

## CHAPTER 130

### Finite Element Simulation of Wave-Induced Internal Flow in Rubble Mound Structures

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#### Abstract

A Finite Element Model for the simulation of the wave-induced flow in a porous structure is presented and applied to three types of structures: homogeneous vertical structure, homogeneous trapezoidal structure and traditional multilayered rubble mound breakwater. The obtained results are related to the motion of the phreatic surface, the pore pressure histories and distributions within the structure as well as to the internal velocity field induced by wave action. The problems to be solved in order to get an improved numerical model are discussed.

#### Introduction

Rubble mound breakwaters still represent the most commonly used type of structure for the protection of coastal areas, harbours and further facilities against wave action. The traditional approach to study the stability of such structures still consists in the use of physical wave models with scales ranging from 1:30 to 1:50. For such scales the REYNOLDS numbers of the wave-induced flow in the core material of the model are generally smaller than the critical value  $Re_D = 10^4$ . For lower  $Re_D$ -values scale effects due to higher viscous forces start to occur. The latter will in turn affect the flow field in the armour and underlayer, so that the hydraulic stability may also be subject to scale effects. Due to the oscillatory nature of the flow and to the large variation of the hydraulic gradients, the tentative approach using distortion factors in the FROUDE scaling of the core material is not satisfactory. On the other hand, the performance of large-scale model tests in a wave flume is too costly and time consuming when used for design purposes or basic research including a large variation of the relevant parameters. For this reason, a research strategy has been adopted which consists in using numerical codes calibrated by experimental data obtained from a limited number of large-scale model tests (WIBBELER / OUMERACI, 1992).

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These calibrated codes are used as research tools for further parameter studies, the final objective being, however, the development of analytical solutions. Since the early seventies several models for the simulation of the wave-induced flow on and in rubble mound breakwaters have been developed. Both analytical solutions and numerical models were described. Despite the relatively high number of existing numerical models, no model exists which has been calibrated by reliable data covering all relevant flow phenomena involved. This paper will first briefly review some of the mathematical/numerical models available and then discuss the results of a Finite Element Model developed at the University of Hannover for the simulation of the wave-induced flow in a rubble mound breakwater.

### Review of Existing Mathematical/Numerical Models

An exhaustive review of the existing mathematical/numerical models for the simulation of wave-induced internal flow in rubble mound structures would be beyond the scope of this paper. Therefore, only a very brief review is given below. As illustrated by Fig. 1, one may distinguish analytical and numerical models. The latter are in turn divided in a) Finite Difference Models (FD), b) Finite Element Models (FE) and c) Hybrid Models using Finite Elements and Finite Difference Techniques (FD/FE).

