CHAPTER 46

The Effect of Sheared Currents on Wave Kinematics and Surface Parameters

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ABSTRACT

Steep, steady non-linear waves have been generated onto strongly sheared currents. Quantitative, full-field velocity measurements have been made for a number of wave/current combinations using the method of Particle Image Velocimetry (PIV). For one of the combinations, the measured kinematics are compared to the estimate obtained by adding the stretched current profile to the predictions from irrotational theory. The steadiness of the conditions is assessed by calculating the vorticity distribution for different cycles in the wavetrain. Finally, the measured wavelengths for a range of wave frequencies are used to estimate the effective current.

INTRODUCTION

Knowledge of representative wave kinematics is crucial in the understanding of coastal processes and the design of offshore structures. Often, waves ride on top of steady currents, generated by the tides or the wind, and its is important that the combination of the kinematics in these cases can be understood and predicted.

High-order methods have been used for some time to predict the properties of steep waves, in the absence of current, and have been extensively verified by laboratory studies. On a uniform current the combined wave-current kinematics can be calculated by changing reference frame to one moving with the current and using a high-order model, after Doppler shifting the wave frequency. Several authors have recently tackled the problem of steep waves on currents of a arbitrary profile [Dalrymple and Heideman, 1989, Chaplin, 1990] and have implemented numerical models, but experimental studies are still rare.

Swan [1990] produced moderate amplitude waves on strongly sheared currents and addressed some of the difficulties posed by Dalrymple and Heideman in

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their cautionary note on tank testing. Other notable experiments include those of Thomas [1990], where good agreement was obtained between laboratory measurements and numerical predictions for weakly sheared currents, and Kemp and Simons [1983] whose study concentrated on the effects near the bed.

There is a severe experimental difficulty encountered when measuring currents with non-uniform profiles near the surface, in the presence of waves. Whichever end of the flume the currents are injected from, the wave action will ultimately disturb the formation of the upper part of the profile. One solution to this problem is to make the measurements of the combined kinematics during the short time window once the waves are established, but before the current formation is affected and reflected waves return. However, even with this approach the steadiness of the conditions must still be questioned.

Experimental Conditions

Experiments were performed in a purpose built wave flume, depth .75m, capable of producing forward or reverse currents with various profiles. The experimental arrangement is shown in figure 1.

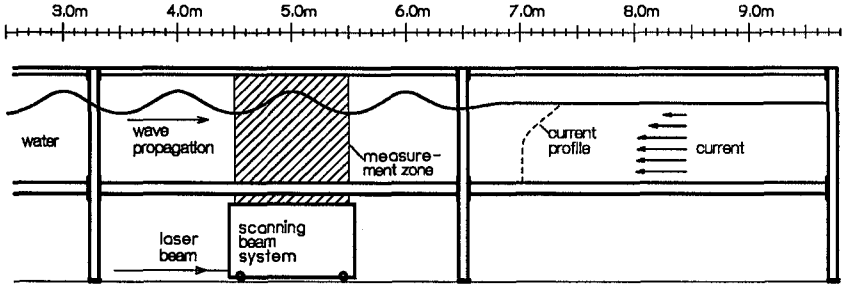

Figure 1: Wave flume used for the kinematic measurements

The main current profile required for the experimental programme was one which was strongly sheared in the direction of wave propagation. The current was generated by introducing a flow opposed to the waves along the bottom half of the flume. In the subsequent analysis, the bulk value of the current was altered to yield the desired profile in the chosen reference frame, and the wave frequency modified with the appropriate Doppler shift. If the frequency in a given frame is f_1, then the frequency measured in a second frame, moving with velocity u compared to the first, is given by

$$f_2 = f_1 - \frac{u}{\lambda} \tag{1}$$

The most strongly sheared current profile used in the tests is shown in figure 2, along with the rms turbulence level. The profile has an approximately linear form,

