CHAPTER 174

Three-Mode Principal Component Analysis of Bathymetric Data, applied to "Playa de Castilla" (Huelva, Spain)

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Abstract

Three-Mode Principal Component Analysis is applied to bathymetric data from a beach nourishment at "Playa de Castilla", Huelva, Spain. This approach is used to separate the temporal and spatial variability of the beach shoreface. The method is shown to describe variations occurring in the cross-shore and in the alongshore direction as well as temporal variations. The results of the analysis show a clear seasonality in the shoreface variations, with bar-berm processes involved in the cross-shore direction and complex sand variations in the alongshore direction. These alongshore variations are induced by the nourished area which avoids the formation of a uniform bar along the beach resulting in a complex sediment redistribution. The results also show an erosion trend in "Playa de Castilla". This erosion, however, is not related to the "spreading out" losses at the nourished beach but with the background erosion.

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1 INTRODUCTION

The configuration of a sandy coast changes both in time and in space. Processes occurring in the nearshore ocean are extremely dynamic, involving the combined action of waves, currents, tides and sediment transport. Furthermore, their individual characteristics change with various scales in both space and time. Here, day-to-day hydrodynamic processes constantly adjust the beach bathymetry through on-offshore and longshore transport of sand due to the wave field, sediment supply and grain size, tide and wave induced currents and sea level.

The situation described above shows that the morphodynamics of nearshore systems are extremely complex and difficult to treat. One approach is the concept of equilibrium shoreface (equilibrium profile and equilibrium shoreline). This idea is based on the premise that the overall shape and morphology of the shoreface will be maintained with some consistency in response to the typical wave and current regime at a particular location. Once the equilibrium shape is determined, the temporal and spatial variations of the shoreface can be related to the variability of the coastal processes.

Different statistical approaches have been applied to beach profile data. Principal Component Analysis (PCA), also known as Empirical Orthogonal Function (EOF) technique, is an efficient method of objectively separating the spatial and temporal scales of variability of a beach. PCA is a technique of linear statistical predictors which represent a large number of data variables by a few spatial, $e_n(s)$, and temporal, $f_n(t)$, empirical orthogonal eigenfunctions which describe most of the variance of a data set $y(s,t)$ by:

$$y(s,t) = \sum_n f_n(t) e_n(s) c_n \tag{1.1}$$

where c_n is a normalizing factor. The eigenfunctions are ranked according to the percentage of the variance defined as the Mean Square Value (MSV) of the data they explain, so that the first eigenfunction explains most of the MSV of the data.

This technique has been previously applied to cross-shore beach profile data: Winant et al (1975), Aubrey (1979), Zarillo and Liu (1988), Medina et al. (1991), and to alongshore profiles data: Losada et al. (1990), Liang and Seymour (1991), who showed that the variations in the longshore direction are as complicated as those in the cross-shore direction. In all these works, only one spatial direction is taken into account (cross-shore or alongshore) when analyzing the temporal evolution of a bathymetric data set. The assumption accepted is that the sediment transport occurring in the nearshore region may

be divided into two independent components by direction: cross-shore and longshore. This analysis may be correct in some particular cases, but it is rendered inadequate when two dimensional movements of sand are expected (*e.g.*, response to coastal structures, beach nourishment...).

In the present study, the beach nourishment data set of "Playa de Castilla" (Huelva, Spain) is analyzed by means of the $three-modePCA$ method in which both cross-shore and alongshore variations and interactions are retained as well as time. The method is used to expand the data in the form:

$$y(x,y,t) = \sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{u}[e_p(x)f_q(y)g_r(t)c_{pqr}] \tag{1.2}$$

2 THE 3-WAY PCA MODEL

Recently, some techniques have been developed to obtain direct solutions for $three-way$ data sets. These dimensions are often referred to as *modes* and the technique is generally referred to as $three-mode$ or $three-wayPCA$. Procedures of this sort were first proposed by Tucker (1966), and extended by Kroonenberg and DeLeeuw (1980) and TenBerge et al. (1987).

If only some eigenfunctions (e.g. k) are used to represent the data, equation 1.1 may be rewritten in matrix form as:

$$Y = ECF' \tag{2.1}$$

where Y is (nxp), E is (nxk), C is diagonal (kxk), F is (pxk) and ()' denotes transpose operator.

