CHAPTER 161

WAVE-INDUCED POREWATER PRESSURE AND SEABED STABILITY

A. Hattori[1], T. Sakai[2], M. ASCE and K. Hatanaka[3]

ABSTRACT

The boundary layer approximation solution by Mei and Foda(1981) is applied to and its applicability is confirmed for the porewater pressure variation in a seabed under a breaking wave in a surf zone . Then the approximate solution is modified to take into account effects of the wave-induced bottom shear which is not negligible in the surf zone. This modified solution is applied to estimate the terms in the right hand side of the momentum equations for the solid skeleton under the breaking wave. The instability proposed by Madsen(1974) is discussed based on the estimated results. It is suggested that the instability just after the wave crest passing is more likely to occur than the instability just before the crest passing. Just after the crest passing, the horizontal gradient of the porewater pressure is large, while the vertical effective stress is small.

INTRODUCTION

There are two kinds of the porewater pressure response to waves. One is the cyclic excess porewater pressure variation. Another is the mean excess porewater pressure buildup. Earthquake induces the mean excess porewater pressure buildup. Here the cyclic porewater pressure variation is treated.

Under the wave trough, the porewater pressure does not decrease so much as the bottom wave pressure(**Fig.1**). The porewater pressure bears more load than the vertical effective stress on the solid skeleton

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1 Engineer, Engr. & Design Div., Civil Engr. Headquarters, Hazama Corporation, Minato-Ku, Tokyo, 107, Japan.
2 Prof., Dept. of Civil Engr., Kyoto Univ., Sakyo-Ku, Kyoto, 606, Japan.
3 Engineer, Construction Div., West Japan Railway Co., Kita-Ku, Osaka, 530, Japan.

bears. In some cases the vertical effective stress becomes zero near the bed surface. Here we call this "momentary liquefaction".

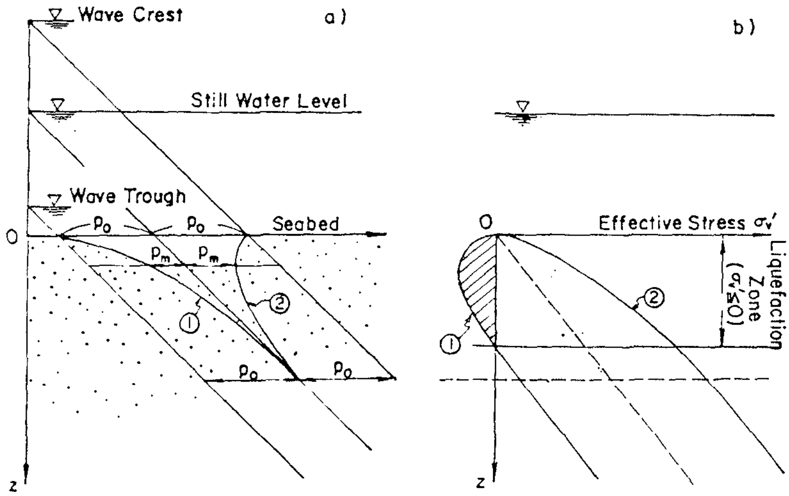

Fig.1 Momentary liquefaction of seabed due to waves (Zen et al., 1987)

The depth of this momentary liquefaction can be calculated by using a theory of the transient porewater pressure variation due to waves. **Fig.2**(Sakai et al., 1992) shows the result by using the boundary layer approximation solution of Mei and Foda(1981). This approximate solution is applicable to a wide range of the wave and soil conditions. The uncoupled analysis proposed by Finn et al.(1983) is applicable only to soft and coarse sand case.

In this figure there are many parameters. k is the permeability coefficient. G is the shear modulus of the solid skeleton. β is the effective bulk modulus of the porewater. It is related to the degree of saturation of porewater S. H is the wave height. z_L is the depth of the momentary liquefaction.

ρ_w is the density of the water. g is the gravity. T is the wave period. h is the water depth. γ' is the submerged unit weight of the solid skeleton. n and ν are the porosity and Poisson's ratio of the solid skeleton.

When the bed material becomes fine, the value of k, so that, the value of the ordinate $kG/\rho_w g^2 Th$ becomes small. Also, when the porewater becomes soft with increasing amount of gas, the value of β

becomes small, so that , the value of the abscissa G/β becomes large. Therefore the finer the bed material and larger the amount of gas in the porewater, the deeper the depth of the momentary liquefaction.

