CHAPTER 18

CHARACTERISTICS OF A SOLITARY WAVE BREAKING CAUSED BY A SUBMERGED OBSTACLE

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ABSTRACT

This study aims to make quantitatively clear the breaking wave characteristics of a solitary wave incident to a submerged obstacle by performing intensive numerical simulations using a boundary integral method. The incident critical wave height, the break point and the breaker height are formulated through regression analyses based on the simulated results and are shown to be determined uniquly by using the regression equations.

INTRODUCTION

A sound knowledge on breaking wave characteristics of coastal waves over a submerged obstacle is clearly important for designing coastal structures. Although many investigations have been made on the characteristics, most of them are concerned with waves on uniformly sloping bottoms. Smith and Kraus(1991) performed laboratory experiments to measure macro-scale properties of breaking waves on barred beaches but simply showed that wave-breaking properties differed on a barred profile. Very little quantitative knowledge is hence avaliable on the breaking wave caused by a submerged obstacle.

It is widely recognized that steep waves on beaches resemble solitary waves. They are in fact representable as a train of solitary waves[Tuchiya & Yasuda, 1984]. It may be hence better to consider each individual wave of coastal waves over a submerged obstacle as an independent solitary wave and to investigate its breaking characteristics, rahter than to examine directly those of the waves. Further, considering a submerged step and dike as submerged obstacles, we could reduce the breaking wave problem of the steep coastal waves over a submerged obstacle to that of a solitary wave incident to the step or dike.

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In this study, we focus our attention on the relation of the geometric quantities of the incident wave and obstacle to the breaking wave characteristics and investigate the following three items and their governing law; i)Critical incident wave height H_c which gives the limiting wave height of the incident wave passing through a obstacle without breaking, ii)Break point and iii)Breaker height. For this purpose, we perform the intensive numerical simulation using a computational model[Yasuda et al.,1990] to describe the time evolution of a solitary wave propagating on a step or dike and formulate three items mentioned above based on the simulated results. The model used here can solve numerically but almost exactly the hydrodynamic equations for fully nonlinear two-dimensional irrotational flows over uneven bottoms and can describe accurately the deformation up to overturning of a solitary wave incident to a step or dike. Its validity has been already verified by comparing the computed results with the experimental measurements[Yasuda et al., 1992]. It could therefore be stated that the results obtained here through the numerical simulation have sufficient accuracy.

MODIFIED SURF-SIMILARITY PARAMETERS

It is well-known that the breaker height and type of periodic waves on a plane slope with the gradient of $\tan\theta$ are described by the surf-similarity parameter $\xi = \tan\theta/(H_0/L_0)^{0.5}$, where H_0 and L_0 are respectively the wave height and wave length of incident waves in deep water. This parameter can be derived as the ratio of the perturbation term due to depth change to the nonlinear term of incident periodic waves from the variable coefficient KdV equation.

We now regard periodic waves with the incident wave height H_1 on a plane bottom with the still water depth h_1 as a train of solitary waves with the same wave height H_1 and rewrite the nonlinear term by using the loweat order solution of the solitary wave on the plane bottom , $\eta = H_1 \mathrm{sech}^2[\sqrt{(3/4)(H_1/h_1)}x/h_1]$, instead of that of the sinusoidal wave, $\eta = (H_0)/2\cos(2\pi x/L_0)$. As a result, we can obtain the following parameter for the solitary wave on a plane slope with the same gradient $\tan\theta$,

$$\xi_s = \tan\theta/(H_1/h_1)^{0.4}. \tag{1}$$

If we then consider the depth change perturbation due to a submerged solid step with the crown height R (Fig.1) instead of that due to the plane slope and rewrite the perturbation term, we obtain ,through some trial and error, the following modified surf-similarity parameter for the solitary wave which has the initial wave height H_1 and is incident to the submerged step.

$$\xi_s^* = (R/h_1)^{0.1}/(H_1/h_1)^{0.4}. \tag{2}$$

The slope gradient $\tan\theta$ is omitted in eq.(2) becouse its influence on the breaking characteristics is almost negligible as explained afterwards.

