows the evolution of the averaged wave height along a channel, including the friction effect, for the following case: h = 0.6m, T = 1.98s, 2l = 2.9m, s = .2m, N = 195 strips and M = 5.

This time the overall agreement between analytical and experimental results is good.

Because of time computation, theoretical results were stopped after 195 strips.



Fig.4.- Wave Height Evolution (T=1.98 s, 2.b=3.0 m, 2.l=2.9 m, s=0.2 m, h=0.6 m, N=195, M=5)

The wave height contourlines for, h = 0.6m, T = 1.373s, 2l = 2.6m, 2b = 3.0m, s = 0.67m, N = 20 and M = 25 are presented in Figure 5. In this case the wave height varies from the input wave height, 1m to 0.3m.



Fig.5.- Wave Height Contourlines

Further analysis has shown, for the same values of the parameters, that the wave height attenuation in a mid-channel section presents an oscillating behavior, due to the resonant conditions caused by the strips.

Finally, it could be also observed that there is also an oscillating behavior present, if we the wave height variations across the channel were analyzed in different sections.

5 CONCLUSIONS

Wave propagation in a channel with roughened sides has been studied, using an eigenfunction expansion method (EFEM).

This method is valid to reproduce the wave field in the channel, showing, as the most important characteristic, the wave attenuation with an oscillating behavior.

The model does also reproduce the resonant conditions and the crosschannel variations, due to the presence of the lateral walls and the thin strips.

Frictional effects have been considered using a semi-empirical approach based on Mei et al. (1974). This effect is included in the matching conditions corresponding to the momentum equation at each strip. By applying the EFEM, the local friction effect proposed by Mei et al. is extended to the whole channel width and depth.

Comparison of the analytical results with the experimental data by Battjes (1965), proves the validity of the method.

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