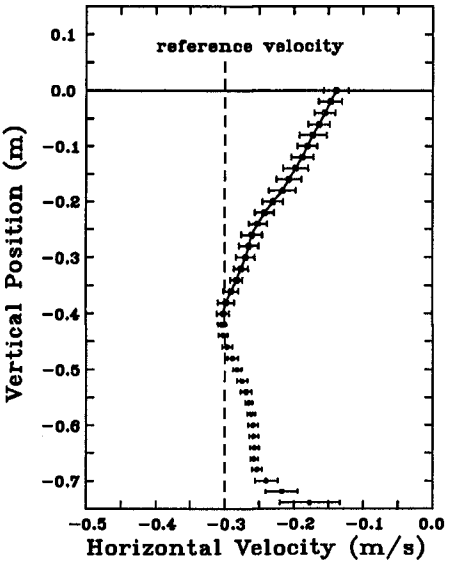

Figure 2: Current profile, sheared in the direction of wave propation, used in the experiments

velocity gradient $0.427s^{-1}$, in the top half of the water depth and was shifted to a frame of reference moving at $-0.3ms^{-1}$ when analysing the combined kinematics. The form of the current near the bed should be disregarded when assessing the profile, as the wave action in this region is small.

Experimental Measurements

In the experimental test sequence, steep waves were run onto the current and kinematic measurements made once the waves were steady and before reflections came back. In addition, estimates of the wavelength were obtained from wavelength records at different positions in the tank and from the photographs of the flow.

The combined kinematics were measured by the method of Particle Image Velocimetry (PIV), in which small seeding particles are multiply exposed by a pulsed light sheet in the flow and their images recorded onto film. From the separations of the images in each part of the film the local velocity at that location can be deduced, and hence the complete flow field established. Further details of the use of the technique in this and other hydrodynamic studies are described in Greated et al [1992].

A typical flow field measured in the study is shown in figure 3 as a vector

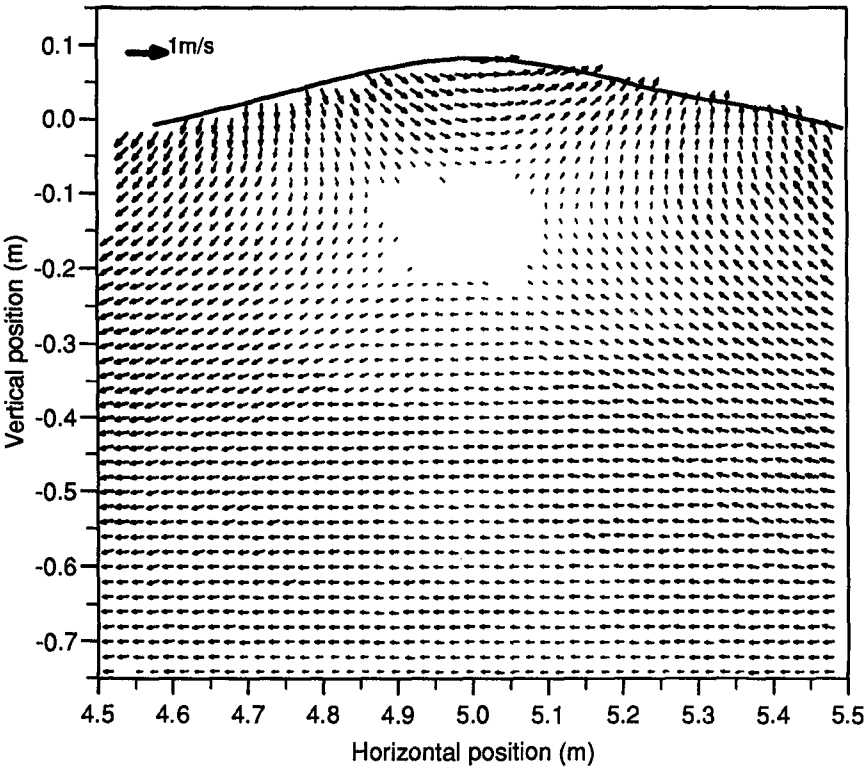

Figure 3: Internal kinematics of a wave on a sheared current measured with PIV

plot. Measurements were achieved up to the surface, and the measurement error was typically less than 2% of the maximum velocity, depending on the velocity gradients present. The missing data in the central part of the wave corresponds to a region of near-zero velocity in the flow, where the crest kinematics of the wave and the opposing current produce an instantaneous stagnation point. This results in overlapping particle images on the PIV photograph, whose separations cannot be resolved. Measurements could be made in this region by using an image shifting technique [Bruce,1992].