Further, considering the depth change due to a submerged symmetric trapezoidal dike with the crown height of R, the crown width of B and the slope gradient of $\tan\theta$ schematically shown in Fig.2, we can obtain another modified surf-similarity parameter,

$$\xi_s'' = [B/h_1 + (R/h_1)/3.5\tan\theta]^{0.2}(R/h_1)/(H_1/h_1)^{0.4}, \quad (3)$$

for the solitary wave with the initial wave height H_1 incident to the rectangular trapezoidal dike. This parameter can be rewritten into the parameter $\xi_s' = (B/h_1)^{0.2}(R/h_1)/(H_1/h_1)^{0.4}$ for the solitary wave incident to a rectangular dike by putting the value of the slope gradient $\tan\theta$ in eq.(3) as infinity. The parameter ξ_s'' can thus be regarded as a unified parameter applicable to both the dikes.

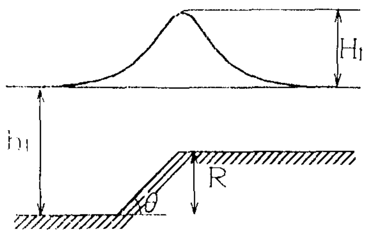


Fig.1 Schematic illustration of solitary wave over a submerged step

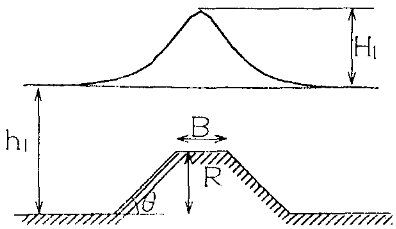
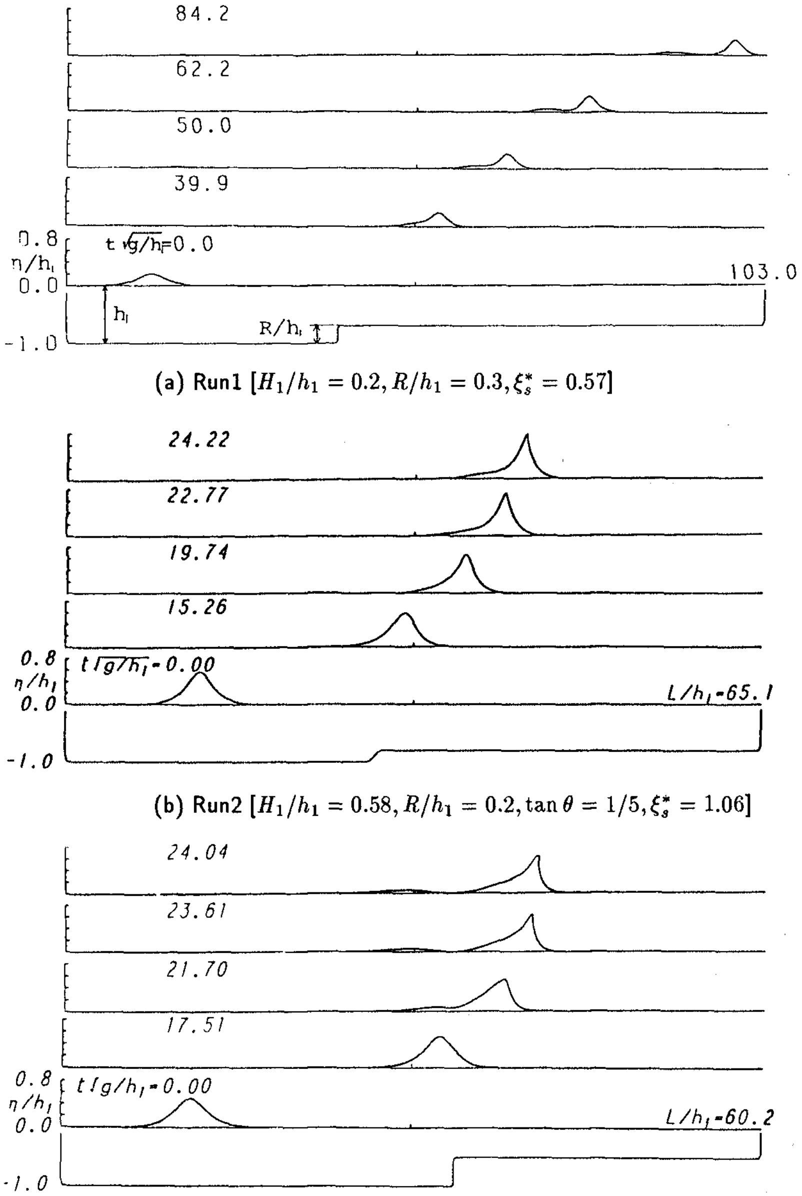


