CHAPTER 68

AN EXPERIMENT AT SEA ON MECHANICS OF THE WAVE GROUPS

Paolo Boccotti (°), Giuseppe Barbaro (°°), Lucio Mannino (°°°)
Department of Fluid Mechanics and Offshore Engineering
University of Reggio Calabria

Abstract

Three field experiments were executed in the Straits of Messina starting in 1990 in order to verify the closed solution for the mechanics of the highest wave groups in a random wind generated sea state (Boccotti, 1984-88-89). The paper presents the first experiment which was concerned with the first part of the theory: the wave groups in an open sea. An array of nine wave gauges and nine pressure transducers supported by vertical piles provided space-time information on waves generated over a fetch of approximately 10 Km.

1 Introduction

A wave with a given very large crest-to-trough height of H, in a random sea state assumed Gaussian, is expected to belong to a well defined wave group whose average configuration in space and time is specified in terms of the autocovariance of the random wave field,

$$\psi(\underline{X},T) = \langle \eta(\underline{x},t)\,\eta(\underline{x}+\underline{X},t+T)\rangle \tag{1.1}$$

where $\eta(\underline{x},t)$ is the surface displacement. Specifically, if the crest of the given very high wave occurs at $\underline{x}_o = (x_o, y_o)$ at time t_o, the mean surface configuration of the wave group is given by

$$\eta_c(\underline{x}_o+\underline{X},t_o+T) = \frac{H}{2}\left\{\frac{\psi(\underline{X},T)-\psi(\underline{X},T-T^*)}{\psi(\underline{O},O)-\psi(\underline{O},T^*)}\right\} \tag{1.2}$$

(°) Professor of civil engineering, director of the Department, via E. Cuzzocrea 48, 89100 Reggio-Calabria, Italy
(°°) Assistant professor
(°°°) Engineer at the Department

where T^* is the abscissa of the absolute minimum of the autocovariance function, which is assumed to exist and to be the first minimum after T=0. The result (1.2), which was got by Boccotti (1984-1988-1989), assumes that H is very large compared with the mean wave height, or with $\sigma=(\overline{\eta^2})^{1/2}$ for the wave field as a whole and, in effect, that the spectrum is narrow in the sense described by Longuet-Higgins (1984), so that η is a damped oscillatory function in $\underline{X}$ and T. Superimposed on the deterministic form (1.2) is of course the "random noise" of the residual wave field whose r.m.s. surface displacement approaches σ as $\underline{X}$ and T increase, but when H/σ is large, the variations in the actual sea surface configuration about η_C surrounding $\underline{x}_o, t_o$ are small compared with η_C itself.

Associated with the configuration (1.2) is a distribution of velocity potential in the water, which to the lowest order in a Stokes expansion is given by

$$\phi_C(\underline{x}_o+\underline{X},z,t_o+T)=\frac{H}{2}\{\frac{\Phi(\underline{X},z,T)-\Phi(\underline{X},z,T-T^*)}{\psi(\underline{O},O)-\psi(\underline{O},T^*)}\} \tag{1.3}$$

where

$$\Phi(\underline{X},z,T)=\langle\eta(\underline{x},t)\phi(\underline{x}+\underline{X},z,t+T)\rangle. \tag{1.4}$$

Note that the hypothesis that H/σ is large is not necessarily inconsistent with the use of the lowest order (linear) terms in the Stokes expansion, provided H remains small with respect to the wave length and the water depth. Note also that, if η and ϕ are taken as solutions to the linear problem, then so are η_C and ϕ_C. This can be demonstrated formally from (1.2) and (1.3).

The covariance functions in (1.2) and (1.3) can be expressed in terms of the spectra; for example

$$\psi(\underline{X},T)=\int_{-\pi}^{\pi}\int_{0}^{\infty}S(\omega,\theta)\cos(\underline{k}\cdot\underline{X}-\omega t)d\omega d\theta \tag{1.5}$$

where $S(\omega,\theta)$ is the directional frequency spectrum and

$$\underline{k}\cdot\underline{X}=\frac{\omega^2}{g}(X\sin\theta+Y\cos\theta). \tag{1.6}$$

