CHAPTER 48

ESTIMATION OF IRREGULAR WAVE KINEMATICS FROM A MEASURED RECORD

Rodney J. Sobey¹

Abstract

This review of rational predictive methodologies for irregular wave kinematics confirms the crucial importance of the complete nonlinear free surface boundary conditions. Four methods, the global design wave method, the global linear superposition method, the local Wheeler stretching method and the local Fourier approximation method, have been compared for a very large measured wave from hurricane Camille. Free surface boundary condition errors are comparable to the wave height for the design wave and Wheeler stretching methods. They are sharply reduced by the local Fourier approximation method, though not eliminated. Linear superposition fails completely in the crest-trough region.

Introduction

The success that has been achieved in the theoretical prediction of regular wave kinematics is not immediately transferable to the prediction of surface and near-surface kinematics in irregular waves. Significant predictive difficulties have been encountered, especially near the crest.

In many situations, analysis is based on measured or simulated water surface time histories at a fixed location. Spatial measurements of the sea state are most rare and the vast majority of measured field and laboratory records are discrete water surface $\eta(t)$ records from wave staffs or accelerometer buoys at a fixed location. The balance of the wave kinematics (velocities, accelerations and pressures) at the fixed location needs to be estimated from the known $\eta(t)$ trace. A closely related problem arises in statistical simulation of a random sea state from the linear Gaussian random wave model. Considerable success is achieved in the prediction of the space and time varying water surface. Unfortunately, this

¹Professor of Civil Engineering, Environmental Resources Engineering Group, University of California, Berkeley, CA 94720, USA

success does not carry through to the near-surface kinematics, where spurious predictions result from the increasing dominance of the higher frequency (but much smaller magnitude) spectral components.

Consideration here will be restricted to a subset of this problem where there is no y variation and the direction of both the wave motion and any coexisting Eulerian current coincides locally with the x axis.

Considerable attention has been given to this problem as water velocities, accelerations and dynamic pressures all reach their maximum magnitudes at the water surface. Unfortunately, predictive methodologies are most vulnerable at the water surface, as a direct consequence of approximations in the imposition of the nonlinear free surface boundary conditions. Any methodology for the prediction of irregular wave kinematics that is both rational and viable must give appropriate attention to the free surface boundary conditions.

Global and local methodologies are distinguished. Local methods do not compromise fidelity in the representation of the free surface boundary conditions in the global interest. Global methods, which closely follow regular wave theory, are less attractive for irregular wave kinematics. This paper will compare the predictive capabilities of two common global method (the design wave approach and linear superposition), a common local method (Wheeler stretching) and the local Fourier approximation method. As interest in irregular wave kinematics inevitably centers on big waves, the comparison will be based on a measured wave segment from Hurricane Camille, Gulf of Mexico, 1969. The selected segment has the highest crest height in the sequence and is among the biggest waves ever recorded.

BACKGROUND IN REGULAR WAVE THEORY

Common approaches to the prediction of irregular wave kinematics closely follow the pattern adopted in classical regular progressive wave theory. The discussion is simplified by recalling the basis of the classical theory.

Attention will be directed to the unsteady formulation of regular wave theory. With the velocity potential function $\phi(x,z,t)$ as the dependent variable, the field equation is the Laplace equation

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \tag{1}$$

where the velocity components (u, w) in the fixed frame are $(\partial \phi / \partial x, \partial \phi / \partial z)$.

This field equation is subject to the following boundary conditions: (i) Bottom boundary condition (BBC), representing no flow through the horizontal bed, is

$$w = 0 \quad \text{at } z = -h \tag{2}$$

.....

where -h is the elevation of the bed.

(ii) Kinematic free surface boundary condition (KFSBC), representing no flow through the free surface, is

$$w = \frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} \quad \text{at } z = \eta(x, t) \tag{3}$$

where $\eta(x,t)$ is the free surface.