If the data array is augmented to include a third dimension, equation 2.1 will also be augmented by the inclusion of an extra term. In matrix form:

$$Y = EC(F' \bigotimes G') \tag{2.2}$$

where $\otimes$ denotes a direct product or Kronecker matrix, E is (nxk), G is (pxk), and F is (rxk). Since this is a matrix equation, both Y and C have to be restated as two-dimensional arrays, Y being $(nxpr)$ and C being $(kxkk)$. E, F and G are columnwise orthonormal matrices and have the same interpretation as the two-mode eigenvectors. However, C, which is now called the *core* matrix, is no longer a diagonal matrix of eigenvalues. One could conceive of

the *core* matrix as describing the basic relations that exist between the various collections of variables, Kroonenberg and DeLeeuw (1980).

The solution to equation 2.2 is based on the observation that the optimal C matrix can be expressed uniquely and explicitly in terms of the data and the component matrices for the three modes. The latter component matrices are optimized by an alternating least squares (ALS). A detailed description of the solution can be found in the paper by Kroonenberg and DeLeeuw (1980).

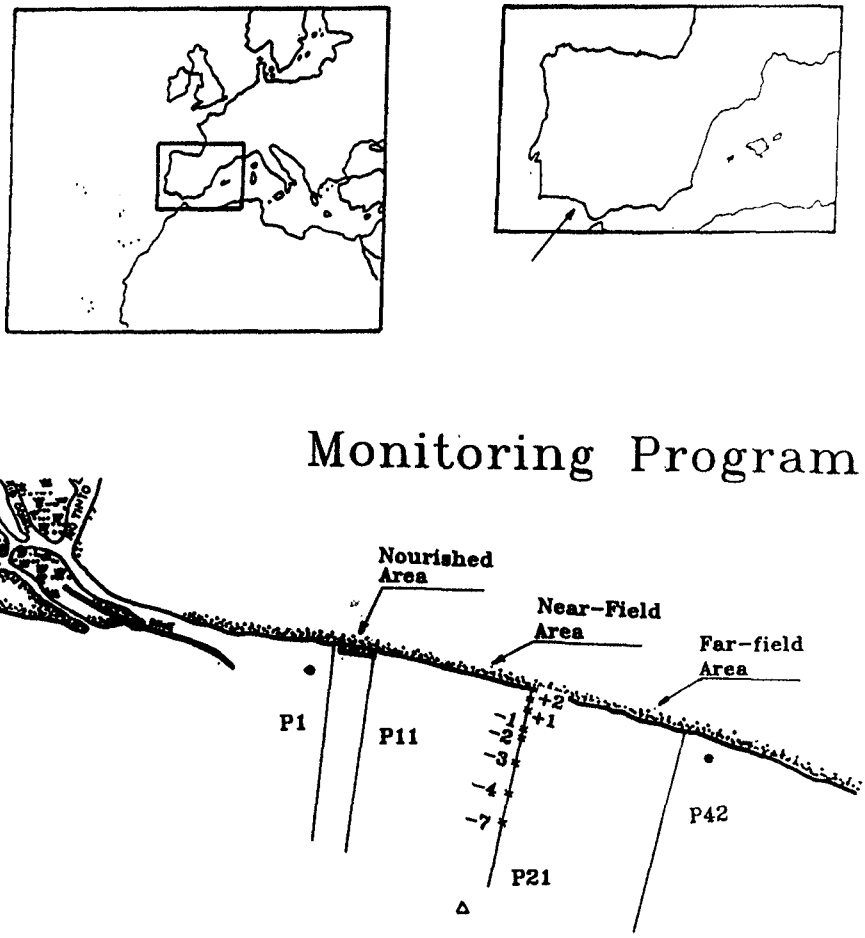

Figure 1: Location Map

3 STUDY SITE AND BATHYMETRIC DATA

The study area is located on the Southwest Coast of Spain (province of Huelva), between the Guadiana and Guadalquivir rivers (fig. 1). "Playa de Castilla" is a sandy beach which extends over 25 km between Mazagón and Matalascañas. Landward the beach is bounded by a rock-sand cliff wich degrades into fine sand under wave action. The coastline at "Playa de Castilla" has been receding at a rate of 1.5 m/yr during the last 30 years (Fernández et al., 1990). The recession is due to two different reasons: first, the littoral drift from West to East, of about 390,000 m^3/yr, and second, the reduction in the volume of sand transported by the rivers to the coast mainly caused by human construction.

