CHAPTER 62

Low Frequency Waves in Intermediate Water Depths

E C Bowers[1]

Abstract

This paper is concerned with calculations and field measurements of low frequency or infragravity waves associated with wave grouping (frequencies in the range of 0.005 to 0.04 Hertz). These waves have periods in excess of wind generated waves and they are assuming particular importance now due to the effect they are thought to have on sediment transport in and near the surf zone. However, the impetus for the work described here originated from a need to quantify the magnitude of these waves in intermediate depths typical of harbour entrances: it being generally accepted that long period waves excite harbour resonances and moored ship movements, leading to berth downtime.

1. Introduction

In intermediate water depths the infragravity waves associated with wave grouping are expected to consist largely of an incoming component bound to groups of shoreward going (primary) waves, sometimes called set-down beneath wave groups, and a free long wave component which will be referred to as surf beat in this paper. In this context surf beat includes both"leaky" modes propagating offshore and trapped, high order, edge wave modes. A full description of set-down in terms of radiation stresses associated with wave grouping was first given by Longuet-Higgins and Stewart (1964) and, in the same paper, they suggested that surf beat was the reflection of the bound long wave which became free of wave groups in the surf zone: this to account for a time lag observed by Tucker (1950) in correlations between long waves measured offshore of a beach and the envelope of the incoming waves.

Subsequently, a description was provided by Symonds et al (1982) of another mechanism for the generation of free long waves or surf beat. Called the moving breakpoint mechanism, the authors showed that gradients in radiation stresses associated with breaking waves, as the break point moves onshore and offshore at wave group periods, would generate long waves. An extension of this mechanism has been developed by Watson and Peregrine (1992). Using non-linear shallow water equations they have shown that the grouping of broken waves, that remains within the surf zone may lead to additional free long wave energy. Schaffer and Jonsson (1990) have compared results from an analytical description of long wave generation, containing the moving break point mechanism and the reflection of the bound long wave, with results of flume experiments carried out by Kostense (1984) and obtained qualitative agreement. Time domain models (List, 1992 and Roelvink 1992) and a frequency space model (van Leeuwen and Battjes 1990) containing both mechanisms of surf beat generation have also been developed and comparisons made with flume and field data.

Free long waves can also be expected to be released by incoming wave groups as they propagate over seabed irregularities (see Mei and Bennousa, 1984, for example).

---

[1]Research Department, HR Wallingford, Wallingford, Oxon. OX10 8BA, UK.

From the above discussion it can be seen that two basic long wave populations associated with wave grouping can be distinguished: the incoming bound long waves and free long waves or surf beats generated via various mechanisms. In principle, the bound long waves can be calculated from a knowledge of the primary waves. Early flume experiments verified that the amplitude of the bound long wave could be predicted in intermediate water depths using a Stokes expansion of the basic wave equations taken to second order in wave amplitude (see Bowers, 1980, for example). This suggested that surf beats could be determined in field data by calculating the bound long wave component and subtracting it from the total long wave energy. With that in mind, a method of analysis of pressure sensor data was developed to separate out the surf beat and bound long wave components in the total long wave spectrum. This analysis was applied subsequently to a number of sites around the UK coast (water depths 4m to 19m) including Port Talbot on the south coast of Wales in 1984, Dover and Shoreham on the south coast of England in 1986, Barrow-in-Furness on the west coast of England in 1988 and Sunderland on the east coast of England in 1988. The method of analysis is outlined in the next section and the results from the 5 sites discussed in a following section.

Similar analyses of field data have been carried out recently by Okihiro et al (1992) and by Herbers et al (1992).

2. Method of analysis

The first point to make is that calculation of the bound long wave has to take the directional spread of primary wave energy into account (Sand 1982). This is well illustrated by flat bed wave basin results obtained recently at HR Wallingford using a shallow water multi-directional wave-maker. Figure 1 shows measured long wave spectra associated with primary waves representing a significant wave height of 8m, a spectral peak period of 15s and water depth of 40m. It can be seen that even a relatively narrow rms spread of 22.5° in short crested primary waves will almost halve the long wave height associated with long crested uni-directional (zero spread) waves. It is also of interest to note that the long wave height is not all that sensitive to the amount of directional spread, with a broad spread corresponding to a $\cos^2\theta$ distribution (32° rms spread) resulting in only slightly smaller long waves than those measured with a primary wave spread of 22.5°. These results indicate that once a small degree of directional spread exists in the primary waves, it produces a considerable reduction in the height of the bound long wave component but thereafter directional spread becomes a less sensitive parameter (see Equations (17) and (18) later). Of course, surf beats due to reflections of the bound long wave from the shingle beaches on the wave basin boundaries would also have been present in these experiments but they can be expected to be a fixed percentage of the bound long wave. Thus, the relative behaviour of the total long wave spectra in Figure 1 can be considered representative of the relative behaviour of just the bound long wave spectra in the model depth equivalent of 40m.