(iii) Dynamic free surface boundary condition (DFSBC), representing constant atmospheric pressure on the free surface, is

$$\frac{\partial \Phi}{\partial t} + \frac{1}{2}(u^2 + w^2) + g\eta = \overline{B} \quad \text{at } z = \eta(x, t) \tag{4}$$

where g is the gravitational acceleration and \overline{B} is the Bernoulli constant. (iv) Periodic lateral boundary conditions (PLBC), imposing wave periodicity in space and time, are

$$\phi(x,z,t) = \phi(x+2\pi/k,z,t) = \phi(x,z,t+2\pi/\omega)$$
⁽⁵⁾

where k is the wave number and ω is the wave frequency.

(v) Wave maintains a stable profile shape (or permanent form), requiring the wave profile to be symmetric in both x and t about the crest.

Because of conditions (iv) and (v), progressive waves of permanent form are steady in a frame of reference moving at the phase speed $C = \omega/k$. The space and time variations of the water surface must follow

$$\frac{d\eta}{dt} = \frac{\partial\eta}{\partial t} + C \frac{\partial\eta}{\partial x} = 0$$
(6)

....

Predictive Capability of Regular Wave Theory

Established regular wave theories (Stokes, Cnoidal, Fourier approximation) generally take advantage of the relative simplicity of the steady formulation where the independent variables are reduced to x - Ct and z. Basis function predictors of the dependent variable (typically the stream function) identically satisfy both the field equation and the bottom boundary condition, together with the permanent form constraint and the periodic lateral boundary conditions. Compatibility conditions designate the wave height and the co-flowing current.

The essential detail of the solution however is determined largely by the free surface boundary conditions, which must be satisfied along the complete water surface. The complexity of the gravity wave problem is manifested through these free surface boundary conditions, which introduce nonlinearity to the problem and are applicable at a free surface that is itself part of the solution. Different orders of the same wave theory are distinguished by the level of approximation to the free surface boundary conditions, higher orders providing enhanced fidelity.

A conflict is immediately identified between the predictive capabilities of regular wave theory and the nature of the solution field. Wave theory consistently predicts that peak magnitudes and response extremes in velocities, accelerations and dynamic pressures are located along the free surface. Unfortunately, this region of peak interest in the kinematics and dynamics exactly coincides with the region of maximum uncertainty in the wave theory predictions.

Appropriate measures of theoretical error may be formulated from the free surface boundary conditions at the water surface. The kinematic and dynamic free surface boundary condition errors are respectively

$$K(x,t) = w(x,\eta,t) - \frac{d\eta}{dt}$$
(7)

and

$$D(x,t) = \frac{\partial \Phi}{\partial t}(x,\eta,t) + \frac{1}{2} \left[u^2(x,\eta,t) + w^2(x,\eta,t) \right] + g\eta - \overline{B}$$
(8)

In application, these will be represented in length units, as K/ω and D/g respectively, for direct comparison with the local water surface elevation.

For waves of even moderate height, the free surface boundary condition errors associated with the lowest order theory are significant. Predictive difficulties for near-surface kinematics must be expected from adoption of linear wave theory. They are indeed experienced. Nevertheless, there remains a reluctance to reject the simplicity and familiarity of the linear theory, and several measures have been proposed to mitigate the near-surface failings.

Higher order theories by definition do a much better job in satisfying the free surface boundary conditions. They have little theoretical difficulties in the prediction of near-surface kinematics in steady waves, though they may often impose a considerable computational burden. Irregular waves are not steady, but it is clear that fidelity in representing the free surface boundary conditions must have a crucial role in any rational predictive methodology for irregular wave kinematics.

Several unfavorable comparisons of regular wave theory and laboratory measurements have received considerable attention. The evidence however is not conclusive and closer scrutiny (Sobey 1989b) suggests that the predictive potential of regular wave theory remains sound. Errors both in measurement and in application of theory can be substantial and more than sufficient to negate any hasty conclusions.