The substitution of (1.5) into (1.2) gives η_C as a function of position and time surrounding $\underline{X}=0$, $T=0$ and leads to the sequence of configurations illustrated in Figure 1, representing a wave group moving along the y-axis, the dominant direction of the spectrum. The spectrum was taken as that used by Hasselmann et al. (1973) with the spreading direction function by Mitsuyasu et al. (1975). In deep water this has the form

$$S(\omega,\theta)=\alpha g^2\omega^{-5}\exp\left[-\frac{5}{4}\left(\frac{\omega_d}{\omega}\right)^4\right]\exp\{\ln(\gamma)\exp[-(\omega-\omega_d)^2/2S^2\omega_d^2]\}\cdot$$

$$\cdot N(n)\,|\cos\frac{1}{2}(\theta-\theta_o)\,|^{2n},\qquad n=n_o\left(\frac{\omega}{\omega_d}\right)^5\quad if\quad \omega\le\omega_d, \tag{1.7}$$

$$n=n_o\left(\frac{\omega}{\omega_d}\right)^{-2.5}\quad if\quad \omega>\omega_d,\qquad N(n)=\left[\int_{-\pi}^{\pi}\cos^{2n}\frac{1}{2}\theta d\theta\right]^{-1},$$

where the parameters γ, S and n_0 are taken respectively as 3, 0.08 and 20, as Hasselmann et al. and Mitsuyasu et al. suggest (n_0=20 applies

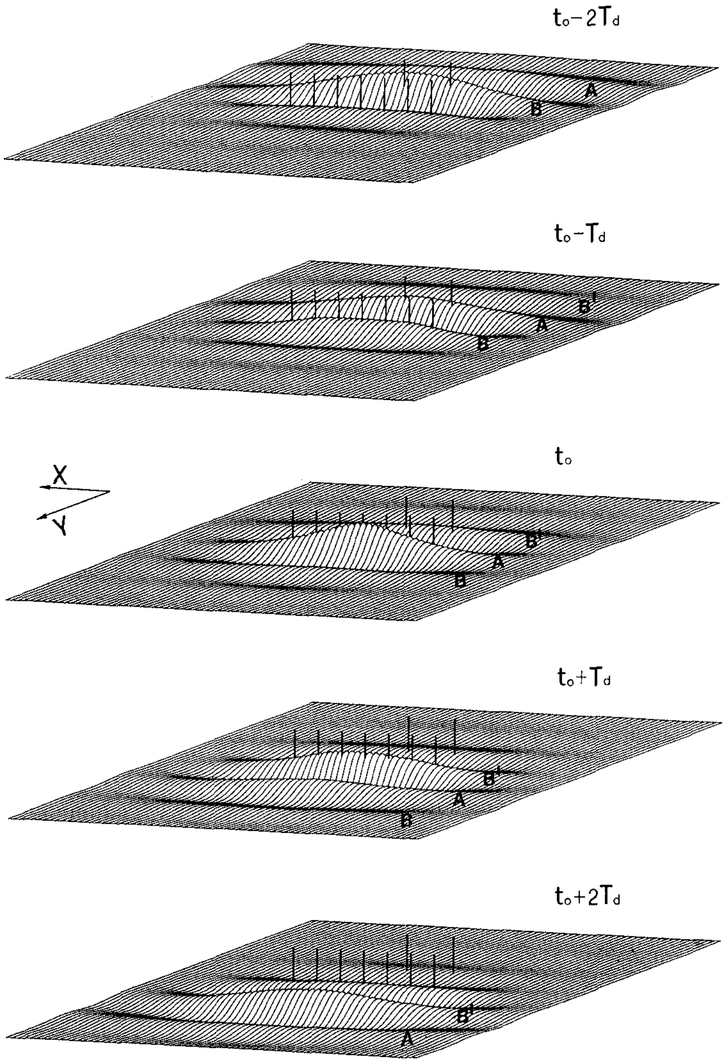

Fig.1 Quasi-determinism of the extreme wave events in a random wind generated sea state, assumed Gaussian, (Boccotti, 1989): an extremely high individual wave at a point x_o, y_o (center of the framed area), with a probability approaching 1, is produced by the transit of the wave group shown by the pictures. The water is deep, the spectrum of the sea state is that described in section 1. The time interval between two consecutive pictures is equal to one wave period T_d and the framed area is large 3 wave lengths L_d along x-axis and 6 L_d along y-axis.

to the conditions of the experiment: fetch of 10 Km and wind speed of about 7 m/s).