An artificial nourishment of the beach of more than 1.500.000 m^3 of sand was carried out in 1989. The total volume of sand was pumped to the updrift edge of the beach forming a protruding area about 2 km long and 115 m wide. The borrowed sand was coarser than the native sand, being $D_{50} = 0.63mm$ and $D_{50} = 0.3mm$ for the borrowed sand and the native material respectively.

In order to study the beach nourishment evolution through time and space, a field measurement program was carried out during the period 1989-1992. The program included wave measurements, sand samples and bathymetry surveys. Bathymetric data were acquired bimonthly from 42 shore-normal profiles located between Mazagón and Matalascañas. Alongshore spacing of the profiles was approximately 500 meters and each profile was surveyed from the beach dune area, seaward to a depth of about 10 metres.

4 RESULTS OF ANALYSIS

The "Playa de Castilla" data described above are used for the $3 - way PCA$ analysis. Of the 42 alongshore profiles, only 20 are examined. The profiles include the nourished part of the beach and 7 km down drift. For the cross-shore profiles, only the nearshore zone from the berm crest seaward to a depth of 4.0 m is selected for its variability. Within this area, very pronounced variations can be found both in the cross-shore and longshore direction. Notice that the transect spacing (500 m) was designed to resolve the long-term "spreading out" losses of the nourishment, but is not adequate to resolve the spacing of rhythmic topography, which can be spaced at several hundred meters or less.

The matrices E, F and G in equation 2.2 are columnwise orthonormal. In other words, the components have length one. In this case, the importance of data is directly reflected by the entries of the *core* matrix, C, and not by the eigenvectors. Bartussek (1973) suggested scaling the orthonormal eigenvectors

of a $3-way PCA$ analogously to the procedure often encountered in standard PCA. One advantage of this scaling is that the scaled eigenvectors obtained are comparable within a mode and over modes. When scaling the eigenvectors (no longer orthonormals but orthogonals) the *core* matrix must also be scaled to leave the model invariant.

In figures 2a, b, c the first three cross-shore, alongshore and temporal eigenvectors are shown. The corresponding Bartussek *core* matrix values and the percentage of variation explained are given in Table 1. Notice that the first eigenvector of the three modes accounts for the 89.36%. This result is not surprising since we are dealing with raw uncentered data and, consequently, the centroid, defined as the mean of the variables, can explain most of the data. For that reason, the first eigenvectors are often highly correlated with mean vectors. Thus, the combination of the first eigenvectors gives a representation of the mean situation or mean bathymetry. Let us examine the meaning of the combinations of other eigenvectors

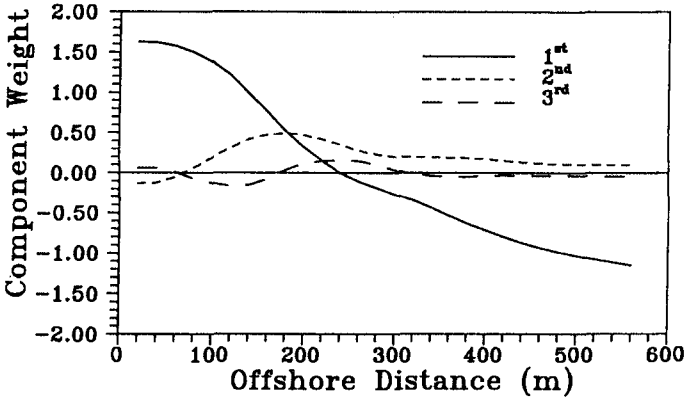

Figure 2a: Cross-shore Eigenvectors

Observing Figure 2a, it is clear that the second cross-shore eigenvector is very important in the upper part of the profile and it will play an important role in determining the upper profile slope. The third cross-shore eigenvector shows the typical S-shape of a berm-bar variability also encountered in standard PCA analysis. In Figure 2b, the nourished beach can easily be observed in the second alongshore eigenvector. The third alongshore eigenvector shows a small value with almost zero mean. In Figure 2c, the second temporal