The influence of current in particular is often not given the attention it deserves. This involves the specification of the appropriate definition (Stokes' first or second) of phase speed and the associated current. The differences are potentially significant and compounded by the fact that many published wave theories have automatically assumed Stokes' first definition of phase speed (and often also zero current) in the problem formulation. Current is assumed to be depth-uniform and any vertical structure is ignored². The adoption of a higher order wave theory may not be realistic where due attention has not been given

²A Slokes theory with a linear velocity profile (i.e. constant vorticity) by Kishida and Sobey (1988) has suggested that vorticity has only a very minor influence. After subtracting out the current profile, the residual "wave" kinematics are little different from those that would have been predicted for a uniform current with the same depth-averaged magnitude. The generality of this observation is presently uncertain.

to the influence of tidal or other ocean currents on the wave kinematics. Alternatively, if the current is not known, higher order precision cannot and should not be expected.

Steady wave theory provides predictions of kinematics in long-crested regular progressive waves. Despite some unfortunate misinterpretation in the literature, steady wave theory does do a credible job where the waves correspond with the assumptions of the theory. The waves must be reasonably long-crested and steady, and the adopted wave theory must be consistent with the field or laboratory conditions. While the basis of wave theory is indeed sound, irregular waves clearly violate the permanent form and periodicity assumptions of the classical steady wave problem. Accordingly, there can be no strong expectation that regular waves.

Generalization to Irregular Waves

The spatial and temporal complexity of irregular waves would initially appear to have little in common with the conspicuous order of regular wave theory. Nonetheless, much of the problem formulation remains appropriate. Irregular waves are by nature unsteady, so that an unsteady formulation is pertinent. The field equation (Eq. 1), the bottom boundary condition (Eq. 2) and both free surface boundary conditions (Eqs. 3 and 4) continue to be applicable; these make up the bulk of the mathematical physics. Neither the periodic lateral boundary conditions, nor symmetry about the crest in x and t are appropriate for irregular waves. Further, application of the KFSBC is inconsistent with the present reliance on measured water surface records at a fixed location, as spatial gradients $\partial \eta/\partial x$ are not available from the measured record.

The crucial aspects of the mathematical physics of the regular wave problem are the nonlinear free surface boundary conditions. The close relationship between regular and irregular wave theory guarantees that the free surface boundary conditions will remain a crucial aspect of irregular wave theories.

GLOBAL APPROXIMATIONS

Methodologies that seek to represent a complete irregular wave, from crest to following crest, from trough to following trough or from zero-crossing to following zero-crossing, are categorized as global. Among global methods, only the design wave and linear superposition methods will be considered here. Fourier-style methods (Dean 1965, Lambrakos 1981) and numerical field solutions (Forristall 1985) have also been proposed.

Design Wave

An obvious candidate is to couple a trough to trough or zero-crossing identification of a wave height and a wave period for an individual wave record with an appropriate steady wave theory prediction for the same height, period, water depth and current. In principle, this is the essence of the design wave approach. As the dominant length and time scales are essentially correct, there is an intuitive expectation that ensuing predictions of crest kinematics have the correct order of magnitude. There can be no expectation however that the predictive capability will consistently exceed order-of-magnitude precision. The associated regular wave theory will predict the water surface profile of the design wave. Regular and irregular wave profiles are unlikely to correspond, so that the free surface boundary conditions will not be satisfied on the actual water surface.

The nature of the design wave approximation is illustrated in Fig. 1, which is based on a measured water surface record from Hurricane Camille in the Gulf of Mexico in 1969. The record sequence is amongst the biggest waves ever recorded. The water depth is 340 ft (103.6 m). In selecting design waves from a measured record, trough to trough identification is often preferred because of its focus on the crest. The height and period are 72.1 ft (22.0 m) and 13.8 s respectively, so that this is a deep water wave. The solid line shows the steady wave theory (Fourier) prediction of horizontal velocity along the predicted water surface, located such that the crest time corresponds with the measured record. Also shown are the KFSBC and DFSBC errors along the measured water surface. These errors are of comparable order to the wave height. The design wave approach will certainly provide order of magnitude estimates of irregular wave kinematics, but consistently better predictions cannot be expected.