The basic phenomena that occur during the course of evolution of the group are not dependent on the detailed shape of the wave spectra. First, the wave group has a development stage in which the height of the central wave grows to a maximum and the width of the wave front reduces to a minimum. Secondly, the individual waves have a propagation speed greater than the envelope. Each wave is born at the tail of the group and then dies at the head of the group, as illustrated by wave A in Figure 1. The wave with the given very large height H proves to be that at the center of the wave group at the apex of the development stage.

2 The field experiment

In order to test the predictions, a small scale field experiment was undertaken during May, 1990 at a location off the beach of Reggio-Calabria on the eastern coast of the Straits of Messina. Figure 2 shows the site of the experiment.

Fig.2 The experiment was executed off the beach at Reggio-Calabria.

An array of nine small towers was installed 20 m from the beach in the configuration shown in Figure 3. These rested on the sea bed and their bases (0.8 m squared) were ballasted by pig iron discs. The water depth ranged within 3 m at the inner row (numbered 1-7) and 4 m at the outer pair (8,9). Each tower supported an ultrasonic wave probe furnished by Delft Hydraulics Laboratory, having range 0.6-2.0 m, and a pressure transducer (full scale 0.175 bar) some 0.5 m below the water surface. The sampling rate was 10 Hz for each gauge and the data were stored in two personal computers. Since the station was equipped to receive data (wave elevation and pressure) from eight towers only, one of the nine had to be disconnected. During the experiment tower six was disconnected because its ultrasonic probe was damaged.

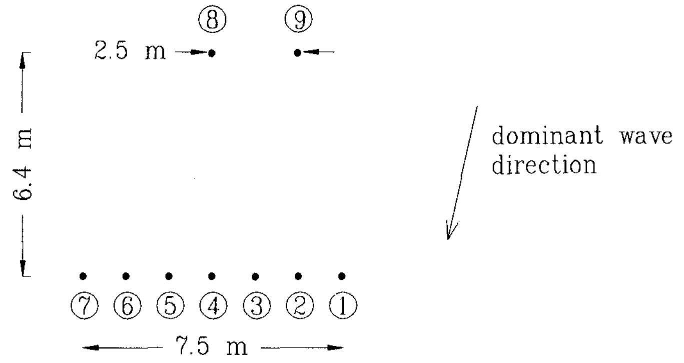

Fig.3 Plan view of the small towers which supported the gauges. The configuration of the towers is shown also in Figure 1 where the dominant wave length L_d is assumed to be 7.5m.

Suitable wind and wave conditions with a steady wind from the North West and the absence of southerly swells, were encountered 10 days after the array was installed. The experiment was conducted over a 12 hour period, starting at 8 AM on May 10, 1990. A total of 64 records was obtained, each of nine minutes duration and containing 280 to 360 dominant waves. The significant height ranged within 0.15 and 0.35 m and the dominant period of the spectrum ranged within 1.7 and 2.5 s, so that the water depth at the gauge locations was in effect deep.

3 Results of the experiment

3.1 The surface displacement

The space-time covariances in equations (1.2) can be found from the measurements by cross-correlation of the time series obtained at the discrete measurement locations. The time series data of a record provide measured auto-covariances as a function T for the various gauge locations and these can be used on the right hand side of equation (1.2) to estimate the surface displacement at these locations in an extreme wave. With $\underline{x}_o$ taken as the location of one of the gauges, the vectors $\underline{X}$ were specified by the relative locations of the other gauges.

A typical example of the result is shown in Figure 4 which was obtained from the time series data of record 30 without smoothing, with $\underline{x}_o$ taken as the location of gauge 4. Thus the figure represents the time histories of the expected surface configuration at the various gauge locations, if an extreme wave of given crest-to-trough height H is encountered at location 4. In this figure, A denotes the wave which is the highest at location 4, and B is the wave immediately before this one.

The direction of the wave can be estimated accurately since the front of wave A in the central position of the group along the traverse of locations 1-7 proves to be nearly straight. The relative phases indicate an angle of incidence of 20° - the front center before point 3 transits point 9, and this is consistent with the fact that in Figure 4, the wave group at point 9 is higher than at point 8.