In this analysis, however, the sinusoidal waves were assumed. In surf zone, the wave profile is not sinusoidal but asymmetric. The bottom wave pressure has also an asymmetric time profile.

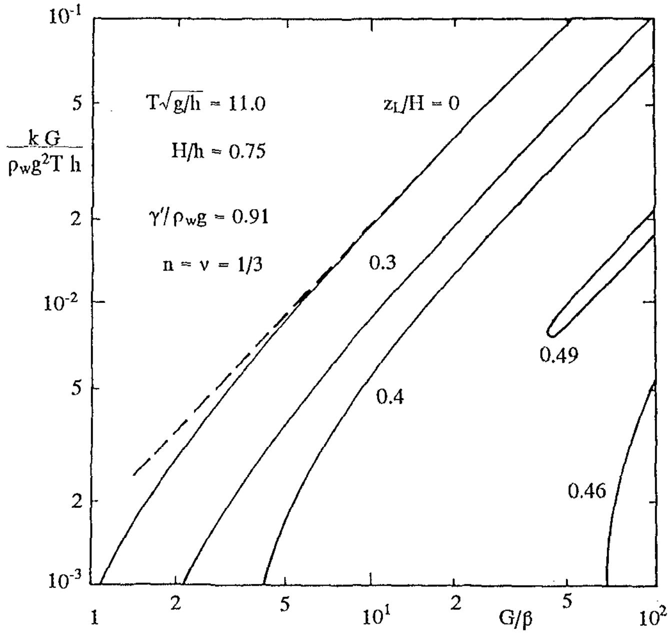

Fig.2 Momentary liquefaction depth in surf zone (Sakai et al., 1992)

APPLICABILITY OF BOUNDARY LAYER SOLUTION FOR ASYMMETRIC WAVES

Here the applicability of the boundary layer approximation solution for asymmetric waves in surf zone is examined.

Zen et al.(1989) measured the porewater pressures under waves in a surf zone at a Japanese Pacific Ocean coast. The measurement was done at the head of an observation pier of the Port and Harbour Res. Inst. of Japanese Ministry of Transport. The water depth h was about 4m(**Fig.3**). The 50% sand grain size was 0.16mm.

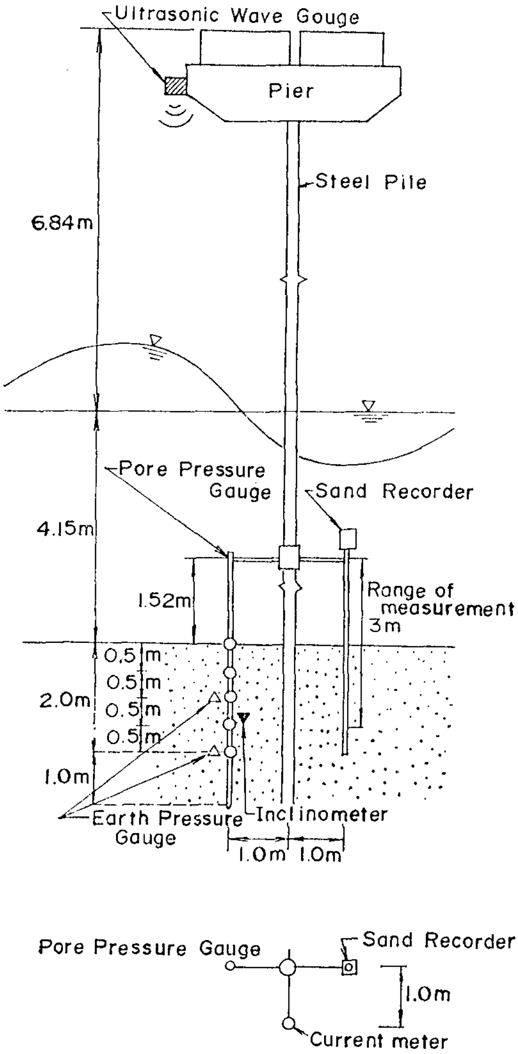

Fig.3 Measuring apparatus of porewater pressures (Zen et al., 1989)

Fig. 4 is one of the records(Fig.19(b) in their paper). The second figure shows the bottom wave pressure. The lowest figure shows the water level variation. The three figures from the third to fifth show the porewater pressures at three levels near the bed surface. The wave No.7 was selected as a typical surf zone wave. The wave period T

was 14.7sec, and the wave height H was 1.9m.