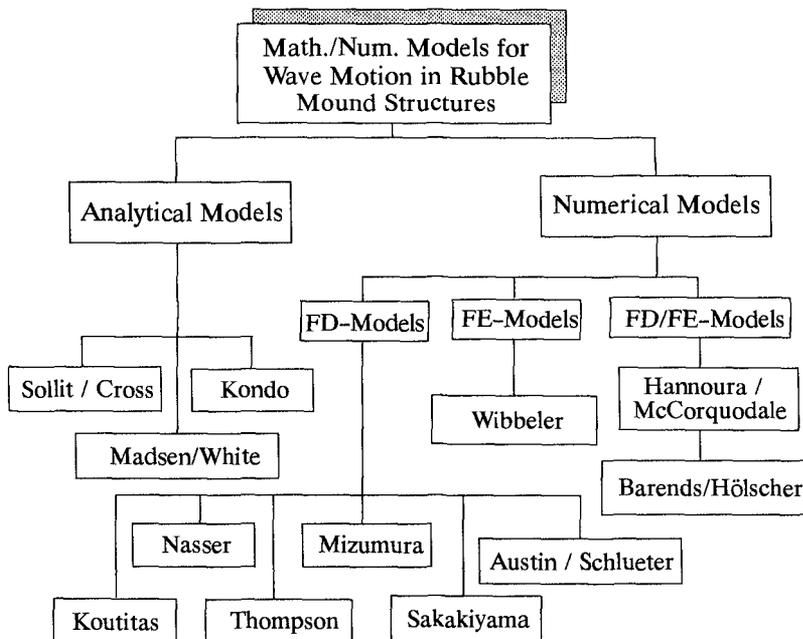


Fig. 1: Existing Models for Wave Motion in Rubble Mound Structures

However there are still a number of problems to be solved in order to come up with a reliable model which can properly simulate the interaction between waves and rubble mound structures, especially the wave-induced internal flow. Among these problems the following may be mentioned:

a) *Description of the external flow field:* The present knowledge on the wave kinematics in front of a rough permeable structure is almost nil.

b) *Interaction between external and internal flow:* Almost nothing is available on the physical processes involved at the boundary between two layers of different porous materials (discontinuities) and their mathematical formulation (coupling problem).

c) *Energy dissipation on the slope:* The question on how to account for this dissipation in case of breaking is still an unresolved problem.

d) *Porous flow equation:* It is still not clear whether a FORCHHEIMER-type equation will apply for the flow in the armour layer and filter layer.

e) *Air entrainment:* The air entrained by waves breaking on the structure is expected to strongly affect the internal flow. No attempt has yet been made to account for this effect in a numerical model.

f) *Internal stability of the porous medium:* Generally the porous matrix is assumed to be fixed. Actually, however, this is not the case.

As a conclusion, there is still a long way to go before a reliable numerical model can be developed which can account for all these aspects. Meanwhile, a relatively simple Finite Element Model for the simulation of wave-induced internal flow in a rubble mound breakwater has been developed at the University of Hannover. This model and some of the computational results obtained so far are discussed below.

### **Description of the Finite Element Model**

#### **Porous Flow Equations**

For the description of the fluid flow within the rubble mound structure non-linear constitutive equations of the FORCHHEIMER-type :

$$i = (a + b|v|) v \quad (1)$$

or of the exponential-type :

$$i = c |v|^{m-1} v \quad (2)$$

can be used, where :

- $v$  velocity vector
- $a$  FORCHHEIMER constants
- $b$  FORCHHEIMER constants
- $i$  hydraulic gradient ( $i = -grad h$ )
- $c, m$  constants of exponential law

The FORCHHEIMER constants and the constants of the exponential law are mainly dependent on the grain size and shape, the roughness of the grain surface and the porosity of the material. These constants have to be determined experimentally. Such experimental studies have been performed for instance by (SHIH, 1990) and (HANNOURA / McCORQUODALE, 1985). From the latter study the following equations can be evaluated:

$$a = 70.0 \frac{\nu}{g n \delta^2} \quad (3)$$

$$b = 0.54 \frac{(0.5 + 0.5f_r)}{g \sqrt{n} \delta} \quad (4)$$

$$\delta = \frac{n d}{6(1-n)S_f} \quad (5)$$

- $a, b$  FORCHHEIMER constants
- $d$  particle diameter
- $\delta$  effective pore hydr. diameter
- $f_r$  roughness factor
- $g$  acceleration due to gravity
- $n$  porosity
- $\nu$  kinematic viscosity
- $S_f$  particle shape factor

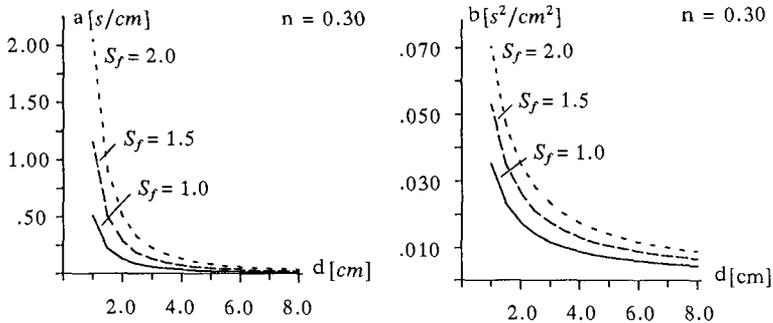


Fig. 2: Effect of Size and Shape of Grain Particles on FORCHHEIMER constants

Furthermore, the velocity vector  $v$  is expressed as a function of the permeability tensor  $K$ . Using a relative permeability coefficient  $k_r$ , the following general linearized constitutive equation is obtained which is used in the computations to evaluate the velocity in the porous media:

$$v = - k_r k_n K i \quad (6)$$

where  $k_n$  is a non-linear permeability coefficient.

**Finite Element Formulation and Evaluation of the Phreatic Surface**

In the present Finite Element Model the piezometric heads  $h$  are used as degrees of freedom ( $h$ -model). The transient motion of the phreatic surface, which is defined by the condition  $p=0$  (pressure head), is described by the following differential equation:

$$\phi \gamma \frac{\partial S}{\partial p} \frac{\partial h}{\partial t} - k_r k_n \operatorname{div} ( K \operatorname{grad} h ) - q = 0 \tag{7}$$

- |          |                     |        |                       |
|----------|---------------------|--------|-----------------------|
| $\gamma$ | specific weight     | $k_r$  | relative permeability |
| $h$      | piezometric head    | $\phi$ | porosity              |
| $K$      | permeability tensor | $q$    | discharge             |
|          |                     | $S$    | saturation            |

A Finite Element formulation of Eq. (7) leads to following equation:

$$\left( \frac{1}{\Delta t} S_e + \Theta K_e \right) h_{n+1} = \left( \frac{1}{\Delta t} S_e - (1 - \Theta) K_e \right) h_n - Q \tag{8}$$

- |  |      |            |                                |
|--|------|------------|--------------------------------|
| with:                                      |      | $B$        | derivatives of shape functions |
|  |      | $\Delta t$ | time step                      |
| $K_e = \int_{\Omega} B^T K B \, d\Omega$   | (9)  | $K_e$      | element matrix                 |
|  |      | $N$        | shape functions                |
| $S_e = \int_{\Omega} S_0 N^T N \, d\Omega$ | (10) | $Q$        | vector of nodal flows          |
|  |      | $S_e$      | saturation matrix              |

Eq. (8) is solved by using a  $\Theta$  - method.

The position of the phreatic surface must be evaluated by an iterative procedure. For these calculations, the domain of the structure is divided into two parts. These are the wet part beneath and the dry part above the phreatic surface. During the iteration, the permeability of the structure above the phreatic surface is set to a very small value, so that it behaves as almost impermeable. In each iteration step the element matrices must be calculated by using the new position of the phreatic surface. This procedure has to be continued until the position of the phreatic surface remains constant. The convergence of the iteration can be improved by use of a transition area between the permeable and impermeable domain (RANK / WERNER, 1986). This transition area is also used to calculate the saturation matrix  $S_e$ . Matrices of elements which are intersected by the phreatic surface have to be calculated by a special integration method (WIBBELER / MEISSNER, 1992).

### The Finite Element Mesh

One of the most important steps of a Finite Element calculation is the discretization of the structure. For the present model, an automatic mesh generator is available. This mesh generator is able to create meshes according to the different layers of the breakwater and the position of the phreatic surface. The reliability of the results can be controlled by the evaluation of an error estimator (MEISSNER / WIBBELER, 1990). The distribution of the approximation error allows the mesh generator to create an improved mesh either by a complete remeshing or by local refinements (adaptive discretization).

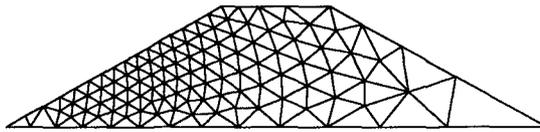


Fig 3: Adaptive Discretization of a Breakwater

### Simulation of Wave Motion

The wave motion in front of the structure and at the seaward slope of the breakwater can be described by using any wave theory or a numerical model as well as by using the wave data directly recorded from hydraulic model tests. The pressure at each boundary slope is prescribed by the function  $h(t)$  where  $h$  denotes the piezometric head (Fig. 4).