Superposition of Linear Waves

A familiar alternative is the superposition of numerous freely-propagating Airy waves, whose amplitudes, frequencies and phases are determined from a discrete Fourier transform of the irregular water surface profile. Consistent kinematics are available from Airy wave theory.

As a direct consequence of the linear superposition approximation, the Fourier transforms all involve relatively simple transformations on the Fourier transform of the irregular water surface profile. The transfer functions (e.g. $\omega CS(kz, kh)$ for horizontal velocity, where $CS(kz, kh) = \cosh k(h + z)/\sinh kh$) are frequency, depth, (wave number,) and elevation dependent, where each of the component Airy waves separately satisfies the linear dispersion relationship. A



Figure 1. Design wave approximation to a measured deep water wave from hurricane Camille. Profile comparisons, horizontal velocity and free surface boundary condition errors along measured water surface.

discrete inverse Fourier transform will predict the time history of the field variable.

Using the FFT algorithm, the numerical procedure is tidy and efficient, but linear superposition does introduce some explicit assumptions regarding the nature of the irregular sea state and its spatial and temporal evolution. The adoption of Airy theory to characterize the separate components assumes that the irregular sea state is composed entirely of free modes, each component being a freely propagating Airy wave that separately satisfies the linear dispersion relationship. There are no bound modes and the FFT algorithm imposes a periodicity in time equal to the duration of the measured record.

In addition, considerable difficulties are encountered at elevations above the MWL, which is the strict upper bound of the Airy solution domain. The hyperbolic function quotients (CS, SS and CC) become exceptionally large for the high frequency (and high wave number) components. Where the record segment is a single irregular wave, the frequency resolution is $\omega_0 = 2\pi/T$, where T is the duration of the record and the approximate period. The frequency of the nth component is $\omega_n = n\omega_0$ and the relative depth $\omega_n^2 h/g = n^2 \omega_0^2 h/g$ rapidly increases with *n*. For $\omega_n^2 h/g > 2.5$, the CS $(k_n z, k_n h)$, SS $(k_n z, k_n h)$ and CC $(k_n z, k_n h)$ hyperbolic function quotients approach $\exp(k_z)$ or $\exp(n^2\omega_0^2 z/g)$. For large positive z in the crest region, these functions enormously magnify the influence of the high frequency components in the Fourier transform or variance spectrum. Kinematic predictions in the crest region become exceptionally large and clearly incorrect. They generally overflow the high number capacity of digital computer software, even in double precision. Any positive value of z is of course outside the strict Airy solution domain, and crest elevations are well above even the water surface of the smaller magnitude higher frequency components.

As these high frequency components generally contribute little to the total variance of the water surface record, low pass filtering of the Fourier transform is a potentially pragmatic response to this difficulty. An application to the Camille record segment is shown in Fig. 2. The fundamental frequency is $\omega_0 = 2\pi/13.8 \text{ s}^{-1}$ or $f_0 = 0.072$ Hz and the record Nyquist frequency is 2.0 Hz. Part (a) shows the measured water surface record together with the horizontal velocity prediction and the KFSBC and DFSBC errors at the measured water surface for a frequency cutoff at 0.1 Hz. This includes only a single frequency component ω_0 . It is effectively a design wave approximation using Airy theory, and the free surface boundary condition errors are broadly comparable to Fig. 1. Part (b) shows the same traces for a 0.15 Hz cutoff. This includes only two frequencies ω_0 and $2\omega_0$, yet predictive problems at the crest are already apparent. The predicted peak horizontal velocity has risen from 14.8 ft/s (4.5 m/s) to 59.6 ft/s (18.2 m/s) and the KFSBC error oscillates off scale. Including also $3\omega_0$ increases the predicted peak horizontal velocity still further to 114.6 ft/s (34.9 m/s).