The second point to make is that relatively long wave records are required to reduce uncertainties in long wave magnitudes. For example, there would only be 10 waves of 2 minute period in a conventional wave record 20 minutes long with the result that large variations in the long wave height would occur from record to record even with a stationary sea state. This problem can be minimised by taking a long enough record for which a full range of different wave grouping patterns has had time to occur leading to representative bound long waves. For example, Bowers (1988) has demonstrated in flume work, with compensation for set-down at the wavemaker, that in moderately long experiments the spectrum of set-down or the bound long wave component in random seas will tend to an "expected" spectrum calculated without taking into account the phases of the primary waves. In short experiments, or short wave records, these phases become important because only certain patterns of wave grouping will have occurred leading to an unrepresentative long wave spectrum. To ensue long records in the field measurements a bottom mounted pressure sensor was programmed to take 2 hour records. This was done by sampling regularly for 5 minutes every 4 hours and when the significant wave height exceeded a present threshold level, a 2 hour record was taken. This technique ensured that long records were only takenat times of relatively high primary wave activity when the associated long waves were worth measuring.

In the measurements reported here, the specially programmed pressure sensor provided information about the one dimensional primary wave spectrum, but no information

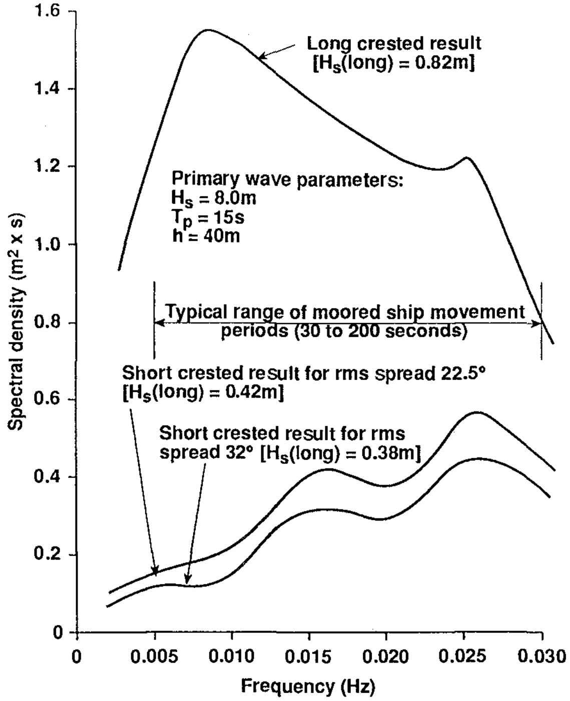

Figure 1 Wave basin long wave spectrum showing effect of short crestedness in primary waves

about directional wave properties. However, the (zero lag) correlation coefficient between the forced low frequency disturbance and the low frequency part of the square of the primary wave pressures is sensitive to the amount of directional spread in the primary waves, with narrow spreads producing higher correlations. Thus, by calculating the correlation coefficient for the total long waves from the pressure sensor data, it is possible to infer the mean directional spread in the primary waves (see Section 2.2). Armed with the one dimensional primary wave spectrum and a mean spread parameter, it is then possible to calculate the "expected" spectrum of the forced low frequency disturbance and subtract it from the measured long wave spectrum to leave the surf beat spectrum.

The bound long wave spectrum based on a second order Stokes expansion (see for example Sand 1982) really applies only to a flat seabed. If the seabed is sloping gently enough, so that the wave system has time to adjust itself to local depths, then the same calculation can be expected to apply to the real situation with the local water depth being used in the equations. While this assumption appears reasonable in most cases for the primary waves, it is less likely to apply to long waves where depth changes can be significant within a wavelength. In what follows, an allowance for seabed slope is made as a correction to the "flat bed" calculation of the bound long wave and it is found that an additional bound long wave component results which lags the main one by 90°. This may explain a lag in the total bound long wave component that has been reported by List (1992) in a numerical model of nearshore surf beat generation. However, for the seabed slopes and intermediate water depths applicable to the field measurements reported in this paper it is found that the additional bound long wave component remains a fraction of the "flat bed" bound long wave and, if included, alters resulting estimates of surf beat height by only a few percent.

2.1 Calculation of bound long waves

For convenience, we represent the seabed by straight parallel contours running perpendicular to the x axis of a right-handed orthogonal co-ordinate system where the x axis points offshore. The plane z = 0 is taken to be the water surface and we assume that a multi-directional sea approaches from offshore, with its mean direction parallel to the x axis. A constant seabed slope α is taken so that water depth(h) is given by,

$$h = x\tan\alpha \tag{1}$$

We assume irrotational wave motion with fluid velocity $\underline{q}$ derived from a velocity potential ϕ,

$$\underline{q} = (u, v, w) = -\underline{\nabla}\phi$$

Incompressibility leads to the basic wave equation

$$\nabla^2\phi = 0 \tag{2}$$

Solutions to (2) are sought subject to the following boundary conditions.

On the seabed z = -h, the normal velocity vanishes, i.e,

$$u\frac{dh}{dx} + w = 0 \tag{3}$$

and on the free surface $z = \eta$ we have the kinetic condition

$$\frac{\partial\eta}{\partial t} + u\frac{\partial\eta}{\partial x} + v\frac{\partial\eta}{\partial y} - w = 0 \tag{4}$$

and Bernoulli's equation (with constant air pressure at the surface)

$$\tfrac{1}{2}\underline{q}^2 + g\eta - \frac{\partial\phi}{\partial t} = 0 \tag{5}$$

We solve the above equations by using a Stokes expansion, retaining linear terms in the equation in lowest order and bringing quadratic terms into next order. With in each order, flat bed solutions are obtained initially and then corrections obtained to allow for depth variations.