Table 1: Frontal Planes of Core Matrix

Fontal Plane Time = 1
Down: Cross-shore. Across: Alongshore

Core Matrix		
1.981	0.007	0.003
-0.006	3.834	0.024
0.023	1.017	6.946

Explained Variation		
89.36	0.00	0.00
0.00	4.09	0.00
0.00	0.03	0.26

Fontal Plane Time = 2
Down: Cross-shore. Across: Alongshore

Core Matrix		
0.022	0.944	0.559
-3.659	-0.475	0.837
2.112	-18.364	-25.445

Explained Variation		
0.00	0.08	0.01
1.73	0.00	0.00
0.06	0.24	0.08

Fontal Plane Time = 3
Down: Cross-shore. Across: Alongshore

Core Matrix		
0.130	-0.474	2.698
0.439	-9.169	34.651
0.115	-6.520	-20.721

Explained Variation		
0.01	0.01	0.03
0.01	0.16	0.41
0.00	0.01	0.02

eigenvector shows a seasonal dependence. The third eigenvector shows a net trend almost linear from negative values to positive ones. The combination of these eigenvectors explains most of the variability of the evolution of the beach (see Table 1). In order to better interpretate the variability they account for, it is useful to examine the profile or the bathymetry that is obtained by the product of one alongshore eigenvector with one cross-shore eigenvector, and use the corresponding temporal function to determine how the obtained profile or bathymetry evolves in time.

The following pair of eigenvectors is analyzed: e_1g_1 and e_2g_2 for time = 1; e_2g_1 and e_3g_2 for time = 2; e_2g_2 and e_2g_3 for time = 3; where e denotes cross-shore eigenvector, g denotes alongshore eigenvector and the subscript denotes the order of the eigenvector. These pairs account for the 96% of the total variance of the data (see Table 1).

- Time = 1, pair $e_1 g_1$ (Fig. 3a). As it was previously anticipated, the product of all the first eigenvectors represents a mean bathymetry. This mean bathymetry shows a uniform beach profile with minor changes in the alongshore direction.

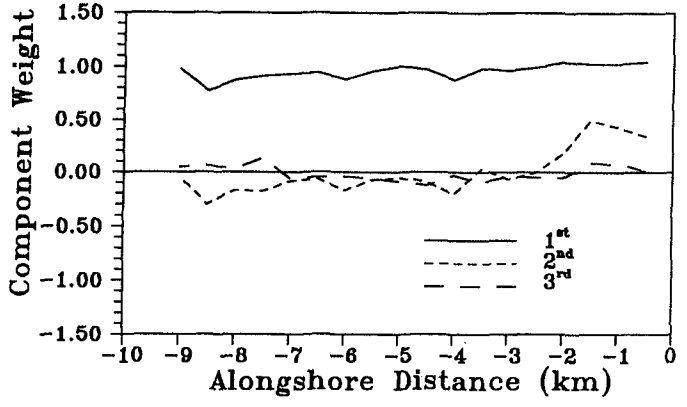

Figure 2b: Alongshore Eigenvectors

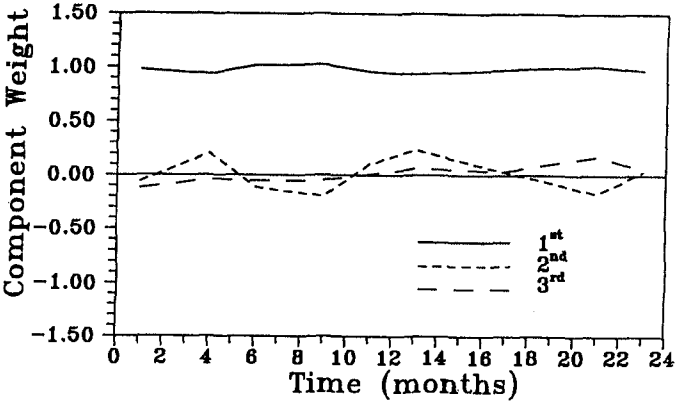

Figure 2c: Temporal Eigenvectors

- Time = 1, pair $e_2 g_2$. This pair is affected by the first temporal eigenfunction (Fig. 2c) which is almost constant in time and, consequently, this pair is

part of the mean bathymetry. The complexity of the bathymetry makes it impossible to achieve a solution just by the product of one cross- shore function and one alongshore function, thus this new pair must be added to the pair e_1g_1 in order to obtain a better representation of the mean situation. Notice that this new pair modifies the upper part of the profile with minor changes in the offshore zone. Further, the modification at the upper shoreface has a different sign at the nourished beach and in the rest of the study area. The final mean bathymetry, achieved by the sumation of the pairs e_1g_1 and e_2g_2 has a much steeper profile at the nourished beach than the native beach (Fig. 3b).

