CHAPTER 52

WAVE, TURBULENT AND MEAN MOMENTUM FLUXES ACROSS THE BREAKING WAVE TRANSITION REGION IN THE SURF ZONE

R. J. Thieke[1], A.M. A.S.C.E.

ABSTRACT

This paper presents an application to the transition region of an integral momentum conservation model incorporating a Reynolds type decomposition for both wave and turbulent fluctuations. With simple parameterizations the model provides both a rough predictive capability for the limiting wave reduction across the transition zone and also an alternative means of examining the partitioning of flow momentum across the region.

INTRODUCTION

Historically, the quantitative prediction of many surf zone processes has suffered from a lack of sufficient understanding of the detailed hydrodynamics of the wave transformation, wave-induced circulation and turbulent velocity fluctuations across the nearshore zone. The processes of wave shoaling, wave breaking, subsequent wave decay in the surf zone, wave setup and wave induced cross-shore mean circulation occur simultaneously and are intimately interdependent. Their complexity has typically resulted in each process being studied in piecemeal fashion. While important advances have been made, there are notable regions where a thorough understanding is still lacking, particularly the transition region following breaking. The physical processes in the transition (or "outer") region play an important role in the establishment of the flow characteristics of the surf zone, however the dynamics of the region are not well understood quantitatively or qualitatively.

---

[1]Department of Coastal and Oceanographic Engineering, University of Florida, Gainesville, FL 32611, U.S.A.

The transition region begins at the breakpoint and extends shoreward, and it is characterized by a rapid decrease in wave height with almost no change in the setup of the mean water level (Svendsen, 1984). The end of the transition region (hereafter referred to as the transition point) is best defined by the point where the setup begins to increase, which is typically accompanied by a decrease in the rate of wave height decay. The entire region represents a relatively rapid reorganization of the wave motion.

The following analysis presents the application of an integral momentum rapidly varied flow analysis to the transition region. A Reynolds type decomposition for both wave and turbulent fluctuations is incorporated. In view of the lack of detailed experimental data within this region, the simplest possible parameterizations of the mean flow and wave properties are adopted. Nevertheless the model provides both a rough predictive capability for the limiting wave reduction across the transition zone and also an alternative means of examining the partitioning of flow momentum across the region.

BACKGROUND

Wave Height Decay and Rapidly Varied Flow

The rapid nature of change in the transition region render it largely unsuitable for a gradually varied flow analysis or a similarity approach. An example is provided by the energy flux conservation models for wave height decay in the surf zone. Such models solve the depth integrated energy and momentum conservation equations, and differ largely only in the chosen description of the breaking wave energy dissipation. Svendsen (1984) found very poor agreement with laboratory data when wave height decay computations were initiated at the breakpoint; agreement was substantially improved by starting the decay computations after the transition region following breaking, beyond which point surf zone waves are largely self-similar. Dally et al. (1985), found that a similar improvement in prediction could be achieved for certain of their comparisons with laboratory data by initiating decay computations after the transition region.

These observations suggest a simple analogy to jet flows: The transition region serves as the "zone of flow establishment" for the surf zone, while the inner surf zone is the "zone of established flow," in which similarity approaches have met with considerable success. The dynamics of these two regions appear quite dissimilar and warrant independent treatment.

Transition Region "Paradox"

Another significant feature of the transition region is the "paradox" observed by Basco and Yamashita (1986) and Svendsen (1984). This paradox is evidenced by the simplified momentum balance in the x (onshore) direction:

$$\frac{dS_{xx}}{dx} + \rho g(h+\overline{\eta})\frac{d\overline{\eta}}{dx} = 0 \tag{1}$$

where: S_{xx} = "radiation stress"

ρ = mass density

g = gravitational acceleration

h = depth to still water level

$\bar{\eta}$ = mean water level setup

The paradox arises since S_{xx} ("the radiation stress") is known to be a function of wave height and hence is decreasing rapidly across the transition region. The first term in Equation 1 would appear to be finite and negative. Measurements indicate that the setup is constant, so the second term is approximately zero; the equality is not satisfied.