For the strongly sheared current, six wave conditions, were studied in detail. These wave cases covered a range of steepnesses (H/λ) from .054 to .098, and a range of water depth to wavelength ratio (h/λ) from .27 to .57. Results for these wave cases and other current profiles have been reported by the authors [Skyner,1992,1993] . The results presented here are for the central wave case on the strongly sheared current (H/λ=.0729,h/λ=.412).

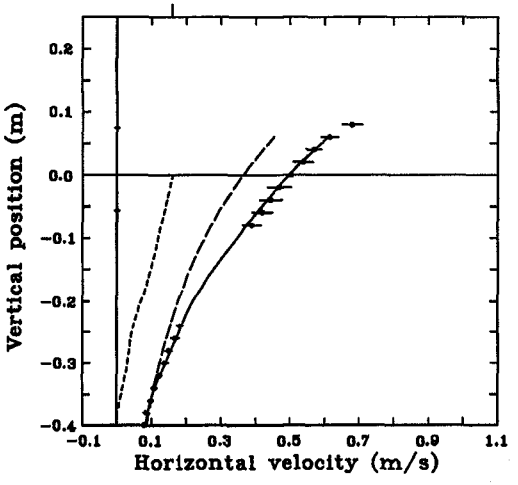

Figure 4: Horizontal velocity profiles under the crest, for a wave on a strongly sheared current. -•- experimental data, with scatter, - - - - undisturbed current, — — irrotational numerical model, ——— prediction from numerical model and stretched current profile

Measured Kinematics

The experimentally measured horizontal velocity profiles under the crest for the central wave condition, in the presence of the strongly sheared current is plotted in figure 4, along with numerical predictions. The graph contains the undisturbed current profile, the velocity profile under the crest for the combined flow, a numerically generated profile for an irrotational wave with the same height and wavelength, and the numerical data combined with the stretched current profile. The irrotational numerical data was generated by a program based on the Fourier approximation method of Rienecker and Fenton [1981]

The method used for obtaining the combined kinematics was that recommended by the UK Department of Energy [1990], when a complete wave/current model is not available. The current value at a given height above the bed is moved to a new location by multiplying the distance from the bed by the factor $(1 + C_N/h)$, where C_N is the crest elevation, obtained numerically, and h the water depth. The recommendation is that no attempt be made to conserve the total mass flux of the current when stretching.

For wave case being considered, a composite vector map was constructed from flow records at different phases, in order to obtain information over a complete wavelength. From this data the average horizontal velocity component was obtained at various levels, and the results are plotted in figure 5. A significant reduction in the average shear is apparent, greater than that which would be

expected if the assumed current stretching applied over the whole wave, when the shear would decrease in the crest but increase in the trough, leading to an average value only slightly less than the undisturbed current value.

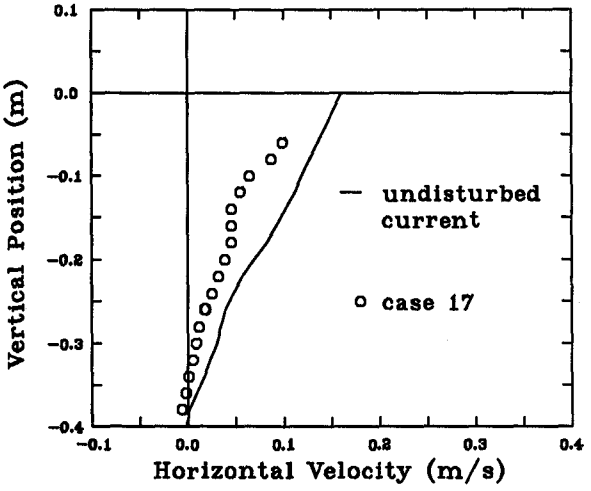

Figure 5: Mean horizontal velocities for medium steepness waves on a strongly sheared current.