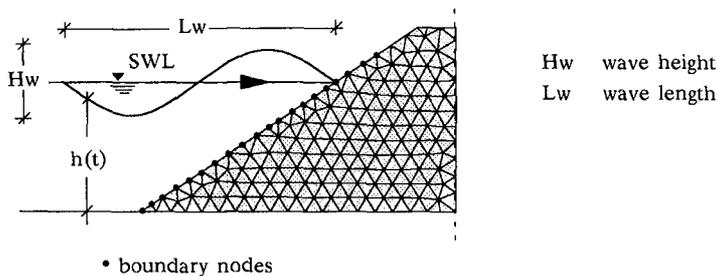


Fig.4: Simulation of Wave Motion

### Flowchart of the Finite Element Model

The main components of the present Finite Element Model are described by the flowchart in Fig. 5.

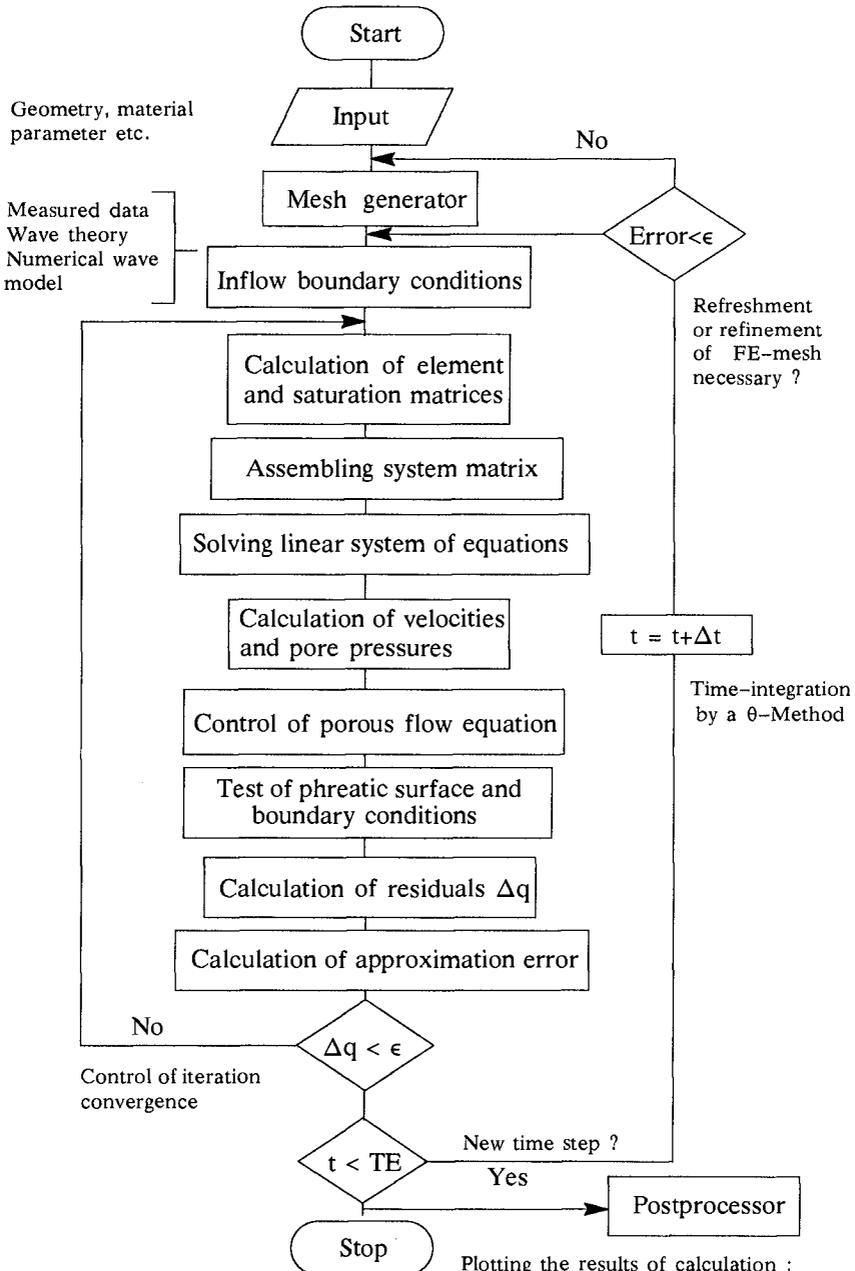


Fig. 5: Flowchart of the Numerical Model

Plotting the results of calculation : Velocity vectors, contours of pore pressure, pore pressure histories etc.

**Computational Results**

The simulation using the FE-Model described above is demonstrated on three types of porous structures: on a homogeneous vertical structure, a homogeneous trapezoidal structure and a multilayered traditional breakwater.

**a) Homogeneous Vertical Structure**

The actual vertical structure and its FE-discretized counterpart are shown in Fig. 6.