Return Flow Modelling

Spatially varying wave height and setup fields have been used as input to a variety of return flow (or "undertow") models which solve for the distribution of the mean flow in the lower portion of the water column (for example Stive and Wind, 1986; Svendsen and Hansen, 1988). Such models follow the qualitative arguments of Dyhr-Nielsen and Sorensen (1970), wherein the return flow is driven by the vertical imbalance between the gradients of radiation stress and setup that is imbedded (but not explicit) in the depth integrated Equation 1. The models typically employ an eddy viscosity parameterization and differ in the description of the wave parameters (particularly the shoreward mass flux in the crest portion of the wave) and the boundary conditions. The mass flux in the crest region is observed to be substantially greater than that of a nonbreaking wave of equivalent height (Nadaoka and Kondoh, 1982) and is dynamically quite significant; the overall magnitude of the local return flow is largely determined by the value of the shoreward mass flux above the trough (which is entered as a "boundary condition"). Svendsen and Hansen (1988) particularly note that this feature of the flow is rather poorly understood.

The nature of the transition region paradox and the importance of this region to both wave height and return flow modelling and indeed to all surf zone processes suggests that additional analysis of this region is warranted. It should be noted that although these effects are sharply evident for the case of monochromatic waves, the "smearing" of the breakpoint in the random wave case tends to mask the importance of the transition region. However Nairn, et al. (1990) found that the inclusion of the lag effect of the transition region was crucial to the accurate modelling of both regular and random wave cases.

INTEGRAL MOMENTUM CONSERVATION EQUATIONS

In the present context, emphasis is placed on the three hydrodynamic mechanisms which are locally crucial to the various surf zone processes of interest, namely, 1) the (generally nonlinear) wave orbital velocity, 2) the mean flow (often wave induced), and 3) the residual turbulent velocity fluctuations. Throughout the breaking process and wave decay in the surf zone, the motion consists of complicated interactions between these three mechanisms, and the total available flow momentum is partitioned and exchanged between them. Consequently this analysis will include each of these component motions in a Reynolds type representation of the instantaneous velocity:

$$u = \bar{u} + u'' + u' \tag{2}$$

where the overbar indicates an average over the wave period (mean flow), and the double and single primes refer to the wave and turbulent fluctuations, respectively. This decomposition can be applied to the fundamental variables in the shorenormal (x) momentum conservation equation. Time averaging the resulting equation over a wave period and depth integrating produces the following integral equation:

$$\frac{d}{dx}\int_{-h}^{\eta_c} \rho \bar{u}^2 \, dz = -\rho g(h + \bar{\eta})\frac{d\bar{\eta}}{dx} - \frac{d}{dx}\int_{-h}^{\eta_c} (s_{xx}'' + s_{xx}') \, dz - \tau_b \tag{3}$$

with the apparent stresses now retaining their depth variation and defined as:

s_{xx}'' = depth varying wave "radiation stress"
s_{xx}' = depth varying turbulent "apparent stress"

The integral on the right hand side of Equation 3 is essentially equivalent to the conventional radiation stress S_{xx}, however in this instance it includes contributions from both wave <u>and</u> turbulent velocity fluctuations. The momentum flux associated with the mean flow is separated from the "radiation stress" and is represented by the integral on the left hand side of Equation 3.

By neglecting the bed shear stress τ_b, Equation 3 can be used to reevaluate the transition region paradox. The paradox appears to stem partially from the conventional linear wave description of radiation stress in which the mean flow and turbulent fluctuations are neglected. Basco and Yamashita (1986) indicate that the paradox merely reflects the redistribution between the velocity and pressure parts of the radiation stress. The above decomposition explicitly including the mean flow further indicates that a reduction in wave height (and hence s_{xx}'') in the absence of a setup gradient must be balanced by an increased mean flow and/or an increase in the turbulence intensity. The measurements of Nadaoka and Kondoh (1982) confirm that both the mean mass flux and turbulence intensity increase across the transition region. The success of the "surface roller" parameterization within the inner surf zone (Svendsen, 1984) also substantiates the requirement of an increase in mean flow.