Clearly, Airy wave superposition, even with an arbitrarily determined cutoff frequency, has no credibility in estimating kinematics in the crest region (Forristall 1985). For elevations below the MWL, linear superposition may continue to provide credible predictions of the wave kinematics.

LOCAL APPROXIMATIONS

Methodologies that seek only to represent the local behavior of an irregular wave are categorized as local. Given that significant problems such as crest kinematics are strongly related to local errors in the free surface boundary conditions, there is evident attraction in such an approach. These methodologies compromise applicability in a global sense in an effort to achieve fidelity in a local sense. Note that this contrasts with the general approach of steady wave theory where local fidelity (especially near the wave crest) is perhaps sacrificed in the global interest.

Stretching Approximations

One form of local approximation, the so-called 'stretching' method of Wheeler (1969), has found considerable favor as a pragmatic approach to the prediction of irregular wave kinematics. Recognizing that the failure of Airy superposition was contributed by extrapolation of the hyperbolic function quotients beyond the upper bound of the Airy solution domain, Wheeler introduced an empirical transformation on the local elevation such that it never exceeds the MWL. The horizontal velocity was predicted as

$$u(x,z,t) = \sum_{j} \omega_{n} \frac{\cosh \alpha k_{n} h}{\sinh k_{n} h} \eta_{n}(x,t)$$
(9)

where $\alpha(x, z, t) = (h + z)/(h + \eta)$, the transformation depending on the local water surface elevation $\eta(x, t)$. Though not defined by Wheeler, consistent definitions for the balance of the kinematics follow directly from the linear superposition approximations.

The stretching transformation shifts the instantaneous water surface to the MWL and avoids the spurious predictions of crest kinematics from direct linear



Figure 2. Linear superposition approximation to a measured deep water wave from hurricane Camille. Horizontal velocity and free surface boundary condition errors along measured water surface.

superposition. It also remains a linear superposition approximation and a linear spectral description of the kinematics. As such, it preserves access to the familiar methodologies of (linear) time series analysis in the time and frequency domains that are common practice in applications such as the dynamic structural analysis of offshore platforms.

Significant problems nonetheless remain. As a result of the stretching transformation, mass and momentum are no longer conserved by the predictive equations for the kinematics. The nature of the residual problems however remains best illustrated in the context of the free surface boundary conditions. The relocation of the local water surface to the MWL does sharply reduce errors in the free surface boundary conditions, as Airy theory imposes the free surface boundary conditions, and the Airy theory does not impose the full free surface boundary conditions, and the omitted nonlinear terms are especially influential in the crucial crest region for the moderate to extreme waves that are common in design.

Smaller magnitude high frequency components in the Fourier transform remain troublesome and Wheeler (1969), for example, adopted a low pass cutoff frequency of 0.3 Hz. An application of the stretching methodology to the Camille record segment is shown in Fig. 3, which shows the measured water surface together with the predicted horizontal velocity, KFSBC and DFSBC traces at the measured water surface; frequencies above 0.3 Hz were excluded from the Fourier transforms. The 0.3 Hz cutoff includes the fundamental frequency $\omega_0 = 2\pi/T$, where T is the duration of the record segment (and the approximate period), together with the first three harmonics $2\omega_0$, $3\omega_0$ and $4\omega_0$. With direct linear superposition (as shown in Fig. 2), the horizontal velocity predictions in the crest region would be extremely large and well off scale, because of the influence the $3\omega_0$ and $4\omega_0$ frequencies on the hyperbolic function quotients in the crest region. The stretching approximation constrains these spurious contributions and



Figure 3 Stretching approximation to a measured deep water wave from hurricane Camille. Horizontal velocity and free surface boundary condition errors along measured WS. Cutoff frequency 0.3 Hz.

provides a prediction of horizontal velocity at the water surface that is similar in magnitude to the design wave approximation in Fig. 1. Whether or not it is a superior approximation is unclear, as both have free surface boundary condition errors of significant magnitude.