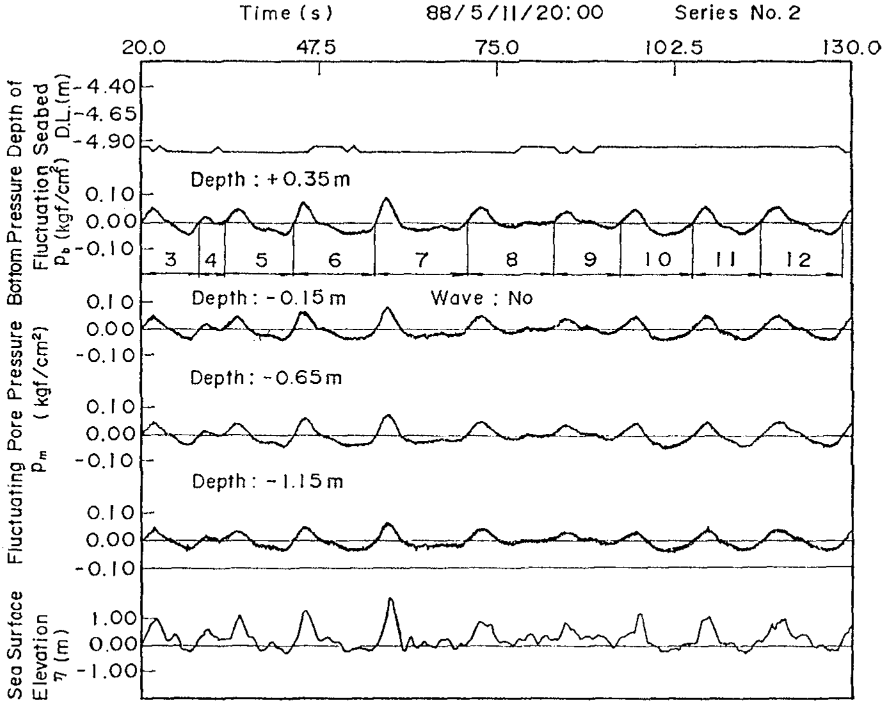

Fig.4 Example of record of bottom wave pressure and porewater pressures(Zen et al., 1989)

In the boundary layer approximation solution, when the bottom wave pressure p_b is given by the small amplitude wave theory as

$$p_b = p_0 \cos(\lambda x - \omega t) = \frac{1}{2}\frac{\rho_w g H}{\cosh(\lambda h)} \cos(\lambda x - \omega t) \ , \qquad (1)$$

the porewater pressure variation p' is given by Eq.(2).

$$\begin{aligned} p' &= p_0 \frac{1}{1+m} \exp\left(\frac{-2\pi z}{L}\right) \cos(\lambda x - \omega t) \\ &+ p_0 \frac{m}{1+m} \exp\left(\frac{-z}{\sqrt{2}\delta}\right) \cos\left(\lambda x - \omega t + \frac{z}{\sqrt{2}\delta}\right) \ . \end{aligned} \qquad (2)$$

Eq.(2) includes two parameters m and δ. m is a non-dimensional parameter proportional to a ratio of G and β(Eq.(3)). δ is the thickness of the boundary layer(Eq.(4)).

$$m = \frac{n}{(1 - 2\nu)} \frac{G}{\beta} , \tag{3}$$

$$\delta = \left(\frac{KG}{\omega}\right)^{\frac{1}{2}} \left(n \frac{G}{\beta} + \frac{1 - 2\nu}{2(1 - \nu)}\right)^{-\frac{1}{2}} . \tag{4}$$

The other parameters are as follows : p_0 is the amplitude of bottom wave pressure, λ ($= 2\pi/L$) is the wave number, L is the wave length, x is the horizontal distance in the wave propagation direction, ω ($= 2\pi/T$) is the wave angular frequency, t is the time, g is the gravity, z is the depth beneath the seabed surface, and $K = k/\rho_w g$.

This solution is linear. The time profile of the bottom wave pressure of the selected wave No.7 was decomposed into its Fourier series components. The porewater pressure variation p' was calculated for each component by using Eq.(2). The typical values in the sandy bed in the surf zone were used for the parameters ($n = \nu = 0.33$, $G = 1.0\text{x}10^8\text{N/m}^2$, and $k = 2.8\text{x}10^{-4}\text{m/sec.}$). The porewater pressure variation of wave No.7 was obtained by summing up those of all components.

