CHAPTER 150

CROSS-SHORE TRANSPORT MODELLING
IN TERMS OF SEDIMENT CONCENTRATIONS AND VELOCITIES

Zhiwen Chen

ABSTRACT

The fluctuations of sediment concentration and velocity during a wave period may play an important role in cross-shore sediment transport. In this study, the contribution of the fluctuations was estimated by measuring time-varying sediment concentration and velocity under laboratory conditions. The results showed that the contribution can only be neglected if the mean flow is relatively strong compared to the wave orbital motion.

1. INTRODUCTION

The phenomenon of cross-shore transport has been widely investigated for a long time. The solution of many real life problems relies on a proper description of the cross-shore transport mechanism [e.g., beach and dune erosion during storm surges; beach profile response to global sea level rise]. Many beach evolution models start by calculating the cross-shore transport distribution along a beach profile.

The cross-shore transport rate through a vertical plane with unit width can be

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expressed in principle by:

\[
q = \int_{0}^{h+\eta} u(z,t) \cdot c(z,t) \, dz
\]

Where:
- \( q \): cross-shore transport rate, [kg/ms]
- \( u(z,t) \): instantaneous cross-shore velocity at level \( z \), [m/s]
- \( c(z,t) \): instantaneous sediment concentration at level \( z \), [kg/m³]
- \( h \): water depth, [m]
- \( \eta \): maximum water surface elevation, [m]
- \( z \): elevation above the bed level, \([z = 0 \text{ at the bottom}], [m]\]
- \( t \): time, [s]
- \( \overline{\text{time-average}} \)

Eq.(1) offers a proper description of cross-shore transport. However, the direct use of Eq.(1) has been proved very difficult because of our entirely insufficient knowledge of the parameters \( u(z,t) \) and \( c(z,t) \). This holds especially inside the surf zone, the most active part of a beach profile. For this reason, coastal engineers in the past have often used more or less integral approaches to calculate cross-shore transport [e.g., Swart (1974) and Dean (1982)].

Recent models for cross-shore transport have attempted to consider a greater degree of detail in the internal mechanism of cross-shore transport [i.e., velocity multiplied by concentration]. For example, those of Bowen (1980) and Bailard (1981) estimated cross-shore transport on the basis of velocity fluctuations alone, indicating that some function of velocity fluctuations is used as a model for concentration variations; those of Stive and Battjes (1984) and Steetzel (1990) calculated cross-shore transport from the product of time-averaged velocities \( \bar{u}(z) \), and time-averaged concentrations \( \bar{c}(z) \), by assuming that sediment transport due to the fluctuations of concentration and velocity is negligible. In the latter case, \( \bar{u}(z,t) \cdot \bar{c}(z,t) \) can be approximated by \( \bar{u}(z) \cdot \bar{c}(z) \). The aim of this paper is to study some fundamental aspects of cross-shore transport modelling in terms of sediment concentrations and velocities, with an emphasis on:

- The temporal behaviour of sediment concentration under waves and a current on an intra-wave scale.
- The relative importance of the fluctuations of sediment concentration and velocity in cross-shore transport.

A description of the experiments is given in Chapter 2. The experimental results are presented in Chapter 3. Conclusions of this study are given in Chapter 4.

2. EXPERIMENTAL SET-UP

The experiments for this study were performed in a small-scale wave flume in the Hydromechanics Laboratory of the Civil Engineering Department of the Delft University of Technology. Regular waves were generated in the flume. A current, directed against the wave propagation, was superimposed on the waves in order to 'simulate' a return flow as caused by wave breaking under field conditions. Measurements of local time-varying sediment concentration and velocity were obtained using an optical concentration meter (OPCON) and an electro-magnetic flow meter (EMS) at a number of elevations above the bottom. The experiments were conducted first over a fixed bed and then over a sandy bed. Five tests were carried out, three under waves and a current and two under waves only.

In the fixed bed tests, artificial triangular ripples were used over the horizontal flume bottom. The ripple height was 20 mm and the ripple length was 80 mm. The measuring cross-section was located in the middle of the flume. During the tests, sediment was fed constantly upstream of the measuring cross-section. The increase in the volume of sediment downstream of the measuring cross-section was measured during a test to check whether a 'steady' sediment transport condition was achieved. A 'steady' condition was required because it took a rather long time to perform the velocity and concentration measurements at many elevations over the water depth. It was found that the transport rate kept approximately constant.

In the sandy bed tests, the middle part of the flume was filled with sand. No sand feeding was applied. The bed level change during a test was measured. Consequently, the net sediment transport rate through the measuring cross-section was computed.