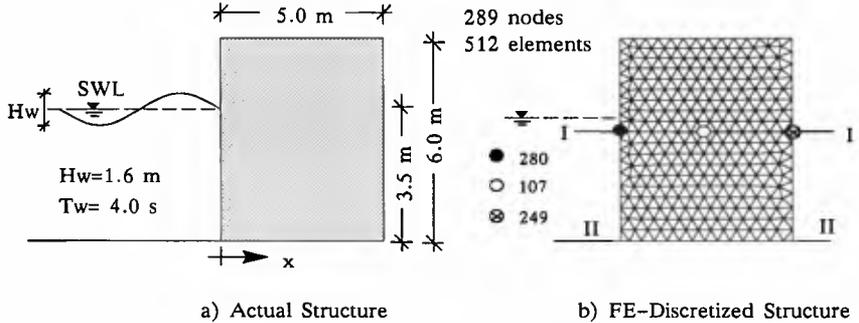


Fig. 6: Homogeneous Vertical Porous Structure

The instantaneous velocity fields and isobars at four successive time steps  $\Delta t = T_w/4$  ( $T_w = \text{Wave Period}$ ) are shown in Fig. 7, providing a good insight in the flow processes within the structure during a wave cycle.

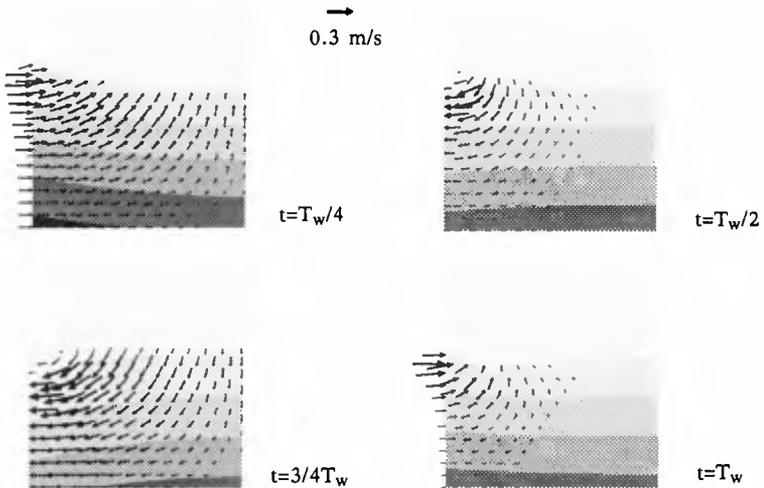


Fig. 7: Velocity Fields and Isobars within the Vertical Structure  
( Wave Conditions see Fig. 6)

Pore pressure histories at two different elevations under still water level are given in Fig. 8, showing a) the rate at which the wave motion is damped at different levels as the wave propagates into the structure and b) the phase shift of the pressure histories at different locations in the direction of wave propagation.