Fig.5 is a comparison between the calculated and measured results. The comparison was done for the maximum value of the porewater pressure. Four kinds of value were applied to the effective bulk modulus of porewater β. The measured values at lower two levels agree well with the calculated curve of $\beta = 3\text{x}10^7\text{N/m}^2$. The measured value at the highest level rather agrees with the calculated curve of $\beta = 6\text{x}10^6\text{N/m}^2$. The porewater pressure at the highest level has a different trend from that of other two levels. Nevertheless the boundary layer approximation solution roughly explains the measured porewater pressure even under the asymmetric wave in the surf zone.

MODIFIED SOLUTION INCLUDING EFFECTS OF WAVE-INDUCED BOTTOM SHEAR

In surf zone, usually the wave-induced bottom friction is not negligible. The solution by Mei and Foda(1981) neglects this effect. Here a modified solution including the effects of the wave-induced bottom shear is derived.

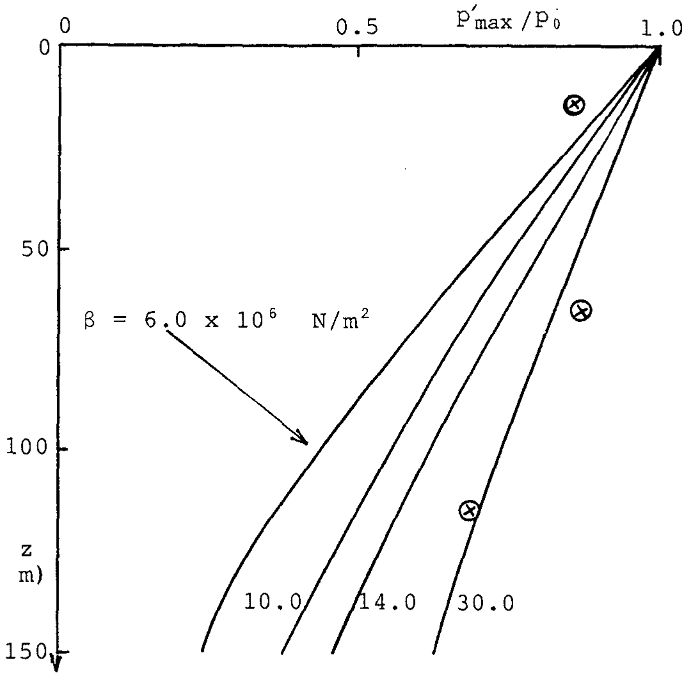

Fig.5 Comparison of calculated porewater pressure(–) with measured porewater pressure(x)

The time profile of the wave-induced bottom shear is not sinusoidal even for sinusoidal waves, because the bottom shear is proportional to the square of the bottom velocity. But here for simplicity, the time profile of wave-induced bottom shear is assumed sinusoidal.

The following boundary conditions are applied:

$$\tau_{xz} = -b\,\exp\{i(\lambda x - \omega t)\},\quad \tau_{zz} = -p_0\,\exp\{i(\lambda x - \omega t)\}\ , \qquad (5)$$

in which τ_{xz} and τ_{zz} are the shear stress and vertical total stress on the bed surface, b is the amplitude of the wave induced bottom shear stress, i is the imaginary unit. The first equation is added to the condition in Mei and Foda(1981).

The porewater pressure variation p', the vertical effective stress variation σ_{zz} and the shear stress σ_{zx} are given as follows :

$$p' = p_0 \frac{1}{1+m} \exp\left(-\frac{2\pi}{L} z\right) \cos \theta + p_0 \frac{m}{1+m} \exp\left(\frac{-z}{\sqrt{2}\delta}\right) \cos\left(\theta + \frac{z}{\sqrt{2}\delta}\right)$$
$$+ b \frac{1}{1+m} \left\{ \exp\left(-\frac{2\pi}{L} z\right) \sin \theta - \exp\left(\frac{-z}{\sqrt{2}\delta}\right) \sin\left(\theta + \frac{z}{\sqrt{2}\delta}\right) \right\} , \tag{6}$$