The crucial objective in this region is thus to quantify the relative magnitudes of wave, mean flow and turbulent momentum fluxes from the point of incipient breaking to the end of the transition region. This breaking process sets up the entire flow in the inner surf zone, within which the previously mentioned gradually varied flow and similarity approaches may be more suitably employed.

SIMPLE RAPIDLY VARIED FLOW MODEL

Further examination of the partitioning of flow momentum is accomplished by the development of a rapidly varied flow analysis of the transition region in direct analogy to the techniques used successfully for hydraulic jumps and bores in channel flows. Because the gradient of setup is zero, Equation 3 can be reduced to a momentum equality across a "shock" which can be considered short enough to neglect bottom friction:

$$\left[\int_{-h}^{\eta_c} \rho \bar{u}^2 dz + \int_{-h}^{\eta_c} (s_{xx}'' + s_{xx}')\, dz\right]_b = \left[\int_{-h}^{\eta_c} \rho \bar{u}^2 dz + \int_{-h}^{\eta_c} (s_{xx}'' + s_{xx}')\, dz\right]_t \qquad (4)$$

The subscript b denotes the breakpoint and the subscript t denotes the transition point. To employ this relationship, a successful parameterization of the quantities in square brackets is necessary.

As a first approximation, the turbulent contributions to Equation 4 are neglected. For simplicity, linear wave theory is used to evaluate the radiation stress and mean flow. A two layer model dividing the flow at the wave trough level is adopted, following Thieke and Sobey (1990). However in this case the simplest possible "block-type" mean velocity profile is considered with the mean velocity considered uniform within each layer and estimated from linear theory for the upper layer (subscript 1) and the lower layer (subscript 2) respectively by:

$$\bar{u}_1 = \frac{g a^2 k}{2\sigma H} = \frac{gHk}{8\sigma} \qquad (5)$$

$$\bar{u}_2 = -\frac{g a^2 k}{2\sigma h_{tr}} = -\frac{gH^2 k}{8\sigma h_{tr}} \qquad (6)$$

Where:

h_{tr} = water depth below wave trough level

k = wave number

σ = angular wave frequency

a = wave amplitude

H = wave height

Linear theory alone cannot capture the increased mass flux of surf zone waves, and guidance is accordingly sought from experimental evidence. The laboratory measurements of Nadaoka and Kondoh (1982) indicate that the mass flux in a surf zone wave is 2-3 times that of a linear wave of equal height. To accommodate this a constant mass flux correction Q is applied to the velocity profile at the transition point. The value Q = 2.5 was selected to reflect the observations. It is recognized that the wave part of the radiation stress as predicted by linear theory is not a completely accurate representation, however the aerated flow in the surf zone has so far prevented any guidance from experimental evidence in this instance.

Substituting the aforementioned parameterizations into Equation 4 yields:

$$\left[\frac{\rho g^2 k^2}{64\sigma^2}(H^3 + \frac{H^4}{h_{tr}}) + \frac{\rho g H^2}{8}(2n - \frac{1}{2})\right]_b = \left[Q^2 \frac{\rho g^2 k^2}{64\sigma^2}(H^3 + \frac{H^4}{h_{tr}}) + \frac{\rho g H^2}{8}(2n - \frac{1}{2})\right]_t \tag{7}$$

Given the conditions at the breakpoint as known, substituting the appropriate parameters on the right-hand side of Equation 7 gives a single equation for the wave height at the transition point, H_t. An initial estimate of one-half the breaking height ($H_t = 0.5\ H_b$) is used together with a Newton-Raphson convergence procedure to solve for H_t.

COMPARISON TO MEASURED DATA

The model was tested against the laboratory wave data of Hansen and Svendsen (1979). Measured conditions at the breakpoint are input to the model on the left hand side of Equation 7. The measured wave height at the transition point was obtained by determining the wave height corresponding to the sharp increase in the gradient of the setup of the mean water level. The controlling parameters for the various test conditions are given in Table 1.