Note that at locations 8, 9, wave B is slightly larger than A, but as it passes to the line 1-7, it decreases, passing from the center to the head of the group while the succeeding wave A grows because it replaces B at the group center. Note also that, in the course from location 9 to location 4, the period of wave B increases as it passes from the center to the head of the group, while the period of wave A decreases as it reaches its maximum height.

Similar calculations have been performed for all the records and the results are consistent with those described above. Specifically, there is an increase in period of the decreasing wave B, a decrease in the growing wave A and a local minimum period coupled to the maximum wave height.

Also Figure 5 shows the time histories of the expected surface waves at the various gauge locations, if an extreme wave of given crest-to-trough height H is encountered at location 4. In this case the auto-covariances in eq(1.2) were calculated from the theoretical spectrum (1.7), with the same dominant period and direction of record 30 ($T_d = 2.17\,s$, $\theta_o = 20°$). We see that the result from the theoretical spectrum (Figure 5) is very close to that from the time series data (Figure 4): all the basic features described earlier are still evident.

3.2 The fluctuating pressure head at the transducer depth

The variations in pressure at a fixed depth are given by $-\rho\partial\phi/\partial t$ or $\rho g\zeta$ where ζ is the fluctuating "pressure head" at this depth, so that the derivative of eq(1.3) with respect to T provides the following relation for the expected fluctuating pressure head at a fixed depth z below an extreme wave

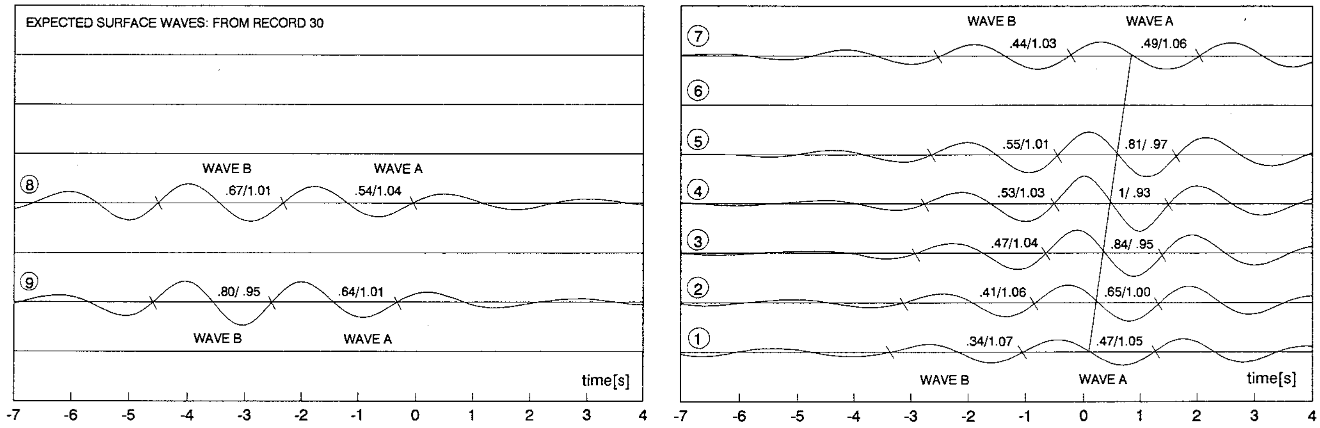

Fig.4 This picture was obtained from equation (1.2), using the time series data of the surface elevation of record 30, without smoothing. It shows the time histories of the expected surface waves at the locations of the gauges, if a wave with a very large height H is recorded at location 4. The first number over each wave denotes the crest-to-trough height, and the second number denotes the wave period; the wave height is scaled to given height H and the wave period is scaled to dominant wave period T_d.