As the expressions for the final bound long wave spectrum are complex we consider just one component associated with two primary wave frequencies ω_1, and ω_2 propagating at angles θ_m and θ_n, respectively, to the x axis,

$$\eta^{(1)} = a_{n2}\cos(\omega_2 t + \underline{k}_{n2}\underline{r} + \varepsilon_{n2}) + a_{m1}\cos(\omega_1 t + \underline{k}_{m1}\underline{r} + \varepsilon_{m1})$$

where

$$\underline{k}_{n2}\underline{r} = k_2\cos\theta_n x + k_2\sin\theta_n y,$$
$$\underline{k}_{m1}\underline{r} = k_1\cos\theta_m n + k_1\sin\theta_m y,$$

Here, a_{n2} (x), a_{m1} (x) are the amplitudes of the two primary wave components and ε_{n2}, ε_{m1} are their random phases, while each wave number satisfies the usual dispersion relationship in terms of the local water depth h(x),

$$\omega^2 = kg\tanh kh.$$

The depth variation can be shown to introduce an additional requirement on each primary component which expresses the conservation of wave energy during refraction over the varying seabed level,

$$\frac{d}{dx}(a^2C_g\cos\theta) = 0$$

where c_g is the group velocity

$$c_g = \frac{\omega}{2k}\left(1 + \frac{2kh}{\sinh 2kh}\right)$$

To next order in the Stokes expansion, the quadratic terms in (4) and (5) are expressed as products of the first order quantities and they lead to terms containing sum and difference frequencies $\omega_2 \pm \omega_1$. The bound long wave component has frequency $\omega_2 - \omega_1$ so we retain just the difference frequency component. Solutions to Laplace's equation (2) are sought for the second order potential $\phi^{(2)}$ subject to boundary condition (3) on the seabed and the following surface condition obtained from (4) and (5),

$$\frac{\partial^2\phi^{(2)}}{\partial t^2} + g\frac{\partial\phi^{(2)}}{\partial z} = \eta^{(1)}\left(\frac{\partial^2 w^{(1)}}{\partial t^2}\right) + g\frac{\partial w^{(1)}}{\partial z}\Big) + 2\underline{q}^{(1)}\ \frac{\partial \underline{q}^{(1)}}{\partial t} \tag{6}$$

Equation (6) shows how the surface perturbation on the right-hand side forces the bound long wave potential $\phi^{(2)}$. Following the pattern of solution outlined above for the primary waves, we calculate the bound long wave first of all neglecting depth variations and then consider the effect of variable depth as a perturbation on this solution. We denote the local depth solution for the bound long wave potential by $\phi_o^{(2)}(x)$ and its correction for depth variation by $\phi_1^{(2)}(x)$. Thus, $\phi_o^{(2)}(x)$ takes the form of the usual "flat bed" second order potential (see Sand, 1982 for example).

$$\phi_o^{(2)} = A_{mn}\cosh\{|\underline{k}^-|\,(z+h)\}\,\sin(\omega^- t + \underline{k}^-.\underline{r} + \varepsilon^-) \tag{7}$$

where,

$$\varepsilon^- = \varepsilon_{n2} - \varepsilon_{m1},$$
$$\omega^- = \omega_2 - \omega_1,$$
$$\underline{k}^- = \underline{k}_{n2} - \underline{k}_{m1},$$

$$A_{mn} = \frac{1}{2}g^2 a_{n2} a_{m1}\left[\frac{e^- + \dfrac{2k_1k_2}{\omega_1\omega_2}\,\omega^-(\cos(\theta_n - \theta_m) + \tanh k_1 h\,\tanh k_2 h)}{(\omega^-)^2\cosh|\underline{k}^-|h \;-\; g|\underline{k}^-|\sinh|\underline{k}^-|h}\right]$$

$$e^- = \frac{k_2^2}{\omega_2\cosh^2 k_2 h} - \frac{k_1^2}{\omega_1\cosh^2 k_1 h}$$

To find $\phi_1^{(2)}$ we consider solutions of (2) of the following form,

$$\phi_1^{(2)} = G_{mn}(x,z)\cos(\omega^- t + \underline{k}^-.\underline{r} + \varepsilon^-) \tag{8}$$

where,

$$G_{mn} = (B_{mn} + z\,E_{mn})\sinh|\underline{k}^-|(z+h)$$
$$+ (C_{mn} + zD_{mn} + z^2F_{mn})\cosh|\underline{k}^-|(z+h),$$

$$2|\underline{k}^-|\,F_{mn} = -k_x^-\,A_{mn}\,\frac{d}{dx}|\underline{k}^-|$$

$$|\underline{k}^-|D_{mn} = -k_x^-\,A_{mn}\frac{d}{dx}|\underline{k}^-|d\ ,$$

$$2|\underline{k}^-|\,E_{mn} = \frac{1}{A_{mn}}\frac{d}{dx}(k_x^-\,A_{mn}^2) + \frac{k_x^- h}{|\underline{k}^-|}\,A_{mn}\frac{d}{dx}|\underline{k}^-|\ ,$$

and,

$$k_x^- = k_2\cos\theta_n - k_1\cos\theta_m$$

The boundary condition (3) leads to a value for B_{mn}

$$2|\underline{k}^-|B_{mn} = -\frac{h}{A_{mn}}\frac{d}{dx}(k_x^- A_{mn}^2) + \frac{k_x^- hA_{mn}}{|\underline{k}^-|}\frac{d}{dx}|\underline{k}^-| \ ,$$

and (6) leads to a value for C_{mn}

$$C_{mn}\left[g|\underline{k}^-|\sinh|\underline{k}^-|h - (\omega^-)^2\cosh\cdot|\underline{k}^-|h\right] + gD_{mn}\cosh|\underline{k}^-|h$$

$$+ gE_{mn}\sinh|\underline{k}^-|h + B_{mn}\left[g|\underline{k}^-|\sinh|\underline{k}^-|h - (\omega^-)2\cosh|\underline{k}^-|h\right] = H_{mn}.$$