Fig.2 Schematic illustration of solitary wave over a submerged dike

BREAKING CHARACTERISTICS OF A SOLITARY WAVE INCIDENT TO A STEP

Wave profiles

Figure 3 describes representative examples of the propagation process of a solitary wave propagating on a bottom containing a solid step. It is easily found that the stage of the deformation caused by the step clearly depends on the value of ξ_s^*. Run 1 shows the propagation process of a solitary wave passing over a step without breaking. Not only a reflected wave but also a shelf is generated by the depth change due to the step . The shelf is not indefinitely carried behind the transmitted wave, but separates from it and finally turns to a secondary solitary wave. This means that when an incident solitary wave is perturbed by a step, it tries to be stable by generating a reflected wave and a shelf turning to a secondary solitary wave and distributing the excess energy to them. Both of Run 2 and 3 show the propagating process leading to wave breaking. The onset of wave breaking is defined here as the instant when the front face at the top of the solitary wave becomes just vertical. The validity of this definition is experimentally confirmed by the fact that the onset almost agrees with the time when the



(a) Run1 $[H_1/h_1 = 0.2, R/h_1 = 0.3, \xi_s^* = 0.57]$

(b) Run2 $[H_1/h_1 = 0.58, R/h_1 = 0.2, \tan\theta = 1/5, \xi_s^* = 1.06]$

(c) Run3 $[H_1/h_1 = 0.5, R/h_1 = 0.6, \xi_s^* = 1.25]$

Fig.3 Time evdution of water surface profile of a solitary wave incident to a submerged step formation of a jet at the crest is initiated in the real fluid[Yasuda et al.,1992]. While the breaker profile of Run 2($\xi_s^* = 1.06$) seems to be a spilling breaker, that of Run 3($\xi_s^* = 1.25$) could be regarded clearly as a plunging breaker because of its remarkably vertical asymmetry. Although the shelf is generated in both the cases, it runs just behind the transmitted main wave until the transmitted wave begins to break. This means that these depth change perturbations caused by the step are too strong for the excess energy distribution by exciting the reflected wave and shelf to prevent the breaking.

Figure 4 shows the comparison with the wave profile at the onset of the wave breaking among a solitary wave incident to a rectangular step and those incident to trapezoidal steps with a face slope of gradient $\tan\theta = 1/5$ and $1/10$. The breaker profiles almost agree with one another except for the reflected waves and shelves. We could hence say that the gradient of the face slope very little affects the breaking characteristics and its influence is negligible, although it affects on the height of the reflected wave and shelf.

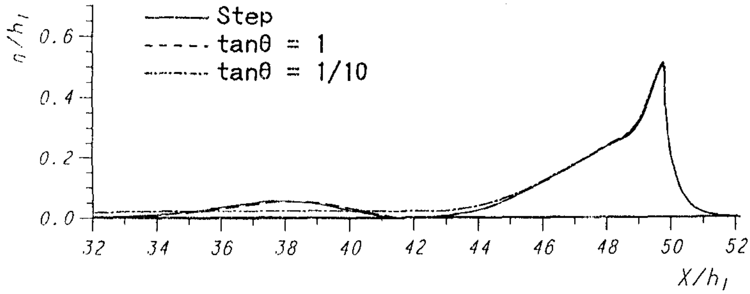


Fig.4 **Comparison of water surface profiles of breaking wave between solitary waves over steps with a vertical face and a slope**

Critical incident wave height

When some investigations are made on the characteristics of breaking of a solitary wave caused by a step, it is required at first to answer the question whether the solitary incident to the step breaks or not and make clear the presence of wave breaking. If the value of the critical incident wave height H_c is available for each dike, the answer to the question can be easily obtained because the value of H_c gives the limiting wave height of the incident solitary wave which can pass through the step without breaking, that is, the solitary wave of which incident wave height H_1 exceeds H_c necessarily breaks on the step.

Figure 5 shows the relation of the presence of wave breaking due to a rectangular step to the relative wave height H_1/h_1 and the relative step height R/h_1.

