Abstract

The mechanism whereby sediment particles are suspended under broken waves in the surf zone is investigated through experimental data analysis and computational modelling. The concentration measurements referred to comprise continuous transmissometer and optical backscatter sensor (OBS) values recorded in the field as well as a large database of time-averaged recordings. The measurements analysis, which included a unique set of laboratory observations derived from a specially designed vertical mixing apparatus, clearly highlight the preeminent role of wave breaker turbulence in the suspension process. The wave period averaged turbulence structure is found to be well predicted by a two equation \((k, \varepsilon)\) turbulence model. With the parametrization of the bottom reference concentration using relevant breaker and bottom generated turbulence variables, a scalar extension of the \(k, \varepsilon\) model is effective in predicting published time-averaged suspended sediment distributions. Analytical relations founded on the assumption of a predominately diffusive turbulent transport regime furthermore display favourable predictive capabilities.

1. Introduction

Suspended sediment concentrations in the surf zone have been measured to be up to several orders of magnitude higher than is typical for an unbroken wave regime. This is particularly evident at the higher elevations above the bed, with suspended
sediment outside the surf-zone often restricted to the limited wave boundary layer.

In assessing the influence of wave breaking on the suspension process, Nielsen (1984) and Shibayama et al. (1986) have identified the action of the large scale vortices generated coincident with wave plunging. These essentially two-dimensional vortices are, however, restricted to a limited transition zone, shorewards of which the surf-zone is characterised by an inner bore region.

Experimental measurements through the inner region (Stive, 1980; Nadaoka and Kondoh, 1982) have revealed the presence of a turbulence generating layer in the vicinity of the surface bore from where the turbulence is transported downwards. Nadaoka et al. (1989) have furthermore identified an eddy structure whereby the surface bore is dominated by a nearly two-dimensional flow structure, bounded below by strongly three-dimensional obliquely descending eddies bringing highly intermittent turbulence to the bottom.

The dominant action of this turbulence for sediment suspension is verified by means of concentration measurements and visualisation studies made by Nadaoka et al. (1988) and Sato et al. (1990) in a wave flume. Computations of the turbulence structure using a 1-equation turbulence model have been shown by Deigaard et al. (1986) to result in favourable prediction of measured breaker zone suspended sediment concentrations. In the present exercise the suspension process is further investigated with reference to field and laboratory measurements as well as quantitative modelling.

2. Concentration Measurements

Continuous

Particular reference was made to a comprehensive set of continuous measurements of suspended sediment recorded by a vertical array of seven transmissometer units. These units were fixed to a frame together with a pressure transducer and time-averaged concentration suction sampler and deployed at an exposed coastal site (Coppoolse et al., 1992). A limiting factor is that transmissometer signals have a tendency to become saturated at elevated concentrations or if air-bubbles become entrained in the flow. However the analysis of a number of surf-zone time series, which also contain saturated signals, appears to demonstrate the importance of breaker turbulence for sediment suspension.

In a separate exercise, continuous measurements of suspended sediment were made by three optical backscatter sensors (OBS) deployed from a scaffold frame placed in the nearshore. The frame, which also had attached two electromagnetic current meters, a pressure transducer and wave staff, was within the surfzone except for the highest tidal conditions. It was found that elevated concentrations through the depth are not necessarily discreetly correlated with the largest wave heights and
velocities, but rather exhibit a sensitivity to a more intensive, but still intermittent, turbulence coincident with the wave groups.

A rather unique set of measurements were obtained from a laboratory experiment in which turbulence is generated from a vertically oscillating grid within a glass-walled tank. The apparatus is based on similar devices used for experiments on density gradient mixing due to diffused turbulence (E. and Hopfinger, 1986). The suspension of sand from a bed of sand at the base of the tank is recorded at various elevations using an optical backscatter sensor. Figure 1 a) is a typical time series of the measured concentrations recorded at 5 mm above the bottom. Despite a relatively constant time averaged concentration, the occurrence of intermittent suspension events due to the turbulence can be clearly seen. Figure 1 b) is a plot of time-mean concentrations evaluated at various elevations. The distribution exhibiting elevated concentrations throughout the entire depth is typical of suspended sediment measurements taken in the surf zone.

![Figure 1](image)

Figure 1  OBS concentrations as a) time series measured 5 mm from the bottom and b) time-averaged at various elevations

**Time averaged measurements**

Analyses of experimental turbulence and suspended sediment measurements (Stive, 1980, Nadaoka et al., 1988, Sato et al., 1990) have indicated that phase-averaged quantities display limited variation over time. This underlines the intermittent nature of the turbulence and suspension process and also confirms the validity of using a time-averaged modelling approach.