The main characteristics of the tests are listed in Table 1.
Table 1 Characteristics of the tests

<table>
<thead>
<tr>
<th>test</th>
<th>condition</th>
<th>$H$</th>
<th>$T$</th>
<th>$h$</th>
<th>$\bar{u}$</th>
<th>$\eta$</th>
<th>$\lambda$</th>
<th>$D_{50}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>w+c, fixed bed</td>
<td>0.065</td>
<td>1.76</td>
<td>0.25</td>
<td>-0.08</td>
<td>0.02</td>
<td>0.08</td>
<td>0.18</td>
</tr>
<tr>
<td>B</td>
<td>w+c, fixed bed</td>
<td>0.065</td>
<td>1.76</td>
<td>0.25</td>
<td>-0.08</td>
<td>0.02</td>
<td>0.16</td>
<td>0.20</td>
</tr>
<tr>
<td>C</td>
<td>w, fixed bed</td>
<td>0.065</td>
<td>1.76</td>
<td>0.25</td>
<td>0</td>
<td>0.02</td>
<td>0.08</td>
<td>0.20</td>
</tr>
<tr>
<td>D</td>
<td>w+c, sandy bed</td>
<td>0.065</td>
<td>1.76</td>
<td>0.25</td>
<td>-0.08</td>
<td>0.01</td>
<td>0.07</td>
<td>0.20</td>
</tr>
<tr>
<td>E</td>
<td>w, sandy bed</td>
<td>0.065</td>
<td>1.76</td>
<td>0.25</td>
<td>0</td>
<td>0.01</td>
<td>0.07</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Where:

$w+c$ : under waves with a current

$w$ : under waves only

$H$ : wave height, [m]

$T$ : wave period, [s]

$h$ : water depth, [m]

$\eta$ : ripple height, [m]

$\lambda$ : ripple length, [m]

$\bar{u}$ : depth-averaged current velocity, [m/s]

$D_{50}$ : diameter, 50% by weight is finer, [mm]

3. RESULTS

3.1 THEORETICAL CONSIDERATIONS

If a periodic water motion is assumed, the velocity and concentration at height $z$ above the bed in a wave period can be described as:

$$u(t) = \bar{u} + u_1 \cos(\omega t - \alpha_1) + u_2 \cos(2\omega t - \alpha_2) + ...$$  \hspace{1cm} (2)
\[ c(t) = \bar{c} + c_1 \cos(\omega t - \beta_1) + c_2 \cos(2\omega t - \beta_2) + \ldots \] (3)

Where:

- \( u_n, c_n \) : amplitudes of the harmonic components
- \( \alpha_n, \beta_n \) : phase angles of the harmonic components
- \( \omega \) : wave frequency, [s\(^{-1}\)]

The sediment transport at height \( z \) is then given by:

\[ \overline{u(t)c(t)} = \bar{u} \bar{c} + \frac{1}{2} u_1 c_1 \cos(\beta_1 - \alpha_1) + \frac{1}{2} u_2 c_2 \cos(\beta_2 - \alpha_2) + \ldots \] (4)

The total cross-shore transport can be found by integrating Eq.(4) over the water depth. As shown in Eq.(4), the transport rate can be decomposed into a number of components. The \( \bar{u} \bar{c} \) term represents the contribution due to the mean flow. The sum of the other terms on the right hand side of Eq.(4) represents the contribution due to the fluctuations.

Under wave action, large \( u_1 \) and \( c_2 \) components can be expected. Due to wave asymmetry, the \( u_2 \) component usually has a large magnitude. Till it can be proved that the \( c_1 \) component is very small or that \( \beta_1 - \alpha_1 \) is in the order of 90° [similar remarks can be made to the third and higher terms in the right hand side of Eq.(4)], one has to be very careful in neglecting the contribution due to the fluctuations.

3.2 TEST RESULTS

Some preliminary results were described by Chen and Van de Graaff (1991). A more extensive description is given below.

**Temporal Behaviour of Sediment Concentration**

The measured time-varying sediment concentration and velocity data were first used to study the temporal behaviour of sediment concentration in a wave period. The test results showed that the concentration at a point varies strongly in time.
at the wave cycle scale. The concentration fluctuations do not repeat in time from one wave period to another. This means that the concentration fluctuations consist of a periodic component and a random component. [However, the random components are of little importance to the net sediment transport, as will be further discussed below].

An example of the periodic fluctuations of sediment concentration, measured over the ripple crest in Test A, is given in Fig.1,

\[ \cdot \ z = 10 \text{ mm}, \ + \ z = 20 \text{ mm}, \ * \ z = 30 \text{ mm} \]

**Fig.1** Velocity and concentration in a wave period [measured over ripple crest, Test A]

As shown, near the bed, three concentration peaks can be found in a wave period. Two are related to the vertical ejections of the high concentration clouds departing from the ripple crest when the flow reverses. The other one is due to the wave orbital motion which carries the high concentration at the flow reversal horizontally from a ripple crest to the neighboring one. In the upper column of the water depth, the concentration fluctuations become insignificant.

The test results under other conditions showed that the time-variations of sediment concentration in a wave period depend very much on the local bed geometry and local flow conditions. A mathematical description is very difficult. This implies that the cross-shore transport modelling based on the separate
descriptions of velocity and concentration as a function of time cannot be accomplished at this stage. In the following discussion, only the product of velocity and concentration [i.e., sediment transport rate] will be considered. Emphasis will be placed on the relative roles played by different forcing agents in sediment transport, especially the mean flow versus the fluctuations.