The circles ○ indicate the solitary waves that break on the step and the crosses + indicate other solitary waves that progress on the step without breaking. The solid line is given by the regression equation,

$$H_c/h_1 = 1.012 - 1.063(R/h_1)^{0.46}, \quad 0.2 \leq R/h_1 \leq 0.6, \tag{4}$$

and indicates the relation of the relative critical wave height H_c/h_1 to the relative step height R/h_1. Although eq.(4) are based on the computed results concerning a solitary wave incident to a rectangular step, it can directly be applied to that incident to a step with the slope grandient over 1/10. Hence, we can now easily answer the aforementioned question almost independently of the slope gradient, only if we substitute the values of the still water depth h_1, the incident wave height H_1 and the crown height R of the step into eq.(4) and examine whether the value of H_1/h_1 exceeds that of H_c/h_1 or not.

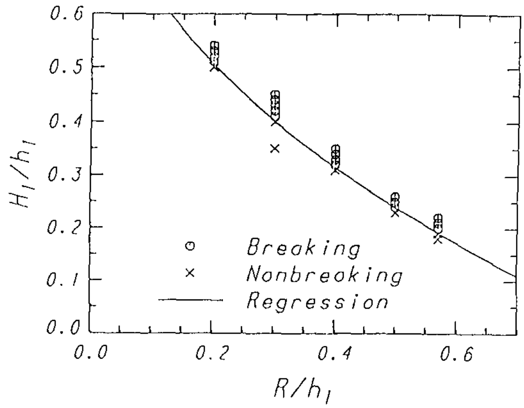


Fig.5 Relation of the presence of breaking caused by a step to the relative wave height H_1/h_1 of an incident solitary wave and the relative step height R/h_1

Break point

We here aim to establish the method to predict the break point of the solitary wave incident to the step and reply to the question, 'Where the incident wave breaks ?'. The break point is defined as the location just under the main crest at the onset of wave breaking and indicated by the break distance X_b from the top corner of the step to the location.

Figure 6 shows the relation between the ratio X_b/h_1 of the break distance to the still water depth and the following parameter γ derived as a function of H_1/h_1 and R/h_1 by a regression analysis,

$$\gamma = (0.78 - H_1/h_1)/(R/h_1)^{-0.83}. \tag{5}$$

The solid line is given by the regression equation,

$$X_b/h_1 = -27.292 + 52.495\gamma^{1.664}, \quad 0.87 \le \gamma \le 1.03. \tag{6}$$

The circles ○ denote the results computed for the solitary wave incident to the rectangular dike. The computed results scatter along the regression curve and hence could be regarded to agree considerably well with it. It is found that the break distance X_b naturally becomes short and the break point tends to approach the top corner of the step as the values of H_1/h_1 and R/h_1 approach 0.78 and 1, respectively. However, the break distance X_b is still greater than ten times of the still water depth h_1 under the present condition($0.87 \le \gamma \le 1.03$). This states that the incident waves never break at once after the perturbation due to depth change has affected them but require propagation distances up to breaking. The results shown in Fig.6 demonstrate that the location of the break point approximately obeys eq.(6) and is determined by the value of γ alone. We can thus easily predict the break point of a solitary wave incident to a step by using eqs.(5) and (6) although not exactly.

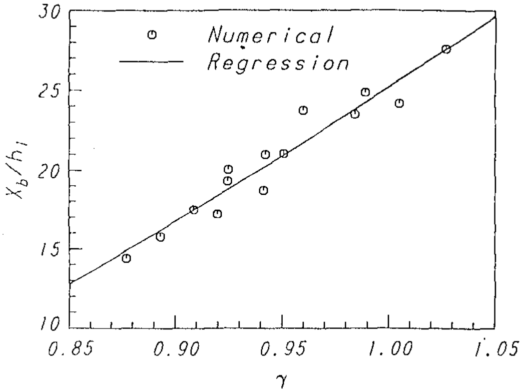


Fig.6 Relation between the break distance X_b from the top corner of a step to the break point and the parameter γ

Breaker height

Breaker height H_b is the wave height at the onset of wave breaking and is given here as the wave height at the instant when the top front face of the main crest becomes just vertical. The breaker height usualy indicates the maximum value during the propagation process. On the other hand, the breaker depth h_b is equal to the submerged crown depth of the step and is given as $h_1 - R$.