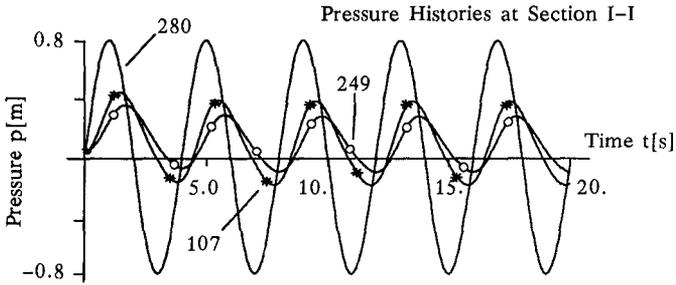


Fig. 8: Pore Pressure Damping and Phase Shift  
(Wave Conditions see Fig. 6a)

The spatial pore pressure distribution at section II-II during maximum wave run-up and maximum wave run-down are given in Fig. 9, showing the possible range of pressure variation within the structure during one wave cycle.

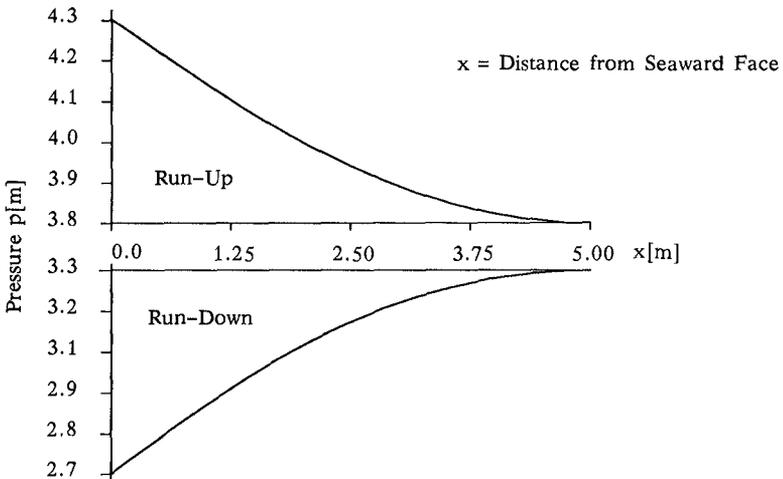


Fig. 9: Pore Pressure Distribution at Section II-II during  
Max. Wave Run-Up and Max. Run-Down

**b) Homogeneous Trapezoidal Breakwater**

Both the actual and discretized homogeneous trapezoidal breakwater are shown in Fig. 10 where also the incident wave conditions used for the computation are given.

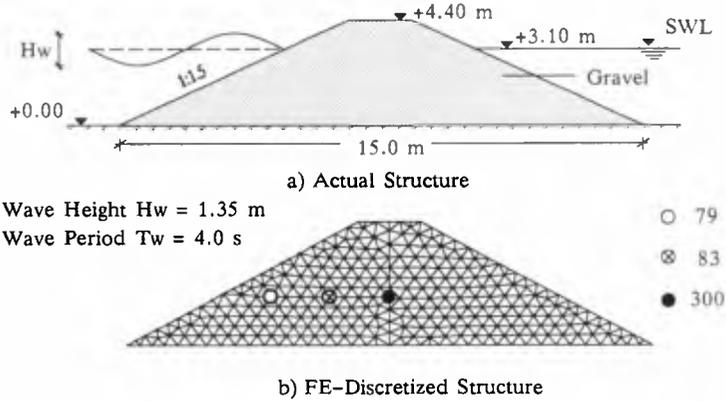


Fig. 10: Homogeneous Trapezoidal Breakwater

In the same manner as for the homogeneous vertical structure, instantaneous velocity fields and isobars at different time steps of the wave cycle can be determined (Fig. 11).