Here, H_{mn} is the term on the right-hand side of (6) proportional to cos $(\omega^- t + \underline{k}^-.\underline{r} + \varepsilon^-)$.

The above set of equations define the additional bound long wave component (8) due to a varying water depth. It can be seen that it has a 90° phase difference with the usual "flat bed" bound long wave (7).

The expressions derived so far relate to just a pair of primary wave components. To define the bound long wave associated with a multi-directional sea it is necessary to sum over all the pairs of wave components with amplitudes that can be defined in terms of the directional wave spectrum S_d, ie

$$a^2_{n2} = 2\,S_d\,(f_2,\,\theta_n)\,df\,d\theta, \tag{9}$$
$$a^2_{m1} = 2\,S_d\,(f_1,\,\theta_m)\,df\,d\theta. \tag{10}$$

Finally, as measurements were made with a bottom mounted pressure sensor (with the diaphragm pointing upwards) we obtain an expression for the forced low frequency disturbance on the seabed from Bernoulli's equation,

$$\eta_s^{(2)} = \frac{1}{g}\left[\frac{\partial\phi^{(2)}}{\partial t} - \tfrac{1}{2}\,(\underline{q}^{(1)})^2\right]_{z\,=\,-h} \tag{11}$$

2.2 Estimating directional spread

This was done using the correlation coefficient R_L (at zero lag) between the low frequency part of the square of the primary wave bed pressures, effectively the square of the seabed wave envelope, and the measured low frequency disturbance on the bed. Denoting the primary wave on the bed by $\eta_p^{(1)}$ and the surf beat by $\eta_b^{(2)}$ we have by definition,

$$R_L = \frac{\int (\eta_p^{(1)})^2(\eta_s^{(2)} + \eta_b^{(2)})\,dt}{(\int(\eta_p^{(1)})^4 dt)^{1/2}\,(\int(\eta_s^{(2)} + \eta_b^{(2)})^2 dt)^{1/2}}$$

Where $\eta_s^{(2)}$ is defined in (11). The above quantity can be calculated from the measured pressure sensor data and examples of R_L are shown in Figure 2 for a range of lags and primary wave heights. It can be seen in all cases that a negative correlation coefficient occurs at zero lag, consistent with a set-down beneath groups of large waves. The Bernoulli pressure in (11) will contribute to this negative correlation but it is insufficient to to explain the magnitude of the measured correlations. In all cases the recordings were made far enough offshore for us to assume that at zero lag surf beat is uncorrelated with both the square of the primary waves and the bound long wave so that R_L simplifies to,