Figure 7 shows the relation between the relative breaker height H_b/h_1 of the solitary wave incident to a step with the slope gradient of $\tan\theta = 1/5, 1/10$ and ∞(vertical) and the modified surf-similarity parameter ξ_s^* defined in eq.(2). The circles ○, triangles △, asterisks $*$ and squares □ in Fig.7 denote the computed results for the step with vertical slope and the slope gradient of $\tan\theta = 1, 1/5$

and 1/10, respectively. The solid line is given by the regression equation,

$$H_b/h_1 = 5.885 - 5.090\xi_s^{*0.133}, \quad 0.2 \le \xi_s^* \le 1.1. \tag{7}$$

It is found that all the relative breaker heights H_b/h_1 agree well with the regression curve independently of the slope gradient($\tan\theta = 1/10 \sim \infty$) and that the values of them are uniquely determined by the parameter ξ_s^* alone. We can hence easily estimate the breaker height of a solitary wave incident to a step with arbitrary slope gradient by substituting the value of ξ_s^* into eq.(7). Further,the so-called breaker depth index H_b/h_b is derived by multiplying the relative breaker height H_b/h_1 into $h_1/(h_1 - R)$.

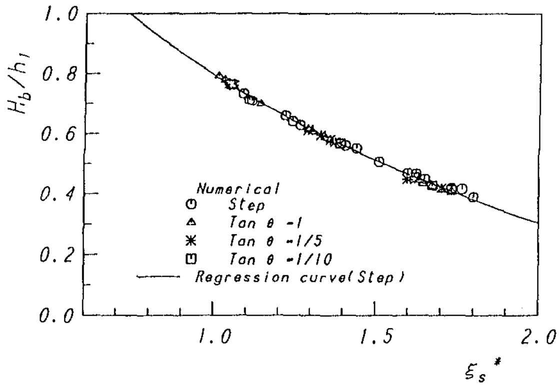
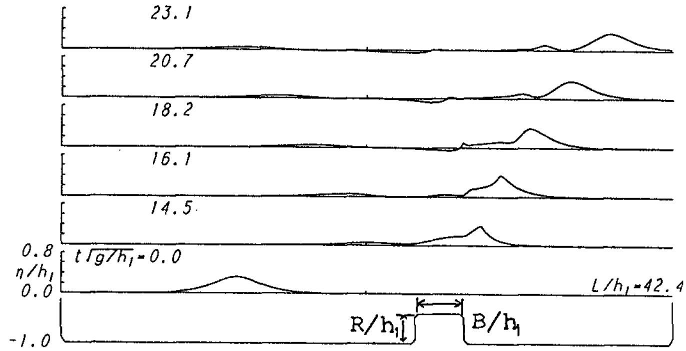


Fig.7 Relation of the breaker height of a solitary wave incident to the submerged step having the slope gradient of $\tan\theta$ = 1, 1/5, 1/10 and ∞(vertical) to the parameter ξ_s^*

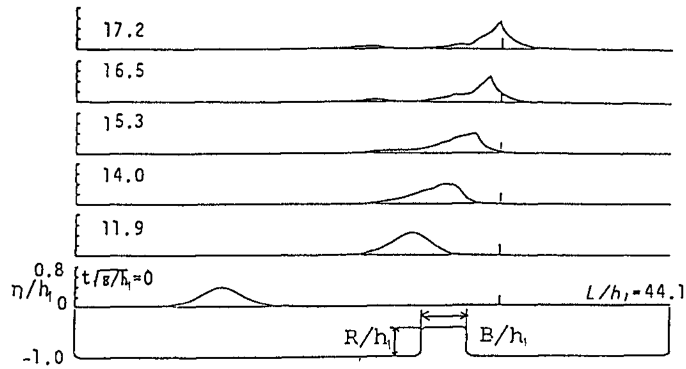
BREAKING CHARACTERISTICS OF A SOLITARY WAVE INCIDENT TO A DIKE