The model results are shown graphically in Figure 1, which compares (in dimensionless fashion) the predicted wave height at the transition point with the corresponding measured values as a function of the breaking wave height.

Note that the predicted curve (which assumes all lost wave momentum is transmitted to the mean flow), provides an effective upper limit to the transition point wave height, and also explains a large portion of the observed behavior. This is shown schematically in Figure 2, where the departure of the predicted wave height from the best fit line through the measured data can be viewed as an approximate representation of the fraction of the wave momentum converted into turbulent momentum flux. The model indicates that the majority of the wave height reduction in the transition region is associated with the generation of the enhanced mean flow, with a smaller contribution toward the turbulent momentum flux.

Test No.	T (s)	H_o (mm)	H_b (mm)	x_b (m)	h_b (mm)	$\overline{\eta_b}$ (mm)
101101	1.0	100	105.1	22.56	132.9	-1.7
A10112	1.0	70	77.1	23.90	93.9	-1.64
081103	1.25	100	121.6	22.39	137.9	-2.36
A08102	1.25	70	86.8	23.82	96.0	-1.56
A08103	1.25	40	56.5	24.88	65.1	-0.98
061102	1.67	100	139.7	21.91	151.9	-2.74
061091	1.67	90	129.1	22.23	142.5	-2.26
061082	1.67	80	117.8	22.36	138.7	-1.94
061071	1.67	70	103.9	22.99	120.3	-1.80
051071	2.00	70	109.1	23.30	111.4	-2.40
051041	2.00	40	72.9	24.63	73.8	-1.62
041071	2.50	70	128.8	22.56	133.0	-2.30
041041	2.50	40	88.0	24.06	89.2	-1.30
031041	3.33	40	95.4	23.94	92.7	-1.48

Table 1: Wave conditions for various laboratory experiments of Hansen and Svendsen (1979) used as input for rapidly varied flow model simulations.

REFINEMENT OF PARAMETERIZATIONS

Higher Order Wave Theories

Accurate modelling of the transition region requires an adequate representation of the initial conditions, that is, the kinematics of incipient breaking waves. Such waves are highly nonlinear and are typically strongly asymmetric with time due to the effect of the sloping bottom. While the use of a steady wave theory (assuming a flat bottom) is strictly not consistent with these conditions, in the absence of any other practical analytical or hybrid numerical-analytical method such a theory can (if appropriately chosen) provide some predictive capability. Given the highly nonlinear nature of near-breaking waves and the obvious inappropriateness of linear wave theory for this region, it is natural to turn to a higher order wave representation.

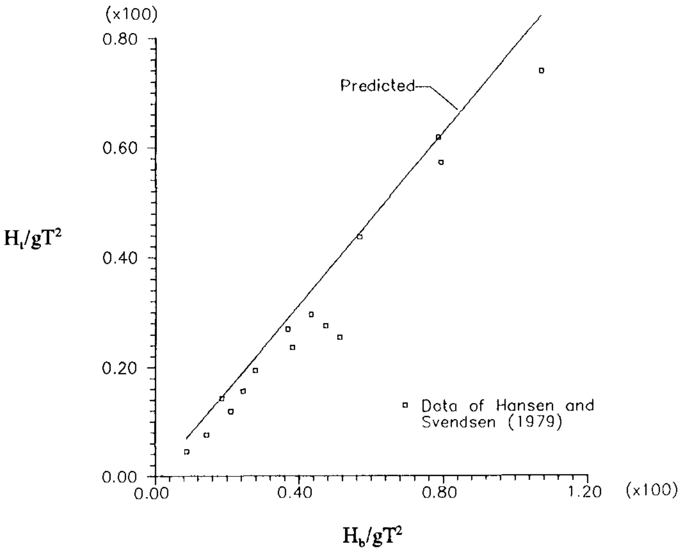

Figure 1: Dimensionless comparison of measured and predicted wave heights H_t at the transition point in terms of the breaking wave height H_b.