$$\sigma'_{zz} = p_0 \left(\frac{m}{1+m} + \frac{2\pi}{L} z \right) \exp\left(\frac{-2\pi}{L} z\right) \cos \theta$$
$$- p_0 \frac{m}{1+m} \exp\left(\frac{-z}{\sqrt{2}\delta}\right) \cos\left(\theta + \frac{z}{\sqrt{2}\delta}\right)$$
$$- b \left(\frac{1}{1+m} - \frac{2\pi}{L} z \right) \exp\left(\frac{-2\pi}{L} z\right) \sin \theta$$
$$+ b \frac{1}{1+m} \exp\left(\frac{-z}{\sqrt{2}\delta}\right) \sin\left(\theta + \frac{z}{\sqrt{2}\delta}\right) , \tag{7}$$

$$\sigma'_{zx} = p_0 \frac{2\pi}{L} z \exp\left(\frac{-2\pi}{L} z\right) \sin \theta + b\left(1 - \frac{2\pi}{L} z\right) \exp\left(\frac{-2\pi}{L} z\right) . \tag{8}$$

Here $\theta = \lambda x - \omega t$. The terms which do not include b are the solution of Mei and Foda itself.

SEABED INSTABILITY UNDER BREAKING WAVES

Madsen(1974) proposed a criterion on the stability of sand bed under breaking waves.

$$- \frac{\partial p'}{\partial x} \geq (\rho_t - \rho_w)\, g \tan \phi \ , \tag{9}$$

$$- \frac{\partial p'}{\partial z} \geq (\rho_t - \rho_w)\, g \ . \tag{10}$$

Eq.(9) is for the horizontal direction, and Eq.(10) is for the vertical direction. Here ρ_t is the density of the saturated bed material, and ϕ is the angle of internal friction of bed material.

He mentioned in his paper as follows : Under a steep front face of breaking wave, the horizontal gradient of the porewater pressure is larger than the vertical gradient. The angle of internal friction is smaller than 45 degree. The horizontal condition(Eq.(9)) is satisfied before the vertical condition(Eq.(10)) is satisfied.

Fig.6 is one of the figures of Tsuruya and Korezumi's paper(Fig.7, 1990). This figure shows a part of the record of the measurement of the sediment concentration near the bed(b) and the porewater pressure(p_m in (e)) near the bed surface and others in an actual surf zone. The figure (d) shows the calculated vertical effective stress(= static stress + p_b - p'). The black part indicates 0 effective stress occurrence. They suggested a relation between the high sediment concentration and the momentary liquefaction of the seabed.

The horizontal and vertical gradients of the porewater pressure are included in the right hand side of the horizontal and vertical momentum equations for the solid skeleton,

$$\rho_s \frac{\partial v_{sx}}{\partial t} = \frac{1}{1-n}\left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{zx}}{\partial z}\right) - \frac{\partial p}{\partial x} + \frac{1}{1-n}\frac{n^2}{K}(v_{wx} - v_{sx}), \qquad (11)$$

$$\rho_s \frac{\partial v_{sz}}{\partial t} = \frac{1}{1-n}\left(\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z}\right) - \frac{\partial p}{\partial z} - \rho_s g + \frac{1}{1-n}\frac{n^2}{K}(v_{wz} - v_{sz}) . \qquad (12)$$

In these equations, ρ_s is the density of the solid skeleton, v_{sx}, v_{sz}, v_{wx} and v_{wz} are the horizontal and vertical velocities of the solid skeleton and the porewater respectively. p is the sum of the static pressure and its variation p'.

To examine Madsen's suggestion, the value of each term in the right hand side of two momentum equations is calculated for the aymmetric wave measured by Zen et al., wave No.7, by using the new solution including the effects of the wave-induced bottom shear, Eq.s (6), (7) and (8).

To emphasize the effects, the wave period is shortened to 7.0sec, and the wave height is multiplied by 1.5. The amplitude of the bottom shear b is taken to be 1/10 of that of the bottom wave pressure p_0. The values of the parameters are as follows : h = 4.0m, ρ_t = 1,910kg/m^3, n = ν = 0.33, G = 1.0x10^{8}N/m^2, β = 1.0x10^{7}N/m^2 and k = 2.8x10^{-4}m/sec. The value of β is determined based on the fact that it is 10^{6}N/m^2 for the degree of saturation of 99%.