The stretching approximation focuses on satisfying the linearized free surface boundary conditions at the measured water surface. This is does exactly for the KFSBC and approximately³ for the DFSBC. But the nonlinear terms become significant, especially the KFSBC term $u\partial\eta/\partial x$ in the crest region. Fig. 3 shows this error to be comparable in magnitude to the wave height.

Local Airy Approximations

Airy wave theory has also been used with locally defined rather than globally defined parameters. Airy theory predicts that the water surface profile is

$$\eta(x,t) = a\cos(kx - \omega t + \theta) \tag{10}$$

where θ is the phase. At a fixed location, the local parameters are the amplitude a, the frequency ω and the net phase $kx + \theta$.

Nielsen (1986, 1989) localized the definition of amplitude, frequency and phase to a moving window of three consecutive water surface observations η_{-} , η_{0} and η_{+} , spaced in time by Δt . Together with Eq. 10, these are sufficient to uniquely define the local amplitude, frequency and phase, as

$$\omega = \frac{1}{\Delta t} \cos^{-1}\left(\frac{\eta_+ + \eta_-}{2\eta_0}\right), \quad kx + \theta = \tan^{-1}\left(\frac{\eta_+ - \eta_-}{2\eta_0 \sin \omega \Delta t}\right), \quad a = \frac{\eta_0}{\cos(kx + \theta)} (11)$$

respectively. The simplicity of this approach is immediately appealing, especially as it accommodates the irregularity by varying the local wave parameters and retains the familiarity and computational simplicity of the Airy wave theory. Unfortunately, Airy theory remains inadequate in the prediction of crest kinematics. Even with the addition of vertical coordinate stretching, it excludes the nonlinear terms in the free surface boundary conditions, especially the $u\partial \eta/\partial x$ term, which are not small in the crest region of moderate and extreme waves.

In addition, Eqs. 11 fail frequently along the water surface on application to strongly irregular waves. $(\eta_+ + \eta_-)/2$ is approximately η_0 , so that the argument of the inverse cosine function in the predictive equation for the local frequency is approximately one. Arguments less than one are always encountered for an exactly linear profile. For a nonlinear wave of approximately permanent form, arguments in excess of one are computed in the neighborhood of the profile MWL crossings. A local frequency cannot be estimated; neither can the local phase and amplitude be estimated, as they depend on the frequency estimate. More significantly perhaps, similar problems are encountered at inflection points and around local minima and maxima in the crest and trough regions, such as are observed in the Camille trace.

(**1 0)**

³But exactly at the smoothed (i.e. after low pass frequency filtering) location of the water surface.

Local Fourier Approximation

As outlined previously, a rational approximation to irregular waves should satisfy the field equation (Eq. 1), the bottom boundary condition (Eq. 2) and the complete form of both free surface boundary conditions (Eqs. 3 and 4); the permanent form and spatial and temporal periodicity constraints are not appropriate. Maximum advantage, at least from a theoretical viewpoint, can be made of a local approximation that does not compromise on these requirements, especially on the free surface boundary conditions along the actual water surface. However, measurements of $\eta(t)$ at a fixed x location provide no information on $\partial \eta / \partial x$. Some compromise is necessary in representing this term, but it should not be allowed to dominate the solution methodology.

A local Fourier approximation (Sobey 1992) provides a pragmatic and rational response to these constraints. As a local approximation method, it enhances fidelity in representation of the crucial free surface boundary conditions and minimizes the influence of the necessary spatial evolution assumption. Further, it is a generalization of the widely successful (but global) Fourier approximation method for regular waves, which has almost universal applicability for both deep and shallow water waves and for coflowing uniform currents. The methodology is extended to complete irregular water surface profiles by means of a moving window of duration τ , which is small in comparison with the local zero-crossing period.

The basis of the method is the representation of the velocity potential function within each window as

$$\phi(x,z,t) = U_E x + \sum_{j=1}^{J} A_j \frac{\cosh jk(h+z)}{\cosh jkh} \sin j(kx - \omega t)$$
(12)

This representation is familiar from global Fourier wave theory (e.g. Sobey 1989a), where $U_{\rm E}$ is the spatially-uniform Eulerian current, h is the water depth, $A_{\rm i}$ are the Fourier coefficients, k is the wave number, ω is the wave frequency and (x,z) is the spatial position in the fixed frame. The current and the water depth define the local propagation medium and must be specified. In Fourier wave theory, ω , k and the $A_{\rm p}$ together with the Bernoulli constant \vec{B} , are a defining set of parameters that have unique values. In the local Fourier approximation method, the defining set of parameters is no longer constant but varies from window to window.

Within each window, the Eq. 12 basis functions exactly satisfy both the field equation throughout the fluid domain and the bottom boundary condition, whatever the numerical values of the defining set of parameters. The defining set of parameters is determined within each window to best satisfy the free surface boundary conditions at the measured elevations of the free surface within the window. In addition, the Bernoulli constant in the DFSBC is not a free parameter, being related to the other solution parameters through an exact integral relationship. In a global approximation, this constraint is implicit in imposition of the DFSBC along the entire water surface from crest to trough.

For each window solution, the given information is the local water depth

h and the local coflowing uniform Eulerian current $U_{\rm E}$, together with a set of water surface elevations η_i , where the i = 1, 2, ... I are distributed over the local window of duration τ .

The temporal and spatial gradients of the water surface in the KFSBC equations remain to be specified. Temporal gradients can be estimated from the water surface time history. Cubic spline interpolation among the measured water surface nodes conveniently provides consistent and smoothly varying estimates of both η and $\partial \eta / \partial t$. The spatial gradient $\partial \eta / \partial x$ is estimated from a locally steady assumption, which imposes Eq. 6 in each local window and relates the spatial and temporal gradients as

$$\frac{\partial \eta}{\partial x} = -\frac{1}{C} \frac{\partial \eta}{\partial t}$$
(14)

where $C = \omega/k$ in the local window. The steady profile assumption is not imposed beyond the local window and does not dominate the solution methodology, as it does for example in the global Fourier-style methodologies.

This equation set is nonlinear and implicit. The primitive unknowns in the local window are ω , k, kx and the A_{i} , of which there are J. There are two independent equations potentially available at each of the η observations within the local window, of which M are selected for the numerical solution. The problem is uniquely defined for M = 3 + J and overspecified for M > 3 + J. In recognition of the certain existence of error bands about the measured water surface elevations, some overspecification is advantageous. Though significantly complicated by the error bands, numerical solution considerations are similar to those encountered in the Fourier steady wave theory; they are discussed in detail by Sobey (1992).

From a strictly numerical viewpoint, the only constraint on the choice of order J and the local window width τ is the $M \ge 3 + J$ requirement. There is a clear expectation however that an appropriate choice of these parameters will be dependent on the physical nature of the water surface time history together with the local resolution of the measured record. With cubic spline interpolation of the water surface record, the time location of individual windows is independent of the order and the window width. Nonetheless, adequate resolution must be provided to capture the temporal variation in the near-surface kinematics.

An application of this methodology to the Camille record segment is shown in Fig. 4. The free surface boundary condition errors are substantially reduced, especially in the crest region where interest is often centered. Overall, errors are small with respect to the wave height, but residual errors persist at profile zero-crossings and around local minima and maxima in the trough region.

Local Polynomial Approximation

Fenton (1986) introduced a local polynomial approximation for the interpretation of submerged pressure records. With some changes to reflect the different context, the local polynomial methodology can be extended to irregular water surface records.



Figure 4 Local Fourier approximation to a measured wave from hurricane Camille. Horizontal velocity and free surface boundary condition errors along measured water surface. J=1, $\tau=0.1$ s.

The wave field is assumed to be locally steady with local phase speed C, such that variations with x and t in the fixed frame can be combined in a locally steady frame as X = x - Ct, as in steady wave theory. The local solution is represented by a truncated polynomial series for the complex potential function

$$\Phi(X,z) + i\Psi(X,z) = \sum_{j=0}^{M} \frac{A_j}{j+1} [X + i(h+z)]^{j+1}$$
(15)

where the a_j polynomial coefficients are real. These basis functions satisfy the field equation and bottom boundary condition exactly. The polynomial coefficients would be determined numerically to best fit the kinematic and dynamic free surface boundary conditions at the known water surface nodes η_i , where the i = 1, 2, ... I are distributed over the local window of duration τ .

The specific equations defining the window solutions closely parallel those for the local Fourier approximation. As ω and k are involved in the local approximation, the local phase speed C is an explicit unknown. Given an estimate of the mean fluid speed in the locally steady frame, such as $\Psi(0,0)/h$ (Fenton 1986), the phase speed is estimated from a local dispersion relationship.

A polynomial variation in the vertical is a feature of the steady Cnoidal wave theory, and it would be expected that this approximation would be most appropriate in shallow water. It may be less satisfactory for deep water waves, where the vertical variation tends to exponential and local Fourier approximation may be more suitable. The local polynomial and Fourier approximations appear complementary.

CONCLUSIONS

Even in regular wave theory, errors are centered on the free surface boundary

conditions. For irregular wave kinematics, a rational predictive capability should give particular attention to the complete nonlinear kinematic and dynamic free surface boundary conditions. Four approaches to irregular wave kinematics, the global design wave method, global linear superposition, local Wheeler stretching and the local Fourier approximation method, have been compared for a very large measured wave from hurricane Camille. Free surface boundary condition errors are comparable to the wave height for the design wave and Wheeler stretching methods. They are sharply reduced by the local Fourier approximation method, though not eliminated. Linear superposition fails completely in the estimation of surface and near-surface kinematics.

REFERENCES

- Dean, R.G. (1965). "Stream function representation of nonlinear ocean waves," Journal of Geophysical Research, 70 4561-4572.
- Fenton, J.D. (1986). "Polynomial approximation and water waves." In Procs., 20th International Conference on Coastal Engineering, Taipei. Vol. 1., ASCE, New York, 193-207.
- Forristall, G.Z. (1985). "Irregular wave kinematics from a kinematic boundary condition fit (KBCF)," Applied Ocean Research, 7, 202-212.
- Kishida, N., and Sobey, R.J. (1988). "Stokes theory for waves on linear shear current," Journal of Engineering Mechanics, 114, 1317-1334.
- Lambrakos, K.F. (1981). "Extended velocity potential wave kinematics," Journal of Waterway, Port, Coastal and Ocean Division, ASCE, 107, 159-174.
- Nielsen, P. (1986). "Local approximations: A new way of dealing with irregular waves." In Procs., 20th International Conference on Coastal Engineering, Taipei. Vol. 1., ASCE, New York, 633-646.
- Nielsen, P. (1989). "Analysis of natural waves by local approximations," Journal of Waterway, Port, Coastal and Ocean Engineering, 115, 384-396.
- Sobey, R.J. (1989a). "Variations on Fourier wave theory," International Journal for Numerical Methods in Fluids, 9, 1453-1467.
- Sobey, R.J. (1989b). "Wave theory predictions of crest kinematics." In Procs., NATO Advanced Research Workshop on Water Wave Kinematics, Molde, Norway. (Eds. Torum, A. and Gudmestad, O.T.) Kluwer Academic Publishers, Dordrecht, 215-231.
- Sobey, R.J. (1992). "A local Fourier approximation method for irregular wave kinematics," *Applied Ocean Research*, 14, 93-105.
- Wheeler, J.D. (1969). "Method for calculating forces produced by irregular waves." In Procs., 1st Annual Offshore Technology Conference, Houston. Vol. 1., 71-82.