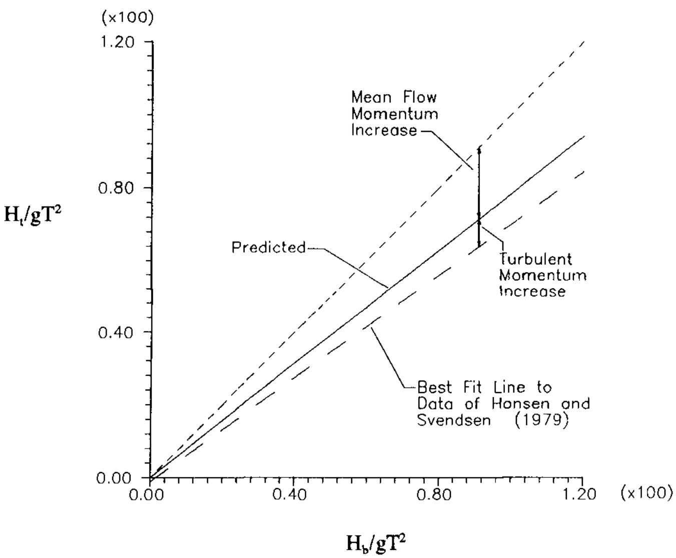

Figure 2: Schematic representation of the transition point partitioning of the increased mean flow and turbulent momentum flux resulting from wave height reduction.

Figures 3a and 3b compare velocity amplitudes measured in the laboratory for waves approaching breaking using a laser doppler anemometer (LDA) (Thieke, 1992), with those predicted by Stokes, Cnoidal and Fourier wave theories of varying order (following Sobey et al. 1987). In each case the velocities are measured at a point just seaward of the breakpoint. Figure 3a (for a relatively short wave) shows that the Fourier theory (to eighteenth order) predicts the positive velocity amplitude tolerably well and the negative amplitude slightly less so. The Stokes wave solutions are not at all useful; the higher order solution shows a clearly recurved profile. Figure 3b (for a somewhat longer wave) again shows that the Fourier wave theory predicts the velocity amplitudes quite well, while a Cnoidal II prediction is quite disparate from the measurements. Higher order Cnoidal predictions diverge quite rapidly under waves of this height (H/h greater than 0.5); Stokes wave solutions at this point were entirely divergent. The performance of the Fourier wave theory in each of these instances suggests that it should yield reasonable predictions of integral properties (mass flux, momentum flux) of waves approaching the breakpoint.

Although some difficulty is encountered in obtaining Fourier wave solutions at the maximum wave heights attained by waves breaking on a slope (which prevents the ready inclusion of such a parameterization in the model), it is generally possible to obtain informative solutions for all but the most extreme near breaking (albeit assumed steady) wave heights.

Evolution of Wave, Turbulent and Mean Momentum Fluxes

The aforementioned predictions from Fourier wave theory, together with existing laboratory measurements in the surf zone, can be used to clarify the relative contributions of the wave, turbulent and mean motions to the total momentum flux as well as the spatial variation of these contributions. Figure 4 shows comparisons of Fourier wave predictions of Eulerian mean velocity with the laboratory LDA mean velocity measurements of Nadaoka and Kondoh (1982) for waves over a plane sloping beach. The example shown is for Case 1, a spilling breaker (wave period T = 1.32 s) on a 1 on 20 slope. Comparisons are shown for three locations. Station P7 (h = 0.347 m) is located just seaward of the breakpoint, while station P4 (h = 0.207 m) is just inshore of the end of the transition region and station P2 (h = 0.107 m) is well within the inner surf zone. The wave heights (scaled from trough and crest elevations) were 0.197 m (P7), 0.161 m (P4), and 0.065 m (P2), respectively. The setup was taken as zero since it was not measured in the experiments. The Fourier prediction of mean velocity in the near breaking wave is quite reasonable, despite the asymmetry and unsteadiness inherent in such a wave. However two observations are significant in the surf zone. The first is that the actual mean flow (as suggested earlier) grows to be much greater than that in a nonbreaking wave of similar height, leading to severe underprediction by the wave theory. Secondly, there are substantial mean vertical velocities present in the surf zone which do not exist outside the breakpoint. Note that these are essential for the maintenance of mass conservation in two dimensions due to the divergence of the horizontal velocity. They cannot be directly predicted by the wave theory.