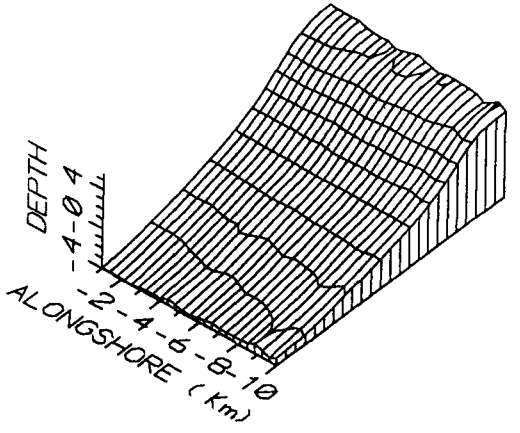

Figure 3a: Mean bathymetry based on e_1g_1

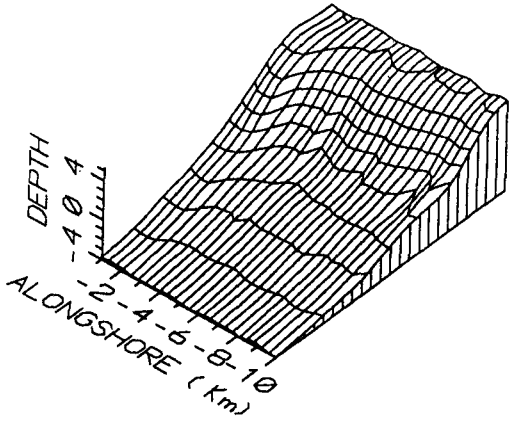

Figure 3b: Mean bathymetry based on e_1g_1 and e_2g_2

- Time = 2, pair e_2g_1. The most important characteristic of the bathymetries associated with the second temporal eigenfunction is the seasonality. The bathymetry achieved by the product of e_2g_1, for instance, must be added or substracted to the mean bathymetry depending on the season (see Fig. 2c). In particular, pair e_2g_1 shows a seasonal variation of the shoreface slope and of the amount of sand in the offshore part of the profile. These changes are almost uniform in the alongshore direction and are related to the erosion of the cliff.

- Time = 2, pair e_3g_2. As previously stated when dealing with the mean bathymetry, the seasonal changes are too complex to be represented by just one product. Consequently, if we want to obtain more information about the seasonal changes, the next pair to be added is e_3g_2. When combining these two eigenvectors, a shift in the location and magnitude of the bar is achived. The complete picture of the seasonal variability can be found by adding pairs e_2g_1 and e_3g_2. When adding those pairs, a clear bar is found in the native beach during the summer period, figure 4a. In the nourished area, however, no bar is formed, as can be observed in figure 4b.

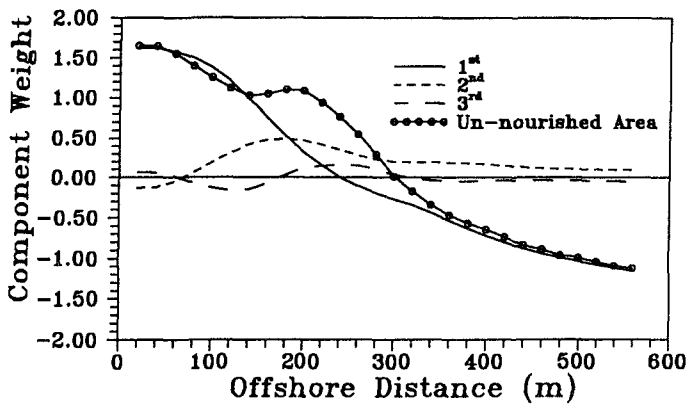

Figure 4a: Seasonal variability. Native area

- Time = 3, pairs e_2g_2 and e_3g_2. Only 0.66% of the total variance can be explained by the third temporal eigenvector. However, this variability is very important since it is associated with a net trend in the data. In the native beach the eigenvectors show a net erosion of the whole profile, see figure 5a. In the nourished area almost no changes have occurred during the period of study, see figure 5b.

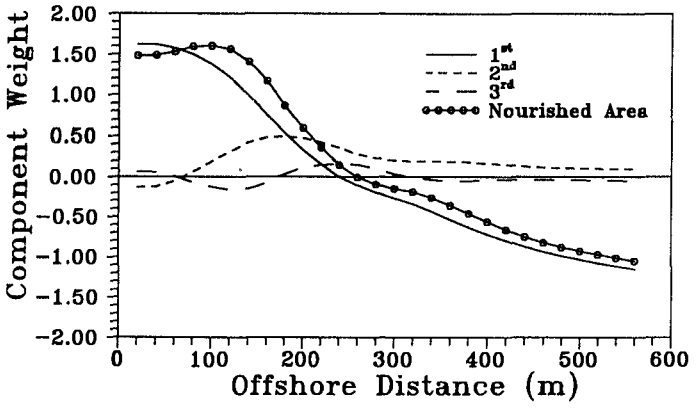

Figure 4b: Seasonal variability. Nourished area

5 DISCUSSION

The transect spacing (500 m) and the survey frequency (bimonthly) were designed to resolve the long-term and long scale evolution of the beach nourishment. The results of the analysis show, however, that there is little evolution but significant seasonal variability.