$$R_L = \left(\frac{M_S}{M_L}\right)^{1/2} R_s \tag{12}$$

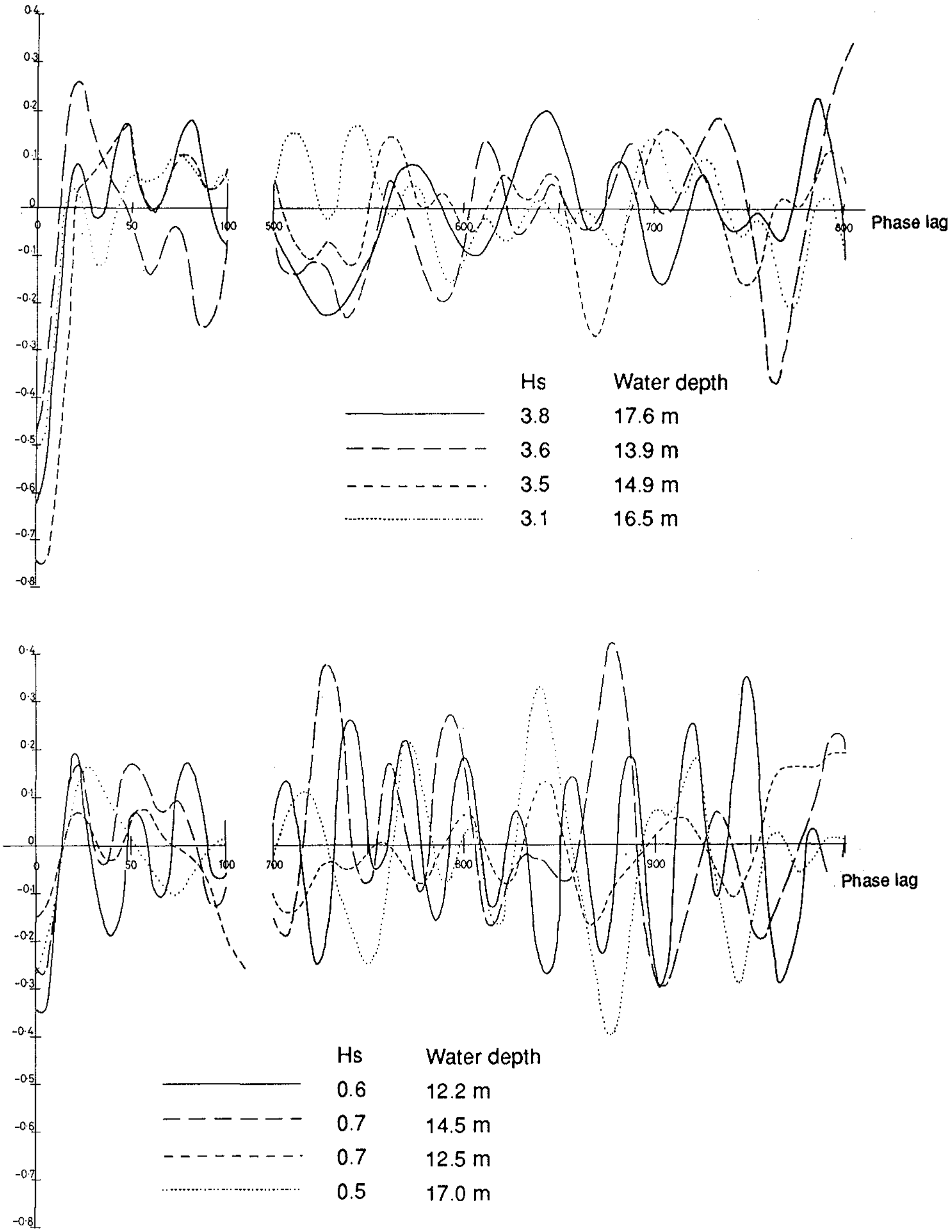

Figure 2 Correlation coefficients for large and small waves at Port Talbot

Where R_s is the correlation coefficient for the forced low frequency disturbance alone,

$$R_s = \frac{\int(\eta_p^{(1)})^2\,\eta_s^{(2)}dt}{(\int(\eta_p^{(1)})^4 dt)^{½}(\int(\eta_s^{(2)})^2 dt)^{½}}$$

and

$$M_s = \frac{1}{T}\int_0^T (\eta_s^{(2)})^2 dt$$

In (12) the correlation coefficient R_L and the mean square of the total low frequency disturbance M_L can be obtained directly from the measurements, while the correlation coefficient (R_s) and the mean square of the forced low frequency disturbance (M_s) can be calculated in terms of the primary wave spectrum using (11), (7), (9) and (10). Only the "flat bed" component of the bound long wave defined in (7) is needed here because the 90° phase difference of the extra potential due to varying water depth means (8) does not contribute to the correlation coefficient at zero lag.

Clearly, both R_s and M_s are functions of the directional properties of the primary waves. We make the following assumption about the primary wave spectrum,

$$S_d(f,\theta) = \frac{1}{(\pi\theta_o)^{½}} \exp\left(\frac{-\theta^2}{\theta_o^2}\right) S(f) \tag{13}$$

where S(f) is the usual one dimensional wave spectrum and θ_o defines a mean spread parameter for the primary waves. Different, and perhaps more familiar, forms for directional spread were tried in the analysis but the results were found to be insensitive to the exact form of the spreading function. The exponential form has the advantage that analytical integration is made possible (see next sub-section).

Equation (13) completes the expressions needed to define R_s and M_s in terms of the unknown θ_o. This mean spread parameter can then be obtained making an initial guess for θ_o and

iterating on this starting value until (12) is satisfied. Of course, even though the bound long wave quantities being used here relate to bed pressures they are expressed in terms of the spectrum S(f) of primary waves at the surface. This is obtained from the measured primary wave bed pressure spectrum by applying the square of the inverse of the (linear potential) depth attenuation factor, frequency component by frequency component. Also, in calculating the bound long wave, its "expected" value is obtained from the primary waves. This ignores the phases of the primary waves, as described above under method of analysis; a process that can be justified for long wave records.

2.3 Calculation of the surf beat spectrum

This spectrum is obtained by simply subtracting the calculated spectrum of the forced low frequency disturbance (11) from the measured long wave spectrum. In doing this, some estimate of the bound long wave due to a varying water depth is needed in addition to the "flat bed" bound long wave. We do this making the shallow water wave assumption, kh<1, when evaluating expression (8). This is justified on the basis that this bound long wave component can be shown to be negligible for deep water waves. After much algebra we find its spectrum $S_G(f^-)$ takes the form,

$$S_G(f^-) = \int S(f)\, S(f + f^-)\, I_G df \tag{14}$$

where,

$$I_G = \left(\frac{h\pi}{2g}\right)^{½} \frac{0.0556 g^5 \tan^2\alpha}{\pi^9 h^7 \Theta_o f\, f^{-4} (f + f^-)^4}$$

It is of interest to compare this spectrum with the spectrum $S_F(f^-)$ of the "flat bed" bound wave calculated under the same shallow water wave assumption,

$$S_F(f^-) = \int S(f)\, S(f + f^-) I_F \, df \qquad (15)$$

where

$$I_F = \left(\frac{h\pi}{2g}\right)^{1/2} \frac{0.2813 g^2 f^-}{\pi^3 h^4 \theta_o f^2 (f + f^-)^2}$$