Wave profiles

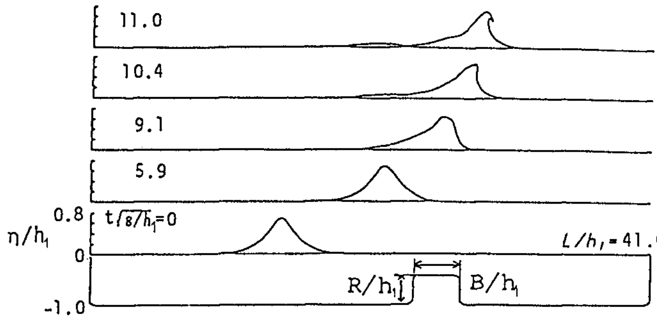
Figure 8 describes the time evolution of a solitary wave propagating on a bottom containing a solid submerged dike. Run 4 shows the propagation process of a solitary wave passing through the rectangular dike without breaking as well as Run 1 shown in Fig.3. Reflected waves of two types are generated in this case; one is a positive reflected wave due to the depth decreasing and another is a negative one due to the depth increasing. However, it is similar with Run 1 that the shelf is generated and finally turns to a secondary solitary wave. This could be said to be a common feature of the solitary wave perturbed by depth change. The breaker profile of Run 2 seems to be a typical spilling breaker, while those of Run 3 and Run 4 could be regarded clearly as plunging breakers because of their vertically asymmetric profiles. However, all of them are in common forward



(a) Run 1

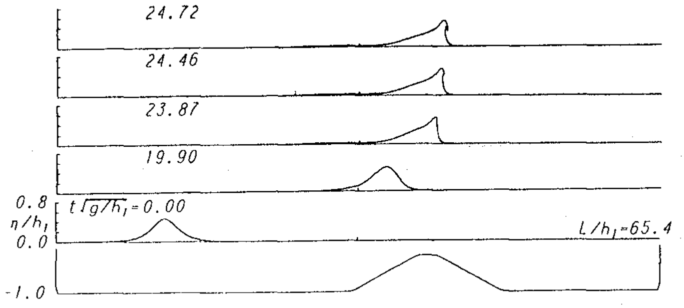


(b) Run 2

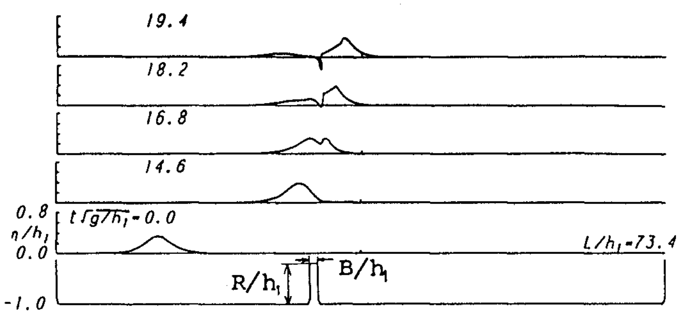


(c) Run 3

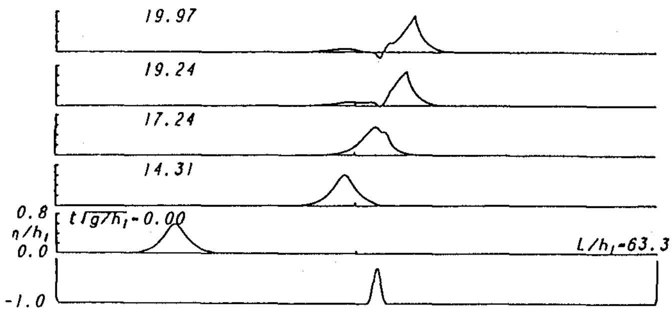
Fig.8 Temporal changes of water surface profiles of a solitary wave passing over a submerged dike



(d) Run 4



(e) Run 5



(f) Run 6

Fig.8 (Continued)

breakers in which the main crests break forward after they have passed through the dike. Whereas the case of Run 5 is remarkably different from them at the point that the surface of the wave on the shelf breaks backward onto the dike. This breaking was firstly found by Cooker et al.(1990) and was called a backward breaker. It is characterized by the backward breaking that occurs before any other breaking event. However, its influence is conjectured to be small and negligible because the height of shelf itself is considerably small in comparison with that of the main crest. Run 6 seems to be the composite type of the backward breaker and the spilling breaker, but this should be strictly classified as a spilling breaker because the forward breaker of the transmitted wave occurs a little faster than the backward breaker of the shelf.