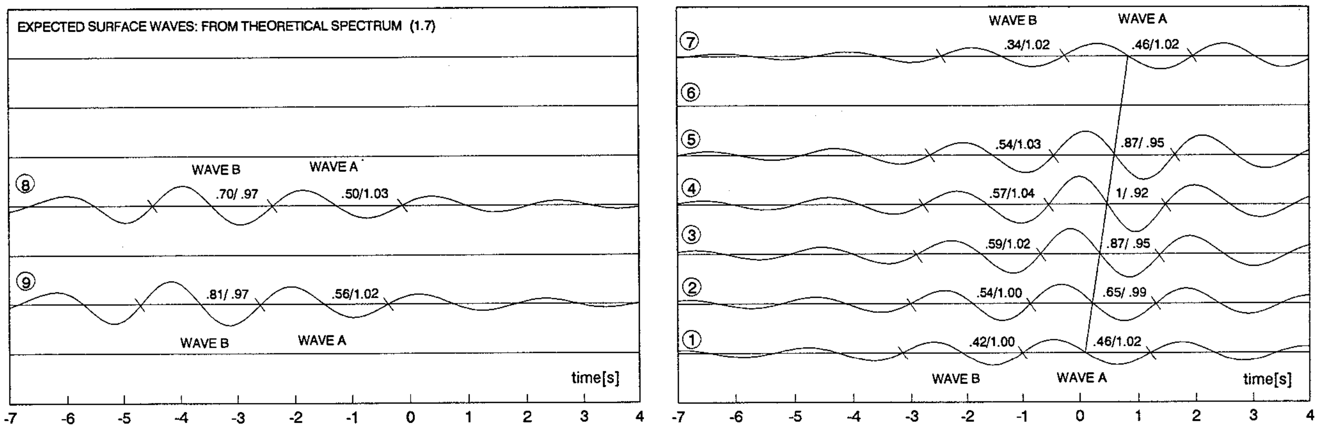

Fig.5 This picture was obtained from equation (1.2), with the auto-covariances calculated from the spectrum (1.7). Like Fig.4, it shows the time histories of the expected surface waves at the locations of the gauges, if a wave with a very large height H is recorded at location 4.

$$\zeta_C(\underline{x}_o + \underline{X}, z, t_o + T) = \frac{H}{2}\left\{\frac{\Pi(\underline{X}, z, T) - \Pi(\underline{X}, z, T - T^*)}{\psi(\underline{0}, 0) - \psi(\underline{0}, T^*)}\right\} \quad (3.1)$$

where Π is the covariance of the surface displacement and the fluctuating pressure head, of the whole record

$$\Pi(\underline{X}, z, T) = \langle \eta(\underline{x}, t)\zeta(\underline{x} + \underline{X}, z, t + T)\rangle . \quad (3.2)$$

The time histories of the expected pressure head waves at the transducer depth at the various gauge locations, if a surface wave of given very large height H is encountered at location 4, were calculated by means of eq(3.1) from the time series data of the measured pressure and surface elevation of record 30. The results are shown in Figure 6. The overall similarity between this and Figure 4 gives confidence in the consistency of the measurements.

Note that the enhancement of wave A during its course from location 9 to location 4 is somewhat smaller at the transducer depth (Figure 6), than at the surface (Figure 4), and this is consistent with the reduction in period during this interval. Also, the abatement of the height of wave B from point 9 to point 4 is somewhat smaller at the transducer depth (Figure 6) than at the surface (Figure 4), which again is consistent with the increase in period between these two points. Finally, the wave direction estimated from the pressure head wave of Figure 6 is the same as that estimated from the surface measurements - the difference is smaller than 1°.

4 Conclusive remarks

The comparison between the extreme wave groups and the predictions in terms of the measured space-time autocovariance, which was the goal of the experiment, is shown in the paper by Boccotti et al. (1992), which also gives a wider overall description of the experiment.

The experiment of May, 1990 revealed that it was possible to work off the beach of Reggio Calabria nearly like in a wave tank, because of the wave characteristics (pure wind waves with typical sizes of the laboratory tanks), of the very small tide excursion and the clearness of the water. That was because we decided to attempt some more complex experiments.

On May, 1991 a reflecting wall of 12x2.2 m was assembled on 1.6 m of water depth and thirty wave gauges were placed before the wall. The experiment essentially aimed to verify the theory of Boccotti (1988-89) as regards the nonhomogeneous wave fields. In particular, the theory shows that a very high wave at a wall forms because a well defined wave group like that of Figure 1, at the apex of the development stage, impacts the wall and is reflected mirrorwise. Also the significant wave height before the wall was analyzed through measurements at growing distances from the wall, like in the laboratory experiment of Hirakuchi et al. (1992). A preliminary illustration of the results was given by Boccotti (1992).