Both spectra in (14) and (15) can be evaluated and then integrated over difference frequencies f^-. For typical parameters we find the ratio N of the bound long wave amplitude due to varying water depth, over the flat bed bound long wave amplitude is,

$$N = \frac{26.4\ T_p^2 \tan\alpha}{h^{3/2}} \qquad (16)$$

Where T_p is the spectral peak period of the primary wave spectrum and the constant has dimensions ($m^{3/2}s^{-2}$). This shows immediately that effects of bed slope will only be important nearshore when the water depth is reduced. It must also be remembered that the Stokes expansions of the type being used in this paper are not valid in very shallow water when the waves become highly non-linear. This will limit the range of validity of the expressions given here. Nevertheless, it is of interest to use (15) to estimate the magnitude of the flat bed bound long wave. Defining the significant value as 4 times the standard deviation we find for a typical wave spectrum,

$$H_s \text{ (bound long wave)} = 0.0413 \left(\frac{h^{1/2}}{\theta_o T_p}\right)^{1/2} \frac{H_s^2 T_p^2}{h^2} \qquad (17)$$

where H_s is the primary significant wave height and the constant has dimensions ($m^{3/4}s^{-3/2}$). This compares with an equivalent expression for uni-directional primary waves,

$$H_s \text{ (bound long wave)} = 0.074 \ \frac{H_s^2 T_p^2}{h^2} \qquad (18)$$

where the constant has dimensions (ms^{-2}). Expression (17) gives bound long waves that, typically, are half the height of those defined by (18) (see also Figure 1). Another difference between multi-directional seas and uni-directional seas is in the shape of the flat bed bound long wave spectrum. It is well known that, in theory, this spectrum has a finite value at zero frequency for uni-directional waves but (15) shows that in shallow water, the bound long wave spectrum in multi-directional waves tends to zero as the difference frequency tends to zero.

Although (17) and (18) are, in theory, limited to shallow water waves they appear, in comparisons with exact calculations, to give reasonable estimates of bound long waves even for $kh \approx 1$.

3. Field data

In applying the method of long wave analysis described in Section 2, exact calculations of the flat bed bound long waves were made using (7). The bound long wave due to varying bed level was then determined using (14). This resulted in only small corrections to the final estimate of surf beat thereby justifying the use of (a conservative) shallow water theory in deriving (14). Examples of these corrections are given for various sites in what follows.

3.1 Port Talbot

In this deployment the pressure sensor was mounted about 4km from the shoreline at the seaward end of the navigation channel leading to the harbour of Port Talbot on the south west coast of Wales. With the large tidal range at the site, depths at the sensor varied from 10m to 19m. Spectral peak periods ranged from 7s to 15s and, typically, significant wave heights were in the range of 1m to 2m but some large wave heights of 4m were measured. Such parameters indicated the main source of wave activity to be the Atlantic as shorter period waves would have been dominant if local fetches were applicable. The seabed slope in the vicinity of the wave recorder was about 1 in 600.

To illustrate the largest correction to the flat bed bound long wave due to a varying seabed level we take the record with the lowest depth. The parameters were,

$$H_s = 1.16\text{m},\ T_p = 12.8\text{s},\ h = 10.2\text{m},\ H_s\ (\text{long wave}) = 0.104$$

Exact flat bed analysis yielded,

$$H_s\ (\text{bound long wave}) = 0.073\text{m with } \theta_o = 10°,\ R_L = -0.60.$$

Here, and in what follows, the term bound long wave is taken to include the Bernoulli pressures in (11) although strictly speaking it should just refer to disturbances due to the second order potentials (7) and (8). Expression (14) leads to an additional bound long wave component with a significant height of 0.016m, ie 22% of the flat bed component. This percentage figure can also be obtained directly from (16). Thus, surf beat height is given by,

$$[(.104)^2 - (0.073)^2 - (0.016)^2]^{½} = 0.072\text{m}.$$

If we had neglected the additional bound long wave component our surf beat height would have been 0.074m and only a small error would have resulted. For the record with the largest water depth (18.9m) the additional bound long wave component was only 10% of the flat bed component and its neglect would have resulted in only a 0.05% error in surf beat height. Because of its small effect on estimates of surf beat it was decided to neglect the additional bound long wave component. However, it should be noted that such a component exists and, according to (16), can be expected to become more important nearer to the shore where it will tend to make the trough of total bound long wave lag behind groups of large waves: an effect also observed by List (1992).

The following table of some of the Port Talbot data gives an idea of the percentage contribution of surf beat to the total long waves measured. It can be seen that surf beat is dominant when primary waves are small but that the bound long wave component begins to dominate when primary wave heights are larger.