(a)

(b)

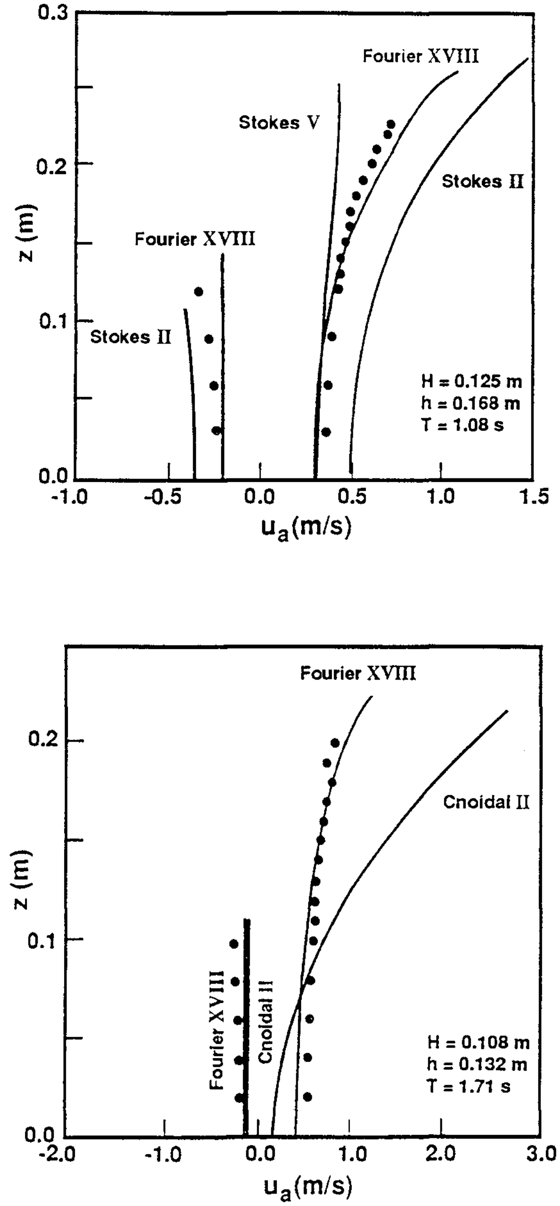

Figure 3: Comparison of LDA measured horizontal velocity amplitudes (•) with wave theory predictions (—).

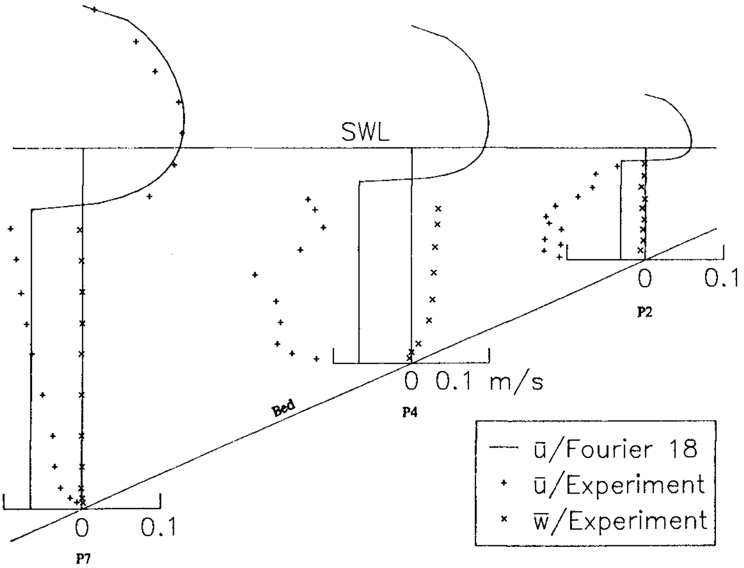

Figure 4: Profiles of predicted Eulerian mean velocities and comparison with data of Nadaoka and Kondoh (1982), Case 1.