Time-averaged concentration profiles recorded seawards of the surfzone have been shown to be relatively well described by an exponential distribution over the depth of $C(z) = C_b e^{-z/\ell}$, where $\ell$ is an integral length shown by Nielsen (1984) to scale on bed ripple dimensions. Although there is some indication (Hardenberg et al., 1991) that the classical exponential relation describes surf-zone concentrations when averaged over relatively long time periods, it is inappropriate for representing the
strong vertical variability in mixing evident on the time scale of the incident waves. In the analysis of a comprehensive database of field and laboratory measurements of suspended sediment concentration profiles compiled by Van Rijn (1991) it is however assumed that the exponential distribution holds over a limited region near the bed. Hence for some 73 sets of experimental surf-zone measurements an exponential regression curve fitted through the lower three data points is extrapolated for the determination of a reference concentration $C_b$ at the bottom.

Previous attempts to parametrize this reference value have focused primarily on the role of turbulence generated by mean shear near the bottom as reflected in the Shields parameter $\theta = \tau/\rho(s-I)gd$. Because of the significance of near surface generated wave breaker turbulence, however, it is of fundamental importance to also incorporate this contribution in any parametrization exercise. The energy dissipation in a breaking wave may be modelled as for a periodic bore. Thus the equivalent energy available for sediment suspension is related to this dissipation ($D \propto H^3/hT$).

Complementing this parameter representing the injection of TKE into the flow is the length scale indicating the relative degree of depth penetration. It is proposed that an appropriate scale for this parameter would be the relative wave height ($H/h$). Combining these scales together with the Shields parameter the best fit regression curve against the database measurements is as shown in Fig. 2 and provides the following expression for predicting the bottom reference concentration:

$$C_b = \rho K^{-0.92} (H/h)^{3.32} (H^3/hT)^{-0.92} \theta^{0.37}$$

where $K = 1.51 \times 10^3 \text{ sm}^{-2}$ is a proportionality constant related to the energy dissipation term. In carrying out the exercise the relative importance of component terms such as the wave height was clearly evidenced.

![Figure 2](image_url)  
**Figure 2**  
Parametrization of bottom reference concentration $C_b$ by current and wave variables
WAVE BREAKER TURBULENCE

The resolution of length scale values (through transformation of the aforementioned exponential relation) for a number of breaker cases included in the Delft database showed interesting tendencies. On the whole magnitudes were relatively small in the immediate vicinity of the bottom, with in most cases a rapid increase to values of between 20 and 40% of the overall water depth at higher elevations. Such magnitudes are in accordance with quantities determined from experimental measurements by Black and Rosenberg (1991) as well as scales determined by George and Flick (private comm.) from field measurements of turbulence.

3. Computational Modelling

System of equations

The time-mean vertical distribution of suspended sediment is described by the time-independent form of the classical advection-diffusion equation:

\[ 0 = \frac{\partial}{\partial z} \left( D_c \frac{\partial C}{\partial z} + w_s C \right) \]  

(2)

where \( C \) is the sediment concentration, \( w_s \) is the sediment fall velocity and \( D_c \) is the eddy diffusivity which is assumed equivalent to the eddy viscosity \( \nu \), defining the turbulent momentum flux. Following the analysis of Prandtl-Kolmogorov the eddy viscosity may be related to the TKE density \( k \) \( (\overline{u_i u_j} = 2k) \) and its rate of dissipation \( \varepsilon \) such that \( D_c = \nu = c^" \frac{k^2}{\varepsilon} \), where \( c^" \) is an empirical constant. For a unidimensional flow the dimensionless form of the time-averaged transport equations for \( k \) and \( \varepsilon \) may be expressed in the following manner:

\[ 0 = \frac{\partial}{\partial z} \left( D_k \frac{\partial k}{\partial z} \right) + P_b + P + G - \varepsilon \]  

(3)

\[ 0 = \frac{\partial}{\partial z} \left( D_\varepsilon \frac{\partial \varepsilon}{\partial z} \right) + \left[ c_{1_\varepsilon} \left( P_b + P + G \left( 1 - c_3 \varepsilon \right) \right) - c_2 \varepsilon \right] \frac{\varepsilon}{k} \]  

(4)

where the above equations have been made dimensionless by characteristic flow parameters such as the mean water depth \( h \) and the wave celerity \( c (= \sqrt{g h}) \). The dimensionless diffusion coefficients are defined as:

\[ D_k = 1/Re + \nu / \sigma_k, \quad D_\varepsilon = 1/Re + \nu / \sigma_\varepsilon \]  

(5)

where the molecular viscosity term \( Re \), the flow Reynolds number, is negligible in comparison to the turbulence contribution.