Critical incident wave height

In order to answer the question whether a solitary to a submerged dike breaks or pass through without breaking, we investigate its critical incident wave height H_c to the trapezoidal dike with the relative crown height R/h_1, the relative crown width B/h_1 and the slop gradient $\tan\theta$. Here, rectangular dikes are included in the trapezoidal ones, because they are regarded as trapezoidal dikes with the vertical slope as mentioned above.

Figure 9 shows the relation between the critical incident wave height H_c/h_1 to each dike and the following parameter γ derived as a function of its geometric quantities , $R/h_1, B/h_1$ and $\tan\theta$, by a regression analysis.

$$\gamma = [(B/h_1) + (R/h_1)/2\tan\theta]^{0.4}(R/h_1), \tag{8}$$

The symbols of triangle $\triangle$, asterisk $*$ and square $\square$ respectively denote the values of H_c/h_1 to the trapezoidal dikes with the slop gradient of $\tan\theta$=1, 1/5 and 1/10 and the circles $\bigcirc$ denote those to the rectangular dikes. These values are obtained by examining the limiting incident wave height of the solitary wave that comes onto each dike and can pass through without breaking. The solid line is given by the regression equation,

$$H_c/h_1 = 0.952 - 0.591\gamma^{0.76}, \quad 10.2 \le \gamma \le 1.4. \tag{9}$$

The computed results agree well with the regression curve and could be said to obey almost eq.(10). This states that the critical incident wave height H_c/h_1 to a dike uniquely relates to a single parameter γ and its value is uniquely determind by γ alone. We can hence easily answer the aforementioned question, only if we calculate the value of H_c/h_1 by subsituting the geometric quantities of the dike, $R/h_1, B/h_1$ and $\tan\theta$, into eqs.(9) and (10) and examine whether the value of the incident wave height H_1/h_1 exceeds the value of H_c/h_1 or not.

Break point

All of the forward breakers simulated here occur behind dikes. If a incident wave breaks on a dike before passing through it, the breaking shoud hence be treated as the breaker due to the step and is thereby excluded here. As well as the case of the rectangular step, we investigate here the relation between the relative break distance X_b/h_1 from the top corner of the rectangular dike to the break point and the following parameter μ derived as a function of H_1/h_1, R/h_1 and B/h_1 by regression analysis,

$$\mu = (0.78 - H_1/h_1)^{0.5}(R/h_1)^{-1.2}(B/h_1)^{-0.13}. \tag{10}$$

Figure 10 shows the relation of the break distance X_b/h_1 to the parameter μ. The circles ○ denote the results computed in this study and the solid bent line is given by the regression equations,

$$\left.\begin{array}{rcl} X_b/h_1 &=& 10.78\mu - 4.14, \;\; 0.6 \leq \mu \leq 1.03 \\ X_b/h_1 &=& 1.94\mu + 5.01, \;\; 1.03 \leq \mu \leq 2.2 \end{array}\right\} \tag{11}$$

It is noticed that the break point of the solitary wave incident to a dike is shorter compared with that of the step(see Fig.4). This cause is conjectured to be the influence of the twice depth change due to the dike on the incident wave. Since the computed results agree well with the regression curve, the relation governed by eq.(11) seems to hold approximately between the incident wave height and the geometric quantities of the dike. We can hence predict the location of the break point of a solitary wave incident to a rectangular dike by using eqs.(10)and (11).

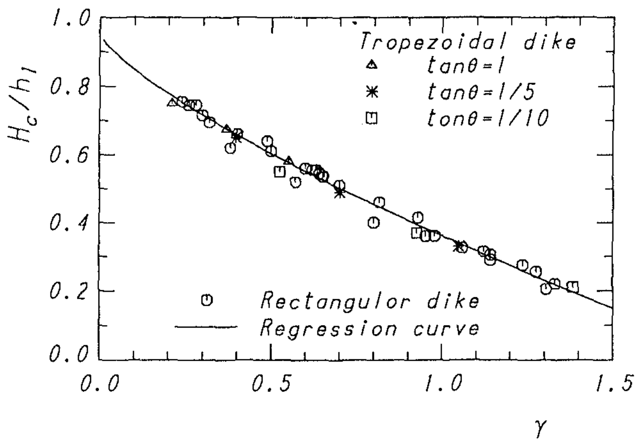


Fig.9 Relation of the critical wave height H_c/h_1 of a solitary wave incident to a submerged rectangular and trapezoidal dike to a parameter γ