Table 1 Results from Port Talbot

Primary H_s(m)	Waves T_p(s)	Measured long waves H_s(m)	Calculated bound long waves H_s(m)	Surf beat H_s(m)
0.49	11.1	0.025	0.006	0.024
0.54	11.1	0.030	0.007	0.029
0.79	6.9	0.032	0.008	0.031
0.80	7.6	0.035	0.011	0.033
2.76	12.8	0.204	0.116	0.168
3.20	13.5	0.374	0.238	0.288
4.12	11.5	0.407	0.349	0.210
4.18	12.0	0.295	0.230	0.185

We know that the bound long wave component will increase with the square of wave height and period to the power 3/2 (see (17)). The above results (as well as those collected at the other sites) indicate that surf beat does not increase as rapidly with the severity of sea state. It is of interest for engineering studies to try and find an empirical relationship for the height of surf beat in terms of primary wave parameters. This was done with the powers of the following three parameters being chosen to minimise scatter in the data,

$$H_s\ (\text{surf beat}) \propto H_s^{\beta}\, T_p^{\gamma} h^{\delta}$$

In the case of Port Talbot, the following relationship was found where wave height and water depth are in metres and wave period is in seconds.

$$H_s \text{ (surf beat)} = 0.0064 \frac{H_s^{1.32} T_p^{1.17}}{h^{0.34}} \tag{19}$$

Scatter in the data was judged by a normalised error parameter and it was found that this error parameter was larger when an empirical relationship of the above type was sort for the significant height of the total long wave component, i.e without first subtracting off the bound long wave energy. This result indicates that the assumptions made about the bound long wave in the method of analysis, are justified. Also, Herbers et al (1992) have provided evidence using bispectral analysis that the bound long wave spectrum in field measurements of long period disturbances, matches the bound long wave spectrum predicted by a second order Stokes expansion.

3.2 Shoreham Harbour

In this case, the pressure sensor was mounted about 2.5km from the shoreline, offshore of the entrance to Shoreham Harbour which lies on the south coast of England. Water depths ranged from 7m to 12m due to the tide and the wave climate consisted mainly of locally generated waves with spectral peak periods from 6s to 10s. Swell, which had propagated up the English Channel from the Atlantic was also present at times with significant heights generally under 1m and spectral peak periods of 12s to 18s.

The seabed in the vicinity of the wave recorder was very flat with a slope of about 1 in 700. The record with the largest slope induced bound long wave had the following parameters,

H_s = 0.6m, T_p = 15.5s, h = 8.8m, H_s (long wave) = 0.055m.

The flat bed bound long wave was 0.024m which, ignoring the slope induced component, gave a surf beat of 0.0495m. Although slope induced bound long wave is 35% of the flat bed component, the inclusion of this component only reduces the surf beat estimate by 1.5% to 0.0488m. This justified neglect of the slope induced component in the other Shoreham records.

The following empirical relationship was found for the resulting surf beat heights,

$$H_s \text{ (surf beat)} = 0.0074 \frac{H_s^{0.93} T_p^{0.99}}{h^{0.06}} \tag{20}$$

3.3 Barrow-in-Furness

Here, the pressure sensor was deployed about 2.5km south of a sand spit, the Isle of Walney, which protects Barrow-in-Furness, on the north west coast of England. Water depths ranged from 4m to 12m due to the tide and locally generated waves occurred with significant heights of up to 2.4m and spectral peak periods of 5s to 8s. Occasional southerly swell from the Irish Sea was able to reach the recorder position with heights of under 1m and spectral peak periods of about 12s.

The seabed slope was very flat at about 1 in 600 and the record with the largest slope induced bound long wave (23% of the 0.10m flat bed component) had a surf beat height of 0.091m which was only 3% less than the surf beat height obtained ignoring the slope induced long wave. This justified neglecting the effect of the slope induced bound long wave.

The following empirical relationship was found for the resulting surf beat heights,

$$H_s\text{ (surf beat)} = 0.0024\ \frac{H_s^{1.08}T_p^{1.59}}{h^{0.36}} \tag{21}$$

4. Discussion and results from other sites

Long wave recording was carried out at two other sites, Dover and Sunderland, but the separation into bound long wave and surf beat proved unsatisfactory in that scatter in the data increased after subtraction of the bound long wave. This may have been due to primary wave reflections from the vertically faced harbour breakwaters affecting the measurements. Ignoring such reflections (which could not be quantified) would tend to lead to overestimates of bound long wave energy under the assumptions made in the analysis and this in turn would under-estimate the surf beat. This problem did not arise for Port Talbot because the breakwaters were of rubble mound construction and highly absorbent of waves. At Shoreham the breakwaters were almost perpendicular to the coastline and directed reflections along and onshore rather than offshore towards the wave recorder while there was only a sandy beach at Barrow.

Averaging the powers of the primary wave parameters in (19), (20) and (21) suggests a variation,

$$H_s\text{ (surf beat)} = K\frac{H_s^{1.11}T_p^{1.25}}{h^{0.25}} \tag{22}$$

where the (dimensional) constant of proportionality K has the not dissimilar values of 0.0044, 0.0066 and 0.0041 for Port Talbot, Shoreham and Barrow, respectively. The largest scatter in surf beat heights occurred for the Barrow Data (see Figure 3).