Sediment Transport Due to Mean Flow and Fluctuations

Role of the random components: The measured time-varying sediment concentration and velocity data were decomposed into period components and random components. It was found from the results that the random components, though relatively large compared to the periodic fluctuations, do not contribute much to the sediment transport.

Role of the higher harmonic components: With the periodic fluctuating components, harmonic components were calculated with Eq.(2) and Eq.(3). Consequently, the transport due to each harmonic component was calculated with Eq.(4). The calculation results showed that the harmonic components with a frequency higher than $2\omega$ together with the random components contribute less than 5% of the total transport.

Role of the first and second harmonic components: The transport contributions due to these components, as indicated in Eq.(4), depend on the amplitudes of the harmonic components the phase differences. The test results showed that the $c_1$ and $c_2$ components at a level are not small compared to the $\bar{c}$ component at the level. Since $u_1$ and $u_2$ are larger than $\bar{u}$, the magnitude of $\frac{1}{2}u_1c_1$ and $\frac{1}{2}u_2c_2$ is not negligible compared to with $\bar{u}\cdot\bar{c}$ [see Fig.2]. This implies that the phase differences may play an important role in the resulting sediment transport.

It was found that significant phase lags occur between the concentration and velocity fluctuations. The distributions of the phase lag over the water depth exhibit, in some cases, fairly distinct trends. The magnitude of $\beta_1-\alpha_1$ decreases with height above the bottom. Under waves with a relatively strong current, the $\beta_1-\alpha_1$ values are approximately $90^\circ$ close to the bottom, suggesting that the transport due to the $u_1$ component close to the bottom is small. Under waves alone, the $\beta_1-\alpha_1$ values close to the bottom increase [order of magnitude over
100°], resulting in an increase in the transport due to the \( u_1 \) component, see Fig.3.

Taking both the contributions of the \( u_1 \) and \( u_2 \) components into account, the sediment transport due to the fluctuations was determined. The test results showed that the transport due to the fluctuations relative to that due to the mean flow may vary over the water depth, both in magnitude and direction. In general, both transports at a level are in the same order. Under the specific wave conditions for this study, the transport due to the fluctuations and due to the mean are in the same direction close to the bottom and in opposite direction in the upper column of the water depth. Therefore, in some cases, the depth-integrated transport due to the fluctuations may be negligible compared to the transport due to the mean flow, even though both transports may be well-matched in magnitude at a specific level above the bed.

**Role of the mean flow:** It appeared from the test results that an increase in the magnitude of the current velocity may alter the relative contributions due to the fluctuations and due to the mean flow [see also Fig.4 and Fig.5, a negative value means that the transport is opposite to the direction of the mean flow]. If the mean flow is relatively weak, both contributions are of the same order of magnitude. If the mean flow becomes strong, the transport by the mean flow may play a dominant role, in that case, the transport due to the fluctuations may be neglected.

**4. CONCLUSIONS**

The test results indicate that, in general, the contribution due to the fluctuations of velocity and concentration should be considered in cross-shore transport calculations. However, under some specific conditions - for example, in a surf zone where the mean flow may be strong due to wave breaking - the contribution of the fluctuations may be neglected as a first order of approximation.
Fig. 2 Vertical distributions of $\frac{1}{2}u_1c_1$, $\frac{1}{2}u_2c_2$ and $\bar{u}c$ over the water depth [measured over ripple crest in Test A]

Fig. 3 Vertical distributions of $\cos(\beta_1-\alpha_1)$ and $\cos(\beta_2-\alpha_2)$ over the water depth [measured over ripple crest in Test A]
Table 2 Sediment transport rates in absolute values

<table>
<thead>
<tr>
<th>test</th>
<th>positions</th>
<th>$q(u)$</th>
<th>$q(\bar{u})$</th>
<th>$q(u) + q(\bar{u})$</th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>C&amp;T</td>
<td>1.21</td>
<td>0.03</td>
<td>1.24</td>
<td>1.22</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>0.84</td>
<td>0.36</td>
<td>0.48</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>0.46</td>
<td>0.05</td>
<td>0.51</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>C</td>
<td>0.31</td>
<td>0.15</td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>C&amp;T</td>
<td>0.42</td>
<td>0.12</td>
<td>0.54</td>
<td>0.31</td>
</tr>
<tr>
<td>E</td>
<td>C&amp;T</td>
<td>0.08</td>
<td>0.07</td>
<td>0.15</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Where:
$q(u)$: transport due to the mean flow, [g/ms]
$q(\bar{u})$: transport due to the fluctuations, [g/ms]
$q$: transport rate obtained from the mass conservation technique, [g/ms]
C: measurement over ripple crest
T: measurement over ripple trough
C&T: average of measurements over crest and over trough

Fig. 4 Vertical distributions of transport rate, in kg/m²s, results of Test A
Fig. 5 Vertical distributions of transport rate at ripple crest, in kg/m²s, results of Tests B, C, D and E
REFERENCES


Bowen, A.J. (1980).

Chen, Z. and J. van de Graaff (1991)

*Models for Beach Profile Response*. Technical Report, No.30, University of Delaware, Newark.