Derived Vorticity

Calculating vorticity from the experimentally measured flow fields is a profitable exercise because it separates the irrotational part of the motion, associated with the wave, from the rotational part, associated with the sheared current. It is possible to extract vorticity from the data obtained with PIV, because the velocity field is known over two spatial dimensions. The vorticity component perpendicular to the measurement plane is

$$\Omega_y = \frac{\partial u_z}{\partial x} - \frac{\partial u_x}{\partial z} \tag{2}$$

A reasonable estimate for the local vorticity can be obtained by numerically differentiating the velocity data, which is available on a regular grid. In order to reduce the effect of turbulence and reveal the underlying trends, the velocity fields obtained from a number of repeats of each wave phase were averaged before calculating the vorticity.

Figure 6 contains a shaded contour plot of vorticity, obtained from the repeats of the flow field illustrated in figure 3. Light areas corresponds to negative vor-

ticity (forward shear), and dark areas to positive vorticity. The blank area in the centre of the wave is due to missing data, no attempt begin made to interpolate.

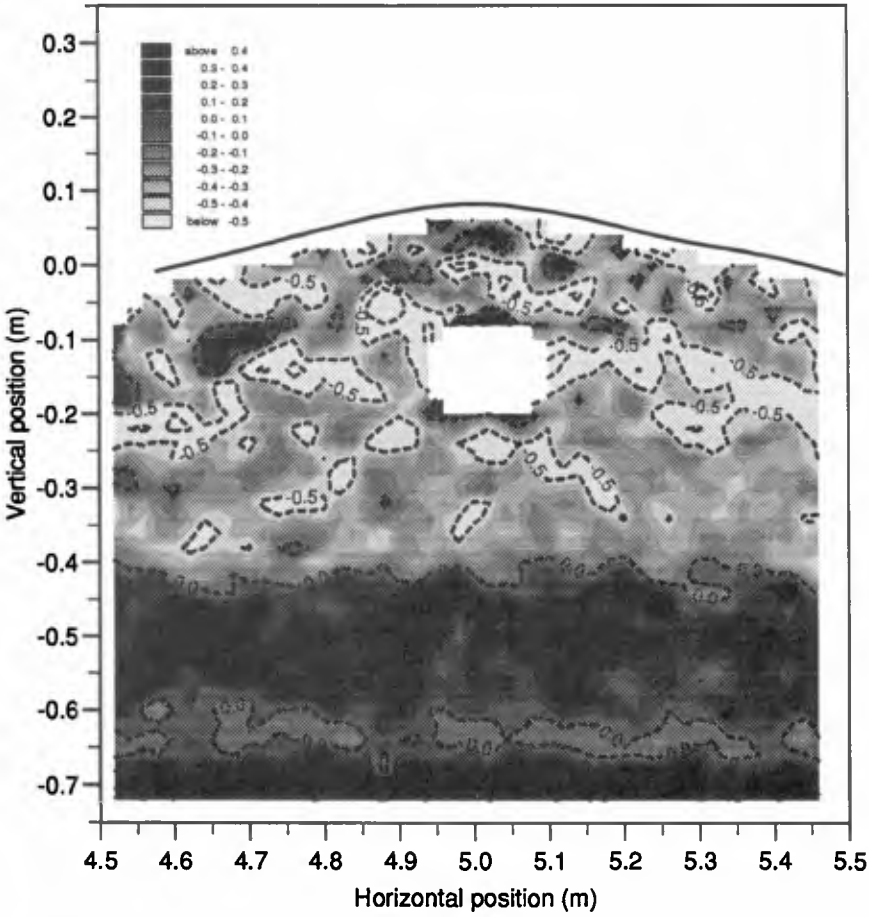

Figure 6: Vorticity contours for the average of 6 repeats of a medium amplitude wave on the strongly sheared current, *first* wave crest. Contour line interval: $0.5s^{-1}$

Comparison with the undisturbed current profile shown in figure 2 reveals that the vorticity field has the expected features. The bed boundary layer is present as strip of high positive vorticity, and there is a major transition from positive to negative vorticity at around $z = -0.4$m. Apart from the small pockets of turbulence, the vorticity in the upper part of the wave is fairly constant with an average value between $-0.3s^{-1}$ and $-0.4s^{-1}$.