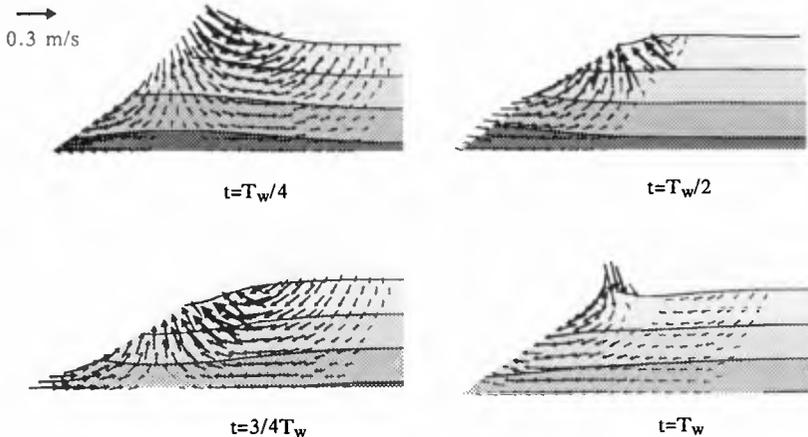


Fig. 11: Velocity Fields and Isobars within the Homogeneous Trapezoidal Structure (Wave Conditions see Fig. 10)

This kind of results is particularly important for the study of the flow-induced forces on the armour units as well as for the stability of the armour layer and breakwater toe. For instance, it is seen from Fig. 11 that not only run-down but also run-up may exert large forces on the armour units. A number of results of numerical tests of this kind can be analysed in order to improve the understanding of the flow processes leading to failures and to enable an analytical description of the forces involved.

The simultaneous pore pressure histories obtained at the three different locations (79,83 and 300 in Fig. 10b) in the breakwater also well illustrate the damping of the wave motion and the phase shift of the pore pressure histories at the different locations in the direction of wave propagation (Fig.12).

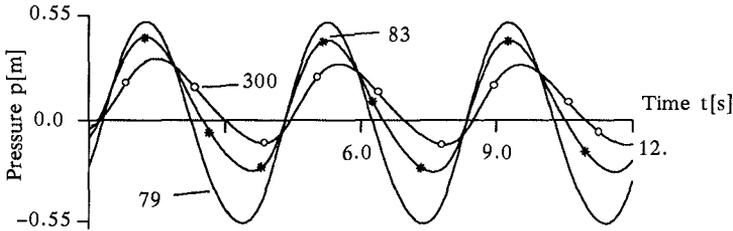
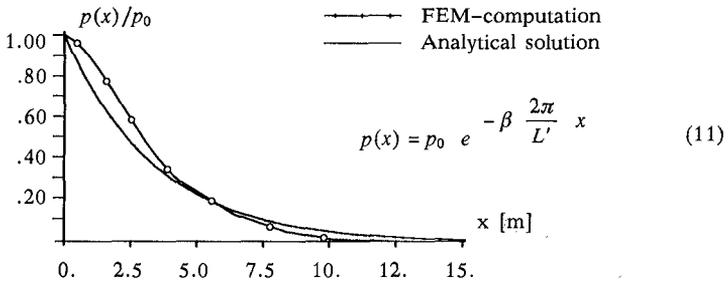


Fig. 12: Simultaneous Pore Pressure Histories  
(Wave Conditions see Fig. 10)

The relative pore pressure  $p(x)/p_0$  distribution in a horizontal plane located slightly below SWL is given for instance in Fig. 13, showing a relative good agreement with the results of large-scale model tests and the analytical solution given by (OUMERACI/PARTENSKY, 1990).