Fig.7 shows the result at the level of 10cm below the bed surface. The bottom wave pressure is also shown in the top figure. The middle figure shows the phase variation of each term in the right hand side of the horizontal momentum equation for the solid skeleton. The horizontal gradient of the horizontal effective stress $\partial\sigma_{xx}/\partial x$ and the vertical gradient of the shear stress $\partial\sigma_{zx}/\partial z$ are as large as the horizontal gradient of the porewater pressure $\partial p/\partial x$. The horizontal

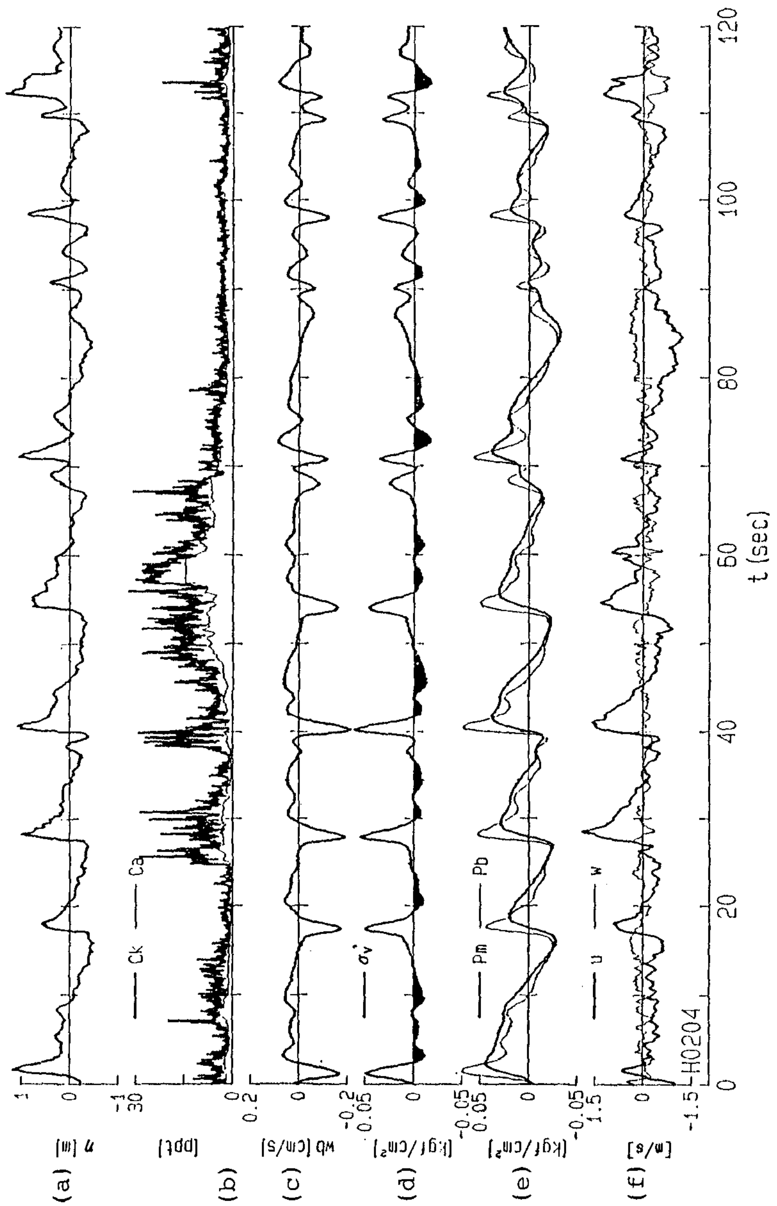

Fig.6 Sediment concentration (*b*) and vertical effective stress in seabed (d) (Tsuruya and Korezumi, 1990)