The available time window for the kinematic measurements of the wave case

being considered corresponded to four wave periods. This portion of the wave elevation timeseries is shown in figure 9 as a continuous trace. The kinematic measurements presented here are for the first wave crest in this "steady portion" of the wavetrain at $t = 12.4$s. PIV measurements were also made for the third and fifth crests in this portion.

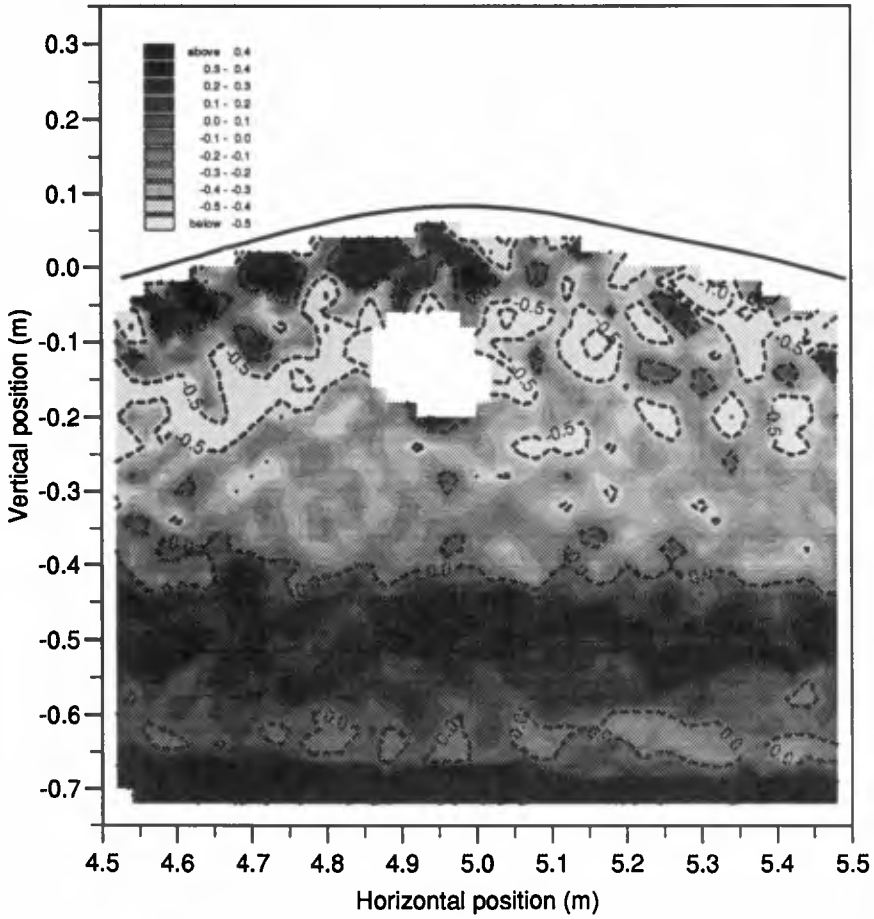

Figure 7: Vorticity contours for the average of 6 repeats of a medium amplitude wave on the strongly sheared current, *third* wave crest. Contour line interval: 0.5s^{-1}

Figure 7 contains the vorticity levels found in the third crest of the steady portion of the wave train. While the vorticity distribution is similar to the earlier cycle of the wave in the middle and lower parts, sizable areas of positive vorticity have appeared near the surface.

In figure 8 the areas of positive vorticity near the surface have increased still further compared to the previous cycles of the wave. The position of the crest is also noticeable retarded compared to its expected position in the middle of the plot.