- $p_0$  : pore pressure at the seaward slope ( $x=0$ )
- $L'$  : wave length in the breakwater
- $\beta$  : damping coefficient which is strongly depth dependent

Fig. 13: Horizontal Distribution of Wave-Induced Pore Pressure

### c) Traditional Multilayered Breakwater

The actual structure which has been tested in the Large Wave Flume (GWK) in Hannover and its discretized counterpart are given by Fig. 14,

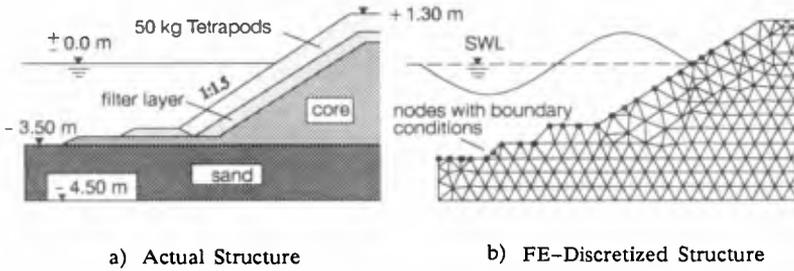


Fig. 14: Traditional Multilayered Breakwater

where only half of the structure is represented (second half is symmetrical to the represented part). In the same manner as for the previous homogeneous structures, the velocity fields in the structure during four steps ( $t=T_w/4$ ,  $T_w/2$ ,  $3/4T_w$ , and  $T_w$ ) of the wave cycle  $T_w$  are obtained in Fig. 15,

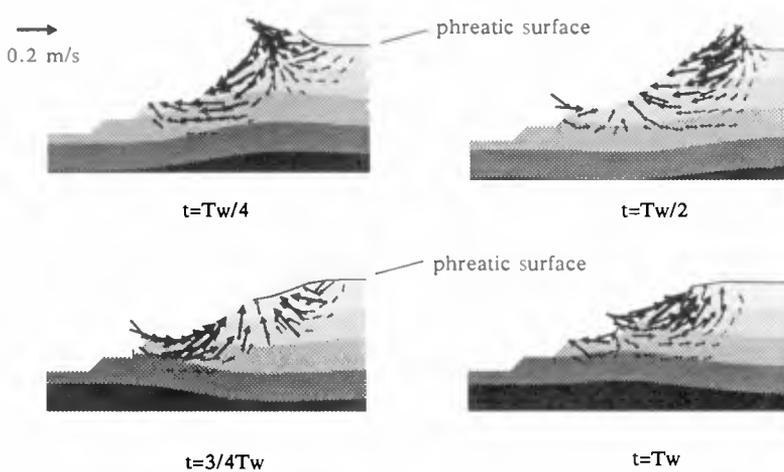


Fig. 15: Velocity Field and Isobars in the Multilayered Structure  
(Wave Conditions see Fig. 14)

which may give some indications on possible critical locations for the stability of the armour layer and the toe of the structure.

Further results of the computation as compared to the experimental data from the large-scale model tests are given in a further paper (WIBBELER / OUMERACI, 1992 ) showing a relatively good agreement between computed and measured wave-induced pore pressures within the structure.

### **Concluding Remarks and Perspectives**

The results of the FE-Model applied to three types of porous structures subject to wave action have shown that:

- a) the wave-induced pore pressure histories at different locations within the structure,
- b) the motion of the internal water table,
- c) the internal velocity field during a wave cycle

can be satisfactorily simulated, despite the simplicity of the model used. This has also been shown more clearly in a further paper (WIBBELER / OUMERACI, 1992 ).

The reasons for some discrepancies between numerical and experimental results are essentially due to air entrainment, singularities due to abrupt changes, unsteadiness of the flow and high turbulence within the first layers. Although the computation yields acceptable results for engineering purposes, there is a considerable need to improve and extend the FE-Model. This concerns particularly the coupling of the internal and external flow field, the improvement of the inflow boundary conditions, the consideration of the virtual mass effects and the simulation of the two-phase flow.

The research within the next years will be directed toward the development of an integrated numerical model for the simulation of the wave-induced flow processes on, at and in the rubble mound structure. The final result will be a reliable research tool for the study of the hydraulic stability of rubble mound structures, including all hydrodynamic effects of internal and external flow.

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