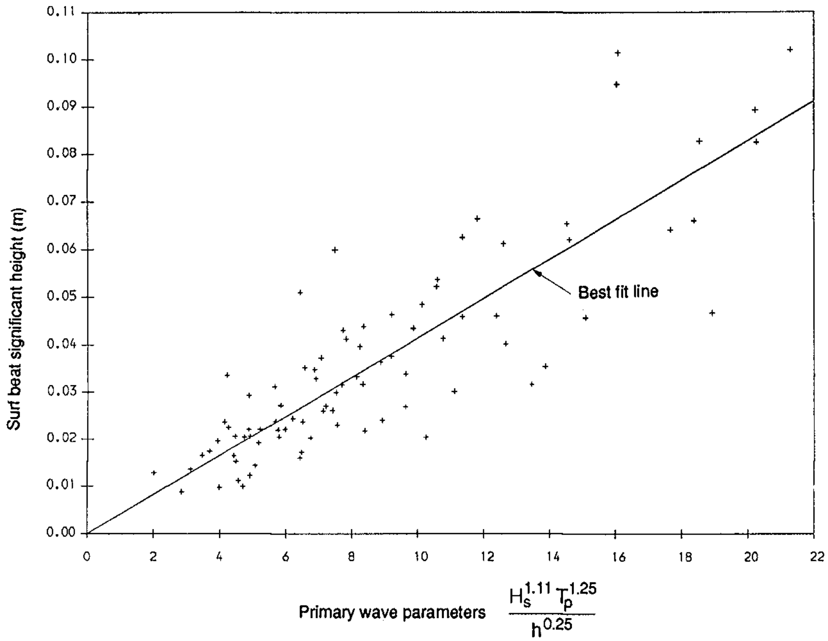

Figure 3 Surf beats off Barrow-in-Furness

This type of result is consistent with the near linear dependence of long wave height on primary wave height found by Tucker (1950) whose measurements appear to have been dominated by surf beat rather than bound long waves. Also, the inverse quarter power of depth (which could of course be fortuitous) is not inconsistent with inverse shoaling of free long waves and high order edge wave modes (Okihiro, 1982).

We can use (22) to predict long wave heights at Shoreham and Barrow for more extreme wave conditions than those measured. [In the case of Port Talbot we already have the data in Table 1]. Limiting primary wave parameters to those where a second order Stokes expansion can be expected to remain valid we obtain the following.

Table 2 Extreme wave predictions for Shoreham in a depth of 12.4m

Primary waves			Significant long wave height (m)		
Return period	H_s(m)	T_p(s)	H_s (bound long wave)	H_s (surf beat)	H_s (total)
10 times/yr	3.3	7.5	0.14	0.16	0.21
1/yr	4.0	8.4	0.27	0.23	0.35
1/10 yrs	4.7	9.2	0.45	0.31	0.55

Table 3 Extreme wave predictions for Barrow in a depth of 13.1m

Primary waves			Significant long wave height (m)		
Return period	H_s(m)	T_p(s)	H_s (bound long wave)	H_s (surf beat)	H_s (total)
10 times/yr	2.4	7.5	0.10	0.07	0.12
1/yr	4.0	9.2	0.32	0.16	0.36
1/10 yrs	4.8	10.0	0.53	0.22	0.57

These results, together with Table 1, show that surf beat dominates the total low frequency wave energy in frequently occurring conditions while bound long waves tend to dominate in more extreme conditions. Thus, surf beat probably consisting mainly of seaward propagating free long waves and along shore propagating high order edge wave modes, has been shown to be an important component of low frequency wave energy in intermediate water depths typical of harbour entrances. This means that both the bound long waves and surf beat need to be well represented in the modelling of harbour resonance and moored ship motions.

Acknowledgements

The author is grateful to Mr J M A Spencer for his help in the writing of the software used for analysis of the data, and to Dr P J Hawkes and Ms C E Jelliman for their help in applying the analysis program to data from the various sites. This work was funded in part by the UK Department of the Environment (CD) and in part by the EC sponsored MAST research programme, G6 Coastal Morphodynamics.

References

Bowers E C (1980). Long period disturbances due to wave groups. 17th ICCE, Sydney, Vol 1, p610, ASCE.

Bowers E C (1988). Wave grouping and harbour design. Proc. Instn. Civ. Engrs, Part 2, Vol 85, p237.

Herbers T H C, Elgar S & Guza R T (1992). Longterm array observations of infragravity waves. 23rd ICCE, Venice, ASCE.

Kostense J K (1984). Measurement of surf beat and set-down beneath wave groups. 19th ICCE, Houston, Vol 1, p724, ASCE.

List J H (1992). A model for the generation of two-dimensional surf beat. J Geophys Res, Vol 97, C4, p5623.

Longuet Higgins M S and Stewart R W (1964). Radiation stresses in water waves: a physical discussion with applications. Deep Sea Research, Vol 11, p529.
Mei C C and Bennousa C (1984). Long waves induced by short wave groups over an uneven bottom J Fluid Mech, Vol 139, p219.
Okihiro M, Guza R T & Seymour R J (1992). Bound infragravity waves. J Geophys Res, Vol 97, C7, p11453.
Roelvink J A, Petit H A H & Kostense J K (1992). Verification of a one-dimensional surf beat model against laboratory data, 23rd ICCE, Venice, ASCE.
Sand S E (1982). Long waves in directional seas. Coastal Eng, Vol 6, p195.
Schaffer H A and Jonsson I G (1990). Theory versus experiment in two-dimensional surf beats. 22nd ICCE, Delft, Vol 2, p1131, ASCE.
Symonds G, Huntley D A and Bowen A J (1982). Two dimensional surf beat: long wave generation by a time-varying breakpoint. J Geophy Res, Vol 87, C1, p492.
Tucker M J (1950). Surf beats: sea waves of 1 to 5 min period. Proc. Roy Soc, Vol A202, p565.
van Leeuwen P J and Battjes J A (1990). A model for surf beat. 22nd ICCE, Delft, Vol 1, p32, ASCE.
Watson C & Peregrine H (1992). Low frequency waves in the surf zone. 23rd ICCE, Venice, ASCE.