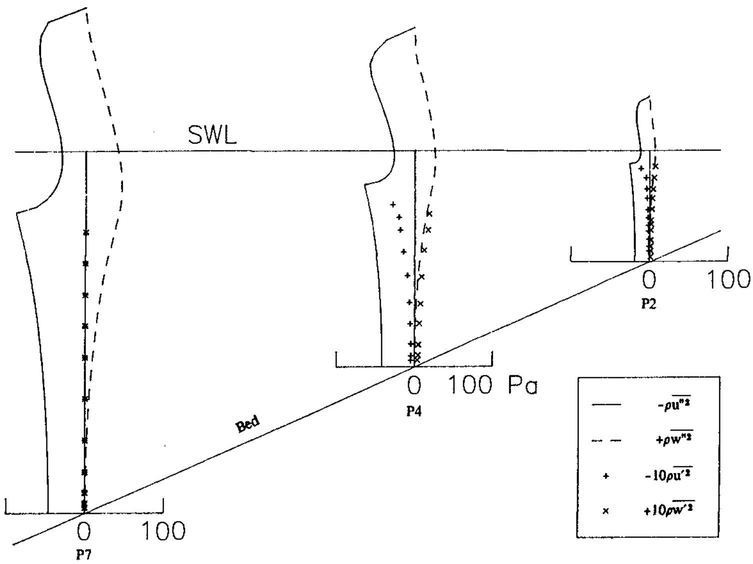

Figure 5: Profiles of predicted wave apparent stress and measured turbulent Reynolds stress from data of Nadaoka and Kondoh (1982), Case 1.

Similar comparisons are made for predictions of wave (double primed) momentum fluxes and laboratory (LDA) measurements of turbulent (single primed) momentum fluxes across the surf zone in Figure 5. It is seen that the turbulent intensities are essentially zero outside the breakpoint, but are "produced" through the transition region and then exhibit decay in the inner surf zone as the shoreline is approached. It is noteworthy that the wave momentum fluxes in the below trough region are approximately an order of magnitude greater than their turbulent counterparts. Although these measurements cannot cover the turbulent aerated crest region due to extensive air entrainment and LDA signal dropout, they indicate that over a large portion of the water column the dominant feature of the momentum balance is still the wave orbital motion, although it is clear that the mean flow has also taken on greater importance than it does outside the surf zone.

Flow Component	Depth Integrated Momentum Flux (in N/m)	
	Station P7	Station P4
$\int_{-h}^{\eta_c} S_{xx}'' dz$	33.0	17.6
$\int_{-h}^{\eta_c} \rho \overline{u}^2 dz$	3.2	10.1
$\int_{-h}^{\eta_{tr}} \rho \overline{u'^2} dz$	≈ 0	0.3
$\int_{-h}^{\eta_{tr}} \rho \overline{w'^2} dz$	≈ 0	0.2

Table 1: Partitioning of total depth integrated momentum flux among wave, mean flow and turbulent components for the experimental test Case 1 (T = 1.32 sec) of Nadaoka and Kondoh (1982). (Mean flow and turbulent components inferred from measured data; wave apparent stress computed from Fourier wave theory)

Figures 4 and 5 together show the global trends of development of surf zone mean and turbulent momentum fluxes, with a limited predictive capability provided by Fourier wave theory. These results can be used to quantitatively analyze the partitioning of total momentum flux between the wave, mean flow and turbulence components across the transition region. For this purpose, locations P7 and P4 of the previous Nadaoka and Kondoh example were again considered. The results of this exercise are given in Table 2. In this instance the wave radiation stress is evaluated from Fourier wave theory, while the turbulent apparent stresses are evaluated from the laboratory measurements and are limited to the below trough region. The depth integrated mean momentum flux is found by integrating the below trough measurements and inferring the remaining contribution through mass conservation constraints.

