CHAPTER 230

MUD TRANSPORT RATE IN MUD LAYER DUE TO WAVE ACTION

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ABSTRACT

In order to calculate mud transport rate, two forms of transport should be considered, which are (1) mud mass transport in mud layer, and (2) suspended mud transport in water layer. The first type of transport (mass transport in mud layer) is greater than the second one in quantity under soft mud or fluid mud condition, therefore the first type of transport is mainly considered. The previously proposed methods to calculate the first type of transport are summarized and discussed. Then a new transport model called visco-elastic-plastic model is derived and some of the results of the model are shown. The model is based on the assumption that the fluid mud layer can be assumed to be visco-elastic fluid and when the magnitude of stress in mud layer exceeds the value of yield stress, the mud layer is modeled as visco-plastic fluid. Results of the new numerical model are compared with laboratory results and some discussions are given.

1. INTRODUCTION

In coastal environment, a part of bottom surface is covered with soft mud. In estuaries, in river mouth area of large rivers

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such as Yangtze river in China or Ganges river in Bangladesh, or in large bay areas, most of the coastal bottom surface is covered with soft mud. In these areas, when surface water waves travel onto a bottom of soft mud, inter-surface wave between water layer and mud layer is generated. In the interface, fluid mud layer with high water content is formed under stormy conditions. The inter-surface wave causes mass transport in mud layer. This type of mud mass transport as well as suspended mud transport in water layer is considered to be the main mechanisms to transport mud in coastal environment. The former (mass transport in mud layer) is lager than the latter in quantity under many conditions of soft mud and therefore it is important. Behavior of suspended mud is also important when we analyze coastal environment problems or the effect of construction works to the environment.

As stated in the previous part of this paper, the mechanism of mud transport can be classified into two types which are (1) mud mass transport in mud layer, and (2)suspended mud transport in water layer. In the followings, the first transport type will be discussed mainly because the first one is greater than the second one in quantity.

Before we start to talk about mud behavior, mud characteristics should be considered. Otsubo and Muraoka (1986) classified mud into two groups according to their characteristics of settling form, flow curve (shear rate - shear stress curve) and resuspension behavior under unidirectional flow. The main factor to control these characteristics is the nature of cation attached to particle surface. The first group is represented by Kaolinite with Al^{3+} Ca^{2+} or H^{+}, and the second group by Bentonite which consist of Na^{+}-Montmorillonite. The first group forms apparent mid-surface between water layer and mud layer when mud particles settle down and it has yield value in flow curve. The second group does not form mid-surface and does not have yield value. Shibayama et al. (1986) suggested that both suspended load and mass transport in mud layer should be considered for the first group and suspended load alone should be considered for the second group. It is also suggested by Shibayama et al. (1986) that in the coastal environment, since the salinity of sea water supplies enough number of cation, major part of mud behaves like the first group. Therefore if we consider mud behavior in the coastal environment, we only consider the first group.

Figure 1 shows the diagram of a model for mud transport due to waves. In the figure, it is stated that mud behavior or mud transport rate is governed by mud characteristics and wave conditions. In order to calculate transport rate, mass transport in mud layer and suspended mud transport should be evaluated. The

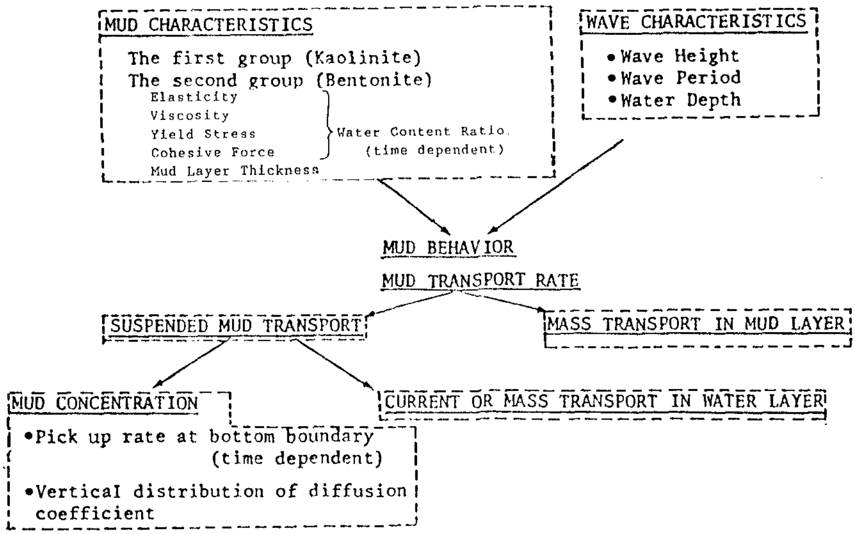


Figure 1: A diagram to evaluate mud transport rate.

suspended transport rate can be evaluated by the information of mud concentration distribution and current or mass transport in water layer. In the followings, mud mass transport in mud layer will be discussed.

2.GENERAL REVIEW OF PREVIOUS STUDIES

The study on the behavior of cohesive bed materials under wave action was started by Gade (1958). He treated mud as viscous fluid and estimated theoretically the decay rate of water surface waves propagating over mud bottom, under the assumption of long waves. Dalrymple and Liu (1978) derived "complete model" which models water layer and mud layer as viscous fluid. They also used boundary layer approximation and obtained an analytical solution of velocity field of mud layer. Recently, Hsiao and Shemdin (1980) and Mcpherson (1980) treated mud layer as visco-elastic fluid. The above four studies were carried out mainly to get the decay rate of water waves which travels over soft mud bottom.

In these five years, several attempts were carried out to estimate mud mass transport rate in mud layer. Nagai et al. (1984) performed experiments using wave flume and gave some qualitative descriptions for mud mass transport. Shibayama et al. (1986) used analytic solution of velocity field which was derived by Dalrymple and Liu (1978) using boundary-layer approximation and got formula to calculate mass transport velocity. Mehta and Maa (1986) formulated multi-layered model with the assumption of visco-elastic fluid property and the results were compared with experiments. Tsuruya et al. (1986) extended the model of Dalrymple and Liu (1978) from Newtonian viscous fluid to Bingham fluid by using a technique of multi-layer model. They divided mud layer into several layers and applied different value of viscosity according to the velocity gradient in each layer. They also calculated mud mass transport rate. However, these two studies, Shibayama et al. and Tsuruya et al., calculated Lagrangean component