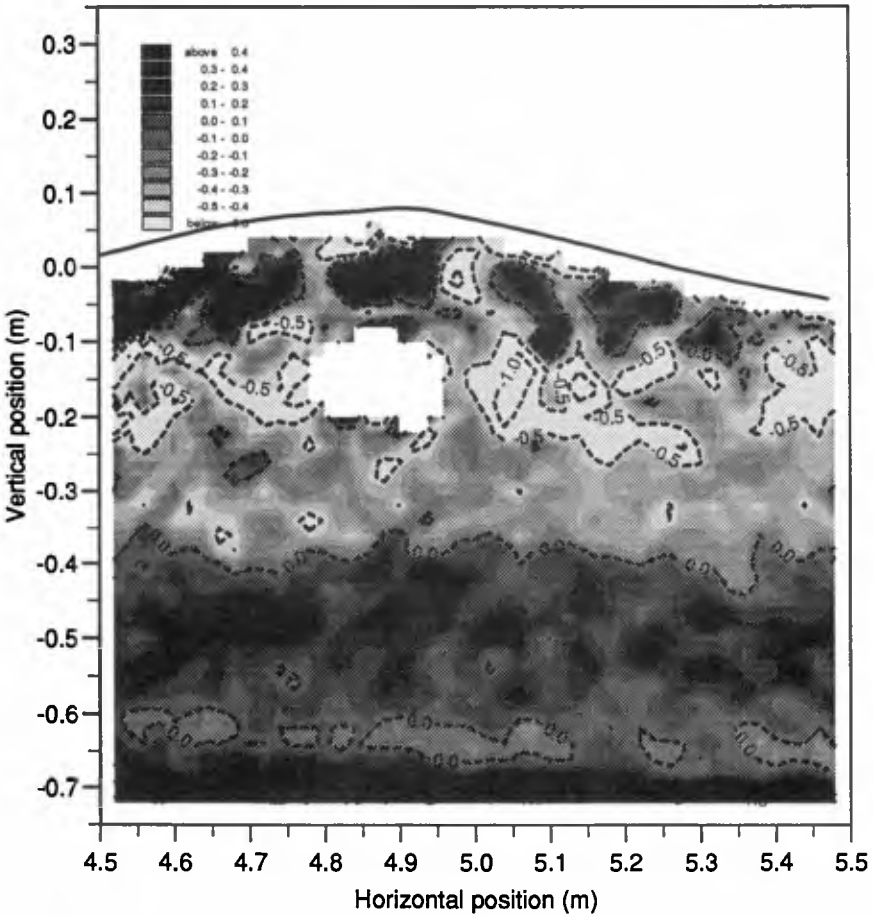

Figure 8: Vorticity contours for the average of 6 repeats of a medium amplitude wave on the strongly sheared current, *fifth* wave crest. Contour line interval: $0.5s^{-1}$

Figure 9 also shows the average vorticity estimated for the top half of the wave (above $z = -0.4$m). These values were calculated for each available wave crest by averaging the vorticity data, shown in figures 6 to 8, with no attempt being made to interpolate missing values. A clear trend is apparent, with the average vorticity moving away with time from the undisturbed current value towards zero.

This is equivalent to a decrease in the shear of the underlying current.