The wave radiation stress is clearly decreasing across the transition region, however it is in a large part balanced by an accompanying increase in the mean momentum flux. The below trough turbulent stresses are of little consequence. Complete closure is not possible due to the lack of information regarding the turbulent apparent stresses in the crest to trough region (a potentially larger contribution) and the neglect of bed shear. Nevertheless the measurements indicate that a substantial portion of the wave height reduction across the region is tied to an increase in mean flow momentum.

CONCLUSIONS

A simple rapidly varied flow model of the breaking wave transition region provides insight into the mechanics of the region and a certain predictive capability, despite the relatively crude parameterizations employed. The model results support the arguments of Svendsen (1984) and Basco and Yamashita (1986) that only part other wave height reduction in the transition region is the result of energy dissipation (i.e. turbulence production). More specifically, the following conclusions may be drawn:

a) A large portion of the wave height reduction across the transition region results from momentum being transferred to the mean flow, with a somewhat smaller reduction resulting from actual energy dissipation.

b) Linear wave theory (with empirical adjustments) and the assumption of all momentum transfer to the mean flow provides a reasonable upper limit for the wave height at the transition point, despite the crude nature of the assumptions.

c) The separation of the "velocity part" of the radiation stress permits the identification of the distribution of the total momentum flux to the wave, turbulent and mean flow contributions.

d) Below the wave trough level the turbulence contributes minimally to the total balance of momentum (above the trough however the contribution is potentially larger and not well documented).

e) The model described is not truly predictive but rather informative (i.e. the length of the transition region is not determined since the region is effectively compressed into a "shock"). Practical empirical measures for including the finite width of the transition region in surf zone modelling are presented by Nairn, et al. (1990).

f) Detailed measurements of the wave, mean flow and turbulence characteristics in the trough-to-crest region are crucial to a complete understanding of the dynamics of the transition region.

ACKNOWLEDGEMENTS

This material is based upon work supported by the National Science Foundation under Award No. CTS-9111076

REFERENCES

Basco, D.R. and Yamashita, T. (1986). Toward a simple model of the wave breaking transition region in surf zones. Proceedings 20th Coastal Engineering Conference, 2, 955-970.

Dally, W.R., Dean, R.G. and Dalrymple, R.A. (1985). Wave height variation across beaches of arbitrary profile. Journal of Geophysical Research, 90,C6, 11917-11927.

Dyhr-Nielsen, M. and Sorensen, T. (1970). Sand transport phenomena on coasts with bars. Proceedings 12th Coastal Engineering Conference, 2, 855-866.

Hansen, J.B. and Svendsen, I.A. (1979). Regular waves in shoaling water, experimental data. Series paper no. 21, Institute of Hydrodynamics and Hydraulic Engineering, Technical University of Denmark.

Nadaoka, K. and Kondoh, T. (1982). Laboratory measurement of velocity field structure in the surf zone by LDV. Coastal Engineering in Japan, 25, 125-145.

Nairn, R.B., Roelvink, J.A. and Southgate, H.N. (1990). Transition zone width and implications for modelling surf zone hydrodynamics. Proceedings 22nd Coastal Engineering Conference, 1, 68-81.

Sobey, R.J., Goodwin, P., Thieke, R.J. and Westberg, R.J. (1987). Application of Stokes, Cnoidal and Fourier wave theories. Journal of Waterway, Port, Coastal and Ocean Engineering, ASCE, 113, 6, 565-587.

Stive, M.J.F. and Wind, H.G. (1986). Cross-shore mean flow in the surf zone. Coastal Engineering, 10, 325-340.

Svendsen, I.A. (1984). Wave heights and set-up in a surf zone. Coastal Engineering, 8, 303-329.

Svendsen, I.A. and Hansen, J.B. (1988). Cross-shore currents in surf zone modelling. Coastal Engineering, 12, 23-42.

Thieke, R.J. and Sobey, R.J. (1990). Cross-shore wave transformation and mean flow circulation. Coastal Engineering, 14, 387-415.

Thieke, R.J. (1992). Integral properties of steep and shoaling waves. (Manuscript).