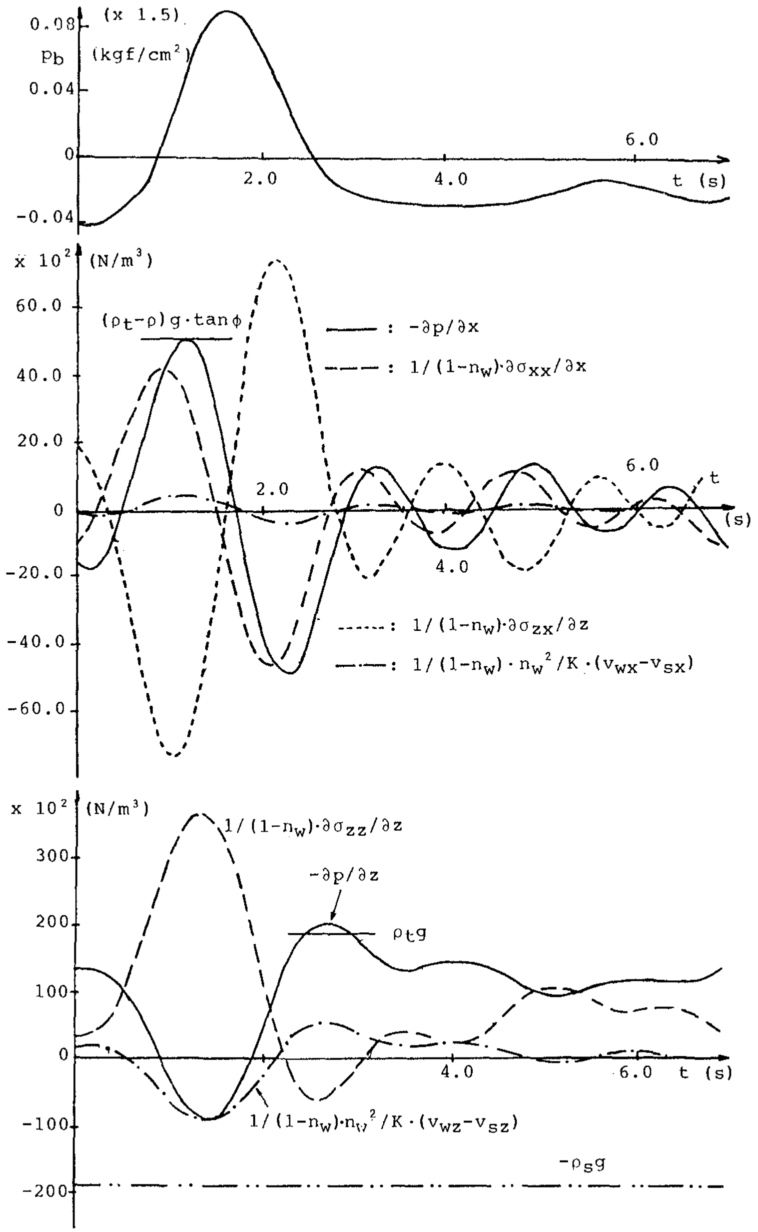

Fig.7 Phase variation of terms in right hand sides of momentum equations of solid skeleton(10cm below bed surface)

drag force due to the relative velocity of the porewater and solid skeleton $1/(1-n)\cdot n^2/K\cdot(v_{wx} - v_{sx})$ is negligible. - $\partial p/\partial x$ becomes maximum at a phase of front of crest (t = 1.2sec), and reaches the critical value of the horizontal momentary failure given by Eq.(9).

The bottom figure shows the phase variation of each terms of the vertical momentum equation for the solid skeleton. The value of term is one order larger than that of terms of the horizontal momentum equation. The vertical gradient of the vertical effective stress $\partial \sigma_{zz}/\partial z$ and the vertical drag force $1/(1-n)\cdot n^2/K\cdot(v_{wz} - v_{sz})$ are as large as the vertical gradient of porewater pressure $\partial p/\partial z$. - $\partial p/\partial z$ becomes maximum at a phase of the back of the crest (t = 2.6sec), and is larger than the critical value of the vertical momentary failure given by Eq.(10). It is found also that at this phase the vertical effective stress becomes zero.

From this result we can say as follows : Under a wave having a very steep front face, a momentary failure of the seabed may occur just before the crest passes. Then a momentary liquefaction occurs near the bed surface just after the crest passes. At the same phase, the absolute value of the horizontal porewater pressure gradient becomes large again as seen in the middle figure. The momentary failure of the seabed is therefore more likely to occur just after the crest passing than just before the crest passing.

CONCLUSIONS

(1) The boundary layer approximation solution for the wave-induced transient porewater pressure variation in seabed by Mei and Foda(1981) is applicable to an asymmetric wave in a surf zone.

(2) This solution was modified so as to take into account effects of the wave-induced bottom shear which is not negligible in the surf zone.

(3) Under a steep breaking wave the momentary failure of the seabed proposed by Madsen(1974) is more likely to occur just after the wave crest passes than just before the crest passes.

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