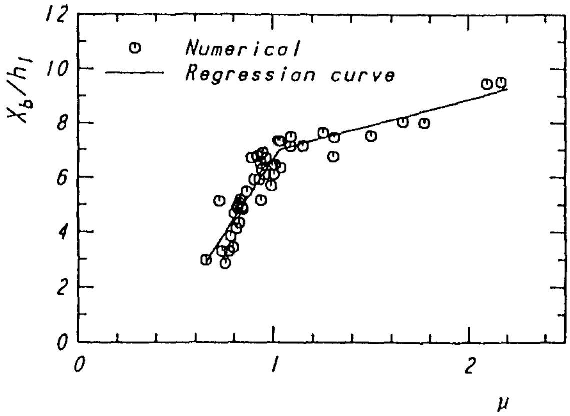


Fig.10 Location of the onset of breaking of a solitary wave caused by a submerged rectangular dike

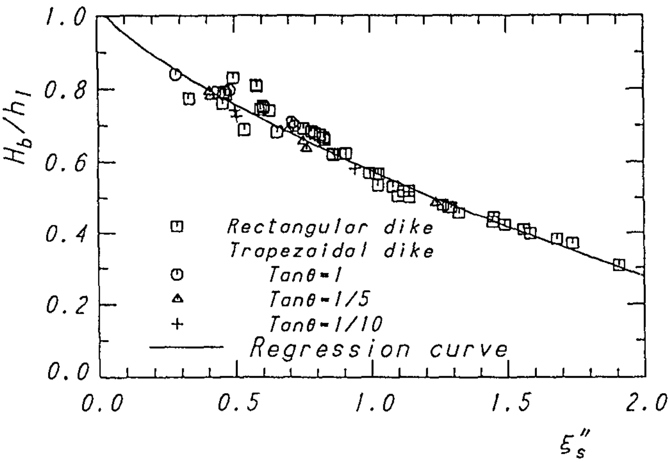


Fig.11 Relation of the parameter ξ_s'' to the breaker height of a solitary wave passing over a rectangular submerged dike or trapezoidal one with face slopes of $\tan\theta=1, 1/5$ and $1/10$.

Breaking wave height

Figure 11 shows the relation between the relative breaker height H_b/h_1 of the solitary wave incident to a submerged dike and the modified surf-similarity parameter for the dike ξ_s'' defined in eq.(3). The symbols of circle ○, triangle △ and cross + denote respectively the values of solitary waves incident to the trapezoidal dikes with the slope gradient of $\tan\theta=1, 1/5$ and $1/10$ and squares □ indicate the values of solitary waves incident to the rectangular dike with vertical slope. The solid line is given by the regression equation,

$$H_b/h_1 = -0.463\xi_s''^{0.133} + 1.039, \quad 0.2 \le \xi_s'' \le 2.0. \tag{12}$$

All the breaker heights are obtained from the computed surface profiles at the break point and they agree very well with the regression curve. This states that the relative breaker height H_b/h_1 of the solitary wave incident to the dike uniquely relates to the modified surf-similarity parameter ξ_s'' and its value is determined by the parameter ξ_s'' alone. We can hence easily know the breaker height H_b/h_1 only if calculating the value of the parameter ξ_s'' to the given incident wave and dike. Further, the breaker depth h_b is equal to the still water depth h_1 because the transitted main crest breaks after passing through the dike. The relative breaker height H_b/h_1 is thereby equal to the breaker depth index H_b/h_b.

CONCLUSIONS

Intensive numerical simulations using a boundary intergral method were performed to make quantitatively clear the breaking wave characteristics of a solitary wave incident to a submerged step or dike. As a result, the critical incident wave height, the location of the break point and the breaker height were formulated as functions of the incident wave height and the geometric quantities of the step or dike through regression analyses based on the simulated results. Further, their values were shown to be determined almost uniquely by the regression equations. It is thus now possible to determine accurately the persence of the wave breaking, the break point and the breaker height by using the regression equations, as far as concerned with a solitary wave incident to a step or dike. Further study is recommended to establish a unified governing law of the breaking wave characteristics of not only a solitary wave but also periodic waves incident to submerged obstacles having arbitrary shapes.

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