Finally, a third experiment dealing with the inertia loads on a big offshore platform has just been completed. The 1:50 scale model of a gravity platform was assembled in 2.5 m of water depth and two sets

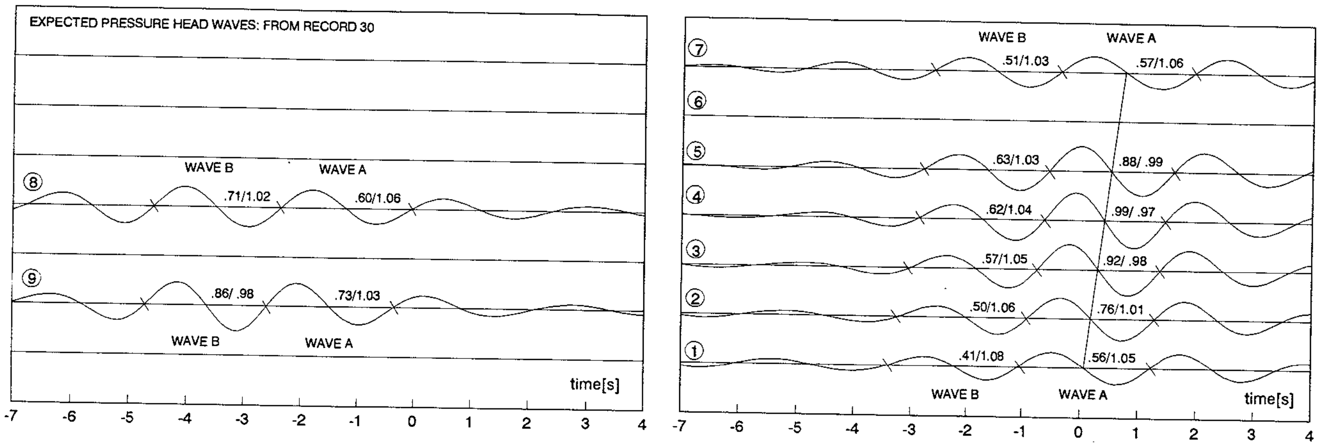

Fig.6 This picture was obtained from equation (3.1), using the time series data of the measured transducer pressure and surface elevation of record 30, without smoothing. It shows the time histories of the expected pressure head variations at transducer depths, if a wave with a very large height H is recorded at the surface at location 4. The wave heights are scaled to $H(\overline{\zeta^2}/\overline{\eta^2})^{1/2}$. The periods are scaled to the dominant period of the surface waves.

of wave gauges and pressure transducers were placed at the platform and far from it, in order to compare the wave forces on the column and on an ideal mass of water with the same shape and volume.

References

Boccotti, P. (1984). Sea waves and quasi-determinism of rare events in random processes, Atti Acc. Naz. Lincei, Rendiconti (Proc. Nat. Acc. Lincei), 76, 2, 119-127.
Boccotti, P. (1988). Refraction, reflection and diffraction of irregular gravity waves, in Excerpta of the Italian Contribution to Hydraulic Engineering, Libreria Progetto Padova Publ., 47-89.
Boccotti, P. (1989). On mechanics of irregular gravity waves, Atti Acc. Naz. Lincei, Memorie (Trans. Nat. Acc. Lincei), 19, 5, 111-170.
Boccotti, P. (1992). Quasi-deterministic mechanics of the highest waves in sea storms, Lectures speciales, Journees Nationales Genie Cotier - Genie Civil, Nantes, France.
Boccotti, P., et al. (1992). A field experiment on the mechanics of irregular gravity waves, accepted for publication in the Journal of Fluid Mech.
Hasselmann, K., et al. (1973). Measurements of wind wave growth and swell decay during the Joint North Sea Wave Project (JONSWAP), Deut. Hydrogr. Zeit, A-8.
Hirakuchi, H., et al. (1992). Characteristics of absorbing directional wavemaker, Proc. 23rd Int. Conf. on Coastal Engng., paper n°187.
Longuet-Higgins, M.S. (1984). Statistical properties of wave groups in a random sea state, Phil. Trans. R. Soc. London, A312, 219-250.
Mitsuyasu, H., et al. (1975). Observation of directional spectrum of ocean waves using a clover-leaf buoy, J. Phys. Oceanography, 5.