The forcing functions are the production \( P_b \) of TKE due to wave breaker induced turbulence and \( P = \nu \overline{(\partial u / \partial z)^2} \) due to mean shear. The buoyancy term is expressed
as \( G = R_i D_c \partial C/\partial z \). This term, which includes a reference dimensionless Richardson number \( R_i = g \Delta \rho / \rho \ h/c^2 \), expresses the effect of turbulence damping due to the gradient in concentration. The choice of empirical constants, discussed in Mocke (1991), are close to classical values (Rodi, 1980) except for \( C_S = 0.3 \).

**Boundary conditions**

The main forcing function for the flow is the production of turbulence in the surface roller. This production is assumed to be imposed at the upper boundary of the flow regime, which is taken at the mean water level. Modelled according to the dissipation in a hydraulic jump the breaker production term may be expressed as:

\[
P_b = A_e \frac{gH^3}{4h^2} D' \tag{6}
\]

where \( A_e \) is a dimensionless dissipation factor expressing the difference between energy dissipation in a wave breaker and hydraulic jump. A series of experimental measurements made by Stive (1984) found this factor to be in the range 1.3~2.0, and together with other data, suggests a dependence of this factor on the wave breaker type. The dimensionless energy dissipation \( D' = h^2/h_c \) (Svendsen, 1984), with \( h_c \) the depth below the wave crest level respectively.

With the assumption of a zone characterised essentially by local equilibrium between diffusion and dissipation of turbulence, the TKE varies over depth as:

\[
k(z) = k_s \exp \left( \frac{(z-h)}{\ell_s} \right) \tag{7}
\]

where \( k_s \) and \( \ell_s \) are respectively the intensity and length scale of the turbulence at a reference surface level taken at the upper boundary \( z=h \). With the rate of dissipation \( \varepsilon \) related to \( k \) through the turbulent length scale \( e \propto k^{3/2}/\ell \), the vertical distribution of \( \varepsilon \) likewise follows an exponential decay:

\[
\varepsilon(z) = \varepsilon_s \exp \left( \frac{3}{2} (z-h)/\ell_s \right) \tag{8}
\]

Substituting the expressions for \( k \) and \( \varepsilon \) in the purely diffusive formulation of (3), the surface value \( \varepsilon_s \) may be related to \( k_s \) through the length scale \( \ell_s \):

\[
\varepsilon_s = \left[ \frac{3}{2} \frac{C_S}{\sigma_k} \right]^{1/2} \frac{k_s^{3/2}}{\ell_s} \tag{9}
\]

In terms of the previously discussed turbulence generating horizontal vortex in the surface roller and as suggested by Battjes (1975), \( \ell_s \) could be expected to scale on the wave height \( H \) \( \ell_s = A \ H \) where the chosen dimensionless length factor
$A_t = 1/3$ provides a length scale approximately equivalent to the wave trough amplitude $(h-d_e)$. It is also consistent with a dimensionless length scale $l/h = 0.2$, which is in close agreement with estimates made from undertow measurements as well as previously mentioned concentration and turbulence measurements.

At the surface boundary, the vertical turbulent and gravity flux of sediment is assumed to be in equilibrium:

$$D_e \frac{\partial C}{\partial z} + w_j C = 0$$  \hspace{1cm} (10)

The lower boundary condition is not applied at the actual bottom but rather at the outer limit of the viscous boundary layer. As detailed in Rodi (1980), the bottom boundary conditions for $k$ and $\varepsilon$ are:

$$k_b = \frac{u_*^2}{\sqrt{c_p}}$$

$$\varepsilon_b = \frac{u_*^3}{k z}$$  \hspace{1cm} (11)

where $u_*$ is the bottom shear velocity, $\kappa$ the constant of Von Karman (~0.4), $z$ the distance from the bed and $c_p$ the diffusive coefficient where production and dissipation are in local equilibrium (=0.09). A constant stress bottom boundary layer is also assumed for the quantification of TKE production due to mean shear. Assuming local equilibrium ($P/\varepsilon = 1$), this production may be approximated as:

$$P = \frac{u_*^3}{k z}$$  \hspace{1cm} (12)

For the determination of $u_*$, reference is made to the bottom roughness and the mean velocity $u$. Where we do not dispose of measurements the mean velocity is reliably estimated from experimental measurements (Stive, 1980) as $\bar{u} = 0.1c$. Although the bottom boundary turbulence source is normally greatly outweighed by the surface breaker contribution, it does act as a limiting condition both in the event of low intensity breaker turbulence as well as by implicitly suppressing the turbulence scales due to wall proximity effects.

The bottom boundary condition for the suspended sediment computations relies on the parametrization relation (1), which provides the bottom reference concentration $C_b$ for the relevant flow condition.

**Resolution procedure**

The set of equations are discretized in an implicit scheme using the finite difference method. A central differencing approach is used to construct a tri-diagonal matrix which can be solved analytically by the Thomas algorithm.
4. **Computational Results**

**Wave breaker turbulence**

Initial model simulations were carried out in the absence of the sediment phase in an effort to evaluate the effectiveness of the turbulence model for predicting the observed dynamic as well as measured turbulence parameters. The experimental arrangements and description of measurements are detailed in Stive (1980) and Stive and Wind (1982), Nadaoka and Kondoh (1982) and Hattori and Aono (1985).

In analyzing laser Doppler anemometer (LDA) measurements, Stive (1980) used an ensemble averaging method to separate the time varying wave motions from the purely turbulence contribution. In analogy with the turbulence characteristics of a plane wake, the turbulent kinetic energy under the breakers was computed from the two components of the turbulence fluctuations as \[ k = \frac{1.33}{2} (u'^2 + w'^2). \]

The measured wave parameters were exploited for the computation of the TKE breaker production rate using expression (6). This rate was adjusted by the measured dimensionless dissipation factor \( A_e \) to obtain the correct energy dissipation for determining the turbulence production forcing function.

Solving the set of equations (3) and (4), predictions of time-averaged kinetic energy are compared to measurements at different positions after breaking for Stive tests 1 and 2 (Figure 3). In general, comparisons were found to be slightly better with the spilling (test 1) rather than the plunging (test 2) wave case. Although the depth averaged value for the predictions compares favourably with that of the measurements, the simulations were found to display somewhat more variation over the vertical. This characteristic is possibly attributable to a small convective contribution present in the experimental case. However, any such comparisons should be made in consideration of the room for optimization of the model constants, the experimental separation technique employed, and the determination of \( k \) by plane wake analogy.

The significant influence of the technique used for separating wave and purely turbulent fluctuations is evident when considering the measurements of Nadaoka and Kondoh (1982). By confining turbulence to a cut-off frequency of 10 Hz, any components with frequencies lower than this value will be neglected. A consequent underestimation of actual values would appear to be evident from intercomparisons with predictions (Figure 4 a)) and the magnitude of Stive values. As is also shown in a purely numerical exercise a reduction of the breaker height by 50% results in favourable correspondence between predictions and measurements. The separation technique employed by Hattori and Aono (1985) is considered equivalent to that used by Stive (1980). Not surprisingly correspondence between predictions and measurements is favourable (Figure 4 b)).
Figure 3  Intercomparison of computed (—) TKE profile and measurements (o) of Stive (1980). Tests 1 and 2.

Figure 4  Intercomparison of computed (—) and measured (o) TKE profiles of a) Nadaoka and Kondoh (1982) and b) Hattori and Aono (1985).

The typical dissipation profile computed for the Stive measurements predicts a curve decaying from the surface in much the same manner as for the TKE. Lacking measurements of the dissipation profile, no comparisons with the experimental case may be made. However, the model was found to at least conceptually confirm the observation by Svendsen (1987) that only a relatively
small percentage of the energy lost in the breaker is dissipated below the wave trough level.

Penetrating downwards from the surface production source the computed length scale was found to display a near linear gradient. This is consistent with the integral length scale growth as measured under an oscillating grid by E and Hopfinger (1986). Approaching the bottom boundary the length scale reduces dramatically to attain a value determined by the imposed boundary conditions.

Due to dimensional considerations, the eddy viscosity profile displays similarities to that for $k$ and $\varepsilon$, indicating an appreciable variation over depth. Although not a directly measurable quantity, Stive and Wind (1986) estimated the eddy viscosity for test case 1 by considering similarity between the flow fields in breaking waves and wake flows. The resulting eddy viscosity is however considered averaged over the depth. The ensemble of computed profiles predicted depth-averaged values in close agreement with these estimations.

suspended sediment

For the computation of the suspended sediment profiles the turbulence equations (3) and (4) are extended by the scalar diffusion equation (2). Model comparisons are somewhat hindered however, by the fact that presently there exists no simultaneous measurements of suspended sediment and turbulence quantities. Furthermore, there is a lack of suspended sediment measurements that also include an accurate assessment of the cross-shore wave energy loss important for determining local dissipation.

Despite these limitations the model was applied to a number of cases where suspended sediment was measured under breaking waves. In these cases, which include both laboratory and field measurements, the given wave height was exploited for determination of the breaker energy production. For the sake of consistency between cases no adjustment of the bore approximation dissipation term is made (i.e. $A \varepsilon = J$). Where the sediment fall velocity of the suspended sediment is not provided, it is estimated from the sediment size ($D_{50}$) characteristics.

Although recognized as potentially significant, little attention has been given to incorporating the turbulence damping effect of a concentration gradient in previous attempts at modelling suspended sediment in the surf-zone. With the inclusion of the buoyancy term $G$ in the equations (3) and (4) however, this contribution can be assessed.

As is in fact shown in Figure 5a) where model simulation with and without the buoyancy term are compared with field measurements of Nielsen (1984), the turbulence damping effect reduces the amount of sediment in suspension. The damping effect on local turbulence is more directly illustrated in the trace of the
mixing length scale shown in Figure 5b), where some reduction in this scale due to the more predominant concentration gradient near the bottom is evident. Superimposed on the predictions are estimates for the relative length scale as estimated from the measurements. The relatively rapid increases in length scale before attaining a magnitude between 20% and 30% of the water depth is evident in both the measured and predicted values.

Figure 5  Predicted (a) concentration profile and (b) turbulent length scale with (−) and without (---) buoyancy effects (Van Rijn, 1991)

Figure 6  Comparisons between predictions (−) and experimental measurements (o) of suspended sediment as obtained from the Delft database (Van Rijn, 1991).
As may be observed from the examples of field and laboratory concentration measurements presented in Figure 6, predictions are generally in reasonable agreement with measurements. The sensitivity of the model to the wave height and bottom friction velocity and reference concentration was found to be important, with adjustments of less than 10% resulting in improved correlations in many cases.

**Analytical relations**

Assuming an exponential decay it is possible to plot profiles of $k$ and $\varepsilon$ knowing the surface boundary values of these parameters and the relevant turbulent length scale. Following substitution of expressions (7) and (8) into the Prandtl-Kolmogorov relation the vertical distribution of eddy viscosity may be approximated by the following expression:

$$\nu_t = \nu_s \exp \left( \frac{1}{2} \frac{(z-h)}{\ell} \right) \quad (14)$$

It is clear that an accurate means of evaluating the surface value $k_s$ would allow reasonable analytical predictions of the vertical profile for all the relevant turbulent parameters. From a joint analysis of $k_s$ (as determined from the computational model) versus the non-dimensionalised breaker production term $P_b^* = P_b \frac{h}{c^3}$ for the Stive (1980) measurements the following relation was derived.

$$\sqrt{k_s} = 0.124 u_* \exp \left( 128.2 \, P_b h c^{-3} \right) \quad (15)$$

Although representing only a limited database of measurements, the above relation provides an intuitively correct approximation of the magnitude of breaker induced TKE at the reference surface level. Exploiting the relation (11) and assuming $A_t = 1/3$, the approximated eddy viscosity profile may be computed over the depth (Fig 6(a)). As may be seen from Figure 6(b), the resulting eddy viscosity profile predicts a concentration distribution in good comparison with measurements over most of the depth.

![Figure 7](image-url) **Figure 7** Intercomparison of analytical (−) and modelled (・・・) (a) $\nu_t$ profiles and (b) concentration profiles (Nielsen case 32, Van Rijn, 1991).
5. **Conclusions**

Through qualitative observation and quantitative computation and modelling, the fundamental significance of wave breaker turbulence for sediment suspension is clearly established. Despite the better understanding of surf zone processes through the increased number of flow measurement and visualization studies carried out in recent years there has been limited progress in quantitative predictive modelling of these processes. In adapting a time-independent diffusive turbulence model to the problem of breaker turbulence, a number of significant flow parameters previously measured in wave flume experiments are satisfactorily predicted.

Measurements of suspended sediment have concentrated on the suspended mass with little detail being available concerning the wave and turbulence properties. However in applying the buoyancy extended model with the given wave heights and a parametrized bottom reference concentration measured suspended sediment profiles are well represented. For further model refinement purposes it would however be desirable to have both turbulence and suspended mass measurements for identical conditions.

With the assumption of a diffusive dominated flow, it was possible to develop analytical relations approximating turbulence profiles from their parameter surface value and an appropriate length scale. With a parametrization of the surface scale of TKE using the term for breaker turbulence production, wave bore characteristics may be used for predicting the vertical distribution of suspended sediment.

**Acknowledgements**

The authors thank M Stive for providing a comprehensive set of experimental data and L Engelbrecht for technical assistance.

**References**