The seasonal changes described by the $3-wayPCA$ method are highly correlated with the seasonal variations of incident wave energy. It is well-known that there is an onshore-offshore movement of nearshore sediments in response to changing wave energy that builds bars and berms in a beach profile. In "Playa de Castilla" there is also a complex sand movement in the longshore direction induced by the protruding nourished area which was built with a coarser grain. The nourished area avoids the formation of a uniform bar along the beach resulting in a complex sediment redistribution. With the alongshore spacing of the transects used in the present study, it is not possible to resolve wave-like sand motions propagating in the longshore direction. This kind of motions have been cited by some authors, but largely ignored in literature. Further studies of these motions should be carried out by means of the 3-way PCA.

The evolution of "Playa de Castilla", described by the third temporal eigenvector and the combinations of pairs e_2g_2 and e_3g_2, shows a net erosion of the beach as reported by (Fernández et al., 1990). However, there is no evidence of "spreading out" losses in the nourished beach. It seems that the effect of using material which is substantially coarser than the native material armors the beach in the nourishment area thereby resulting in less transport from the nourished area.

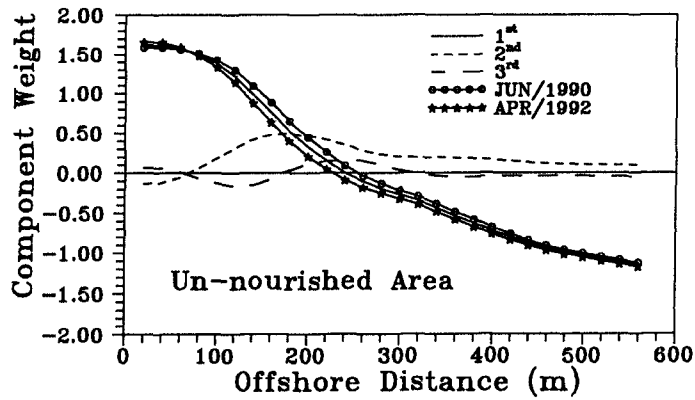

Figure 5a: Net trend. Native area

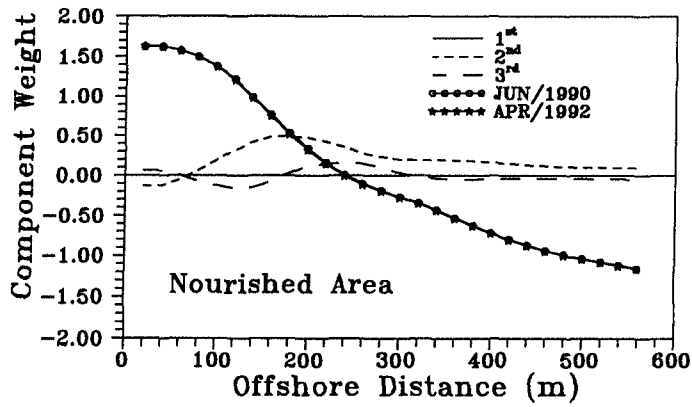

Figure 5b: Net trend. Nourished area

6 CONCLUSIONS

Three-Mode Principal Component Analysis has been applied to bathymetric data from a beach nourishment at "Playa de Castilla", Huelva, Spain. The method has been shown to be of great value in analyzing the variations of the shoreface during a two year period. The ability of the method to jointly analyze the cross-shore, alongshore and temporal variations and to separate these variations into orthogonal eigenvectors allow a better understanding of the processes involved. The results of the analysis show that most of the sand movement in "Playa de Castilla" has a seasonal dependency with involved bar-berm processes in the cross-shore direction and a complex sand variability in the alongshore direction induced by the protruding nourished area. The method has also shown an erosion trend in the beach. This erosion, however, is not related to the "spreading out" losses at the nourished beach but background erosion.

7 ACKNOWLEDGEMENTS

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