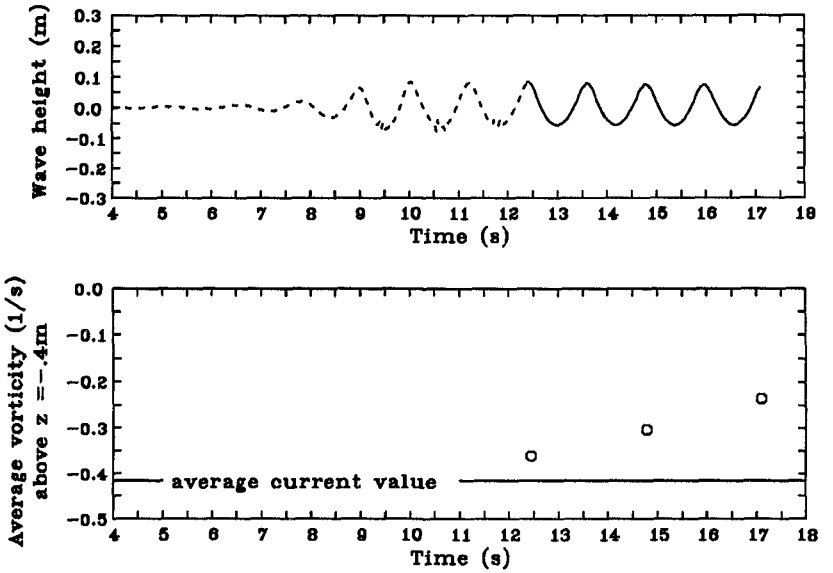

Figure 9: Average vorticity value obtained for different crests in the wavetrain, along with the wave elevation timeseries where the continuous trace represents the "steady" portion of the wave

Wavelength

The measured wavelengths for the cases selected for the kinematic measurements were found to correspond, within experimental error, to those predicted from irrotational theory, given the wave height and wave frequency, after Doppler shifting the frequency to a frame moving with the surface current. From a sequence of tests, sweeping through a range of frequencies of small amplitude waves, the effective current was calculated from the measured wavelength using Doppler theory. The uniform current with the equivalent effect on the wavelength is plotted against wave frequency in figure 10. The values of the surface current and the current at half the depth are also shown.

Discussion

The estimate for the kinematics made by adding the stretched current to irrotational predictions assumed that the underlying current was not effected by the wave action, other than by being stretched according to the phase of the wave. However, calculation of the vorticity for different cycles in the "steady" portion

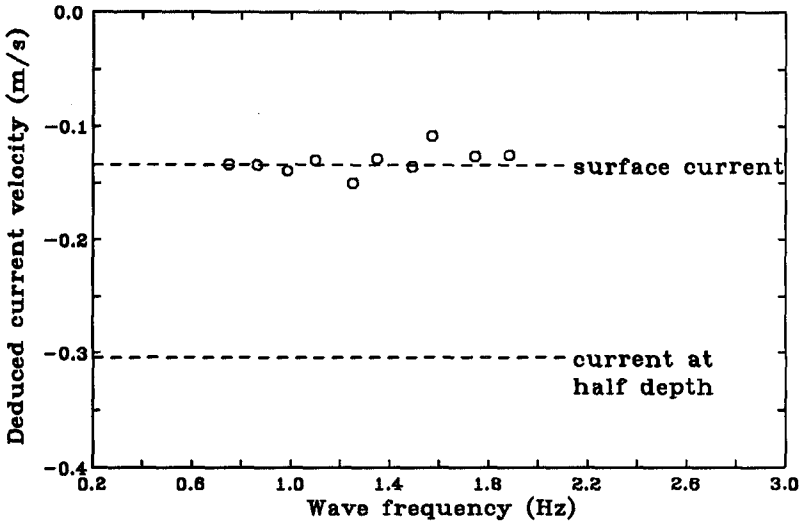

Figure 10: Equivalent uniform current velocity deduced using Doppler theory from measured wavelength on sheared current

of the wave train indicates that the rotational part of the fluid motion is changing with time. This may be due to the presence of turbulence influencing the wave/current interaction. However, the presented results should be viewed with some caution, as although care was taken in the experiments when establishing the wave train that the leading wave did not break, it is possible that interactions occurring at the front of the wave group may have been advected back to the measurement zone, effecting the later wave cycles.

Conclusions

For the conditions considered, the kinematics in the crest were found to be well predicted by combining the results of an irrotational, high-order numerical model with the stretched current profile. In addition, it was found that the wavelength was accurately predicted using irrotational theory, after Doppler shifting the wave frequency to the frame of reference of the surface current. However, there was strong evidence that the conditions were not steady, particularly the change in the vorticity field between the cycles in the wave train.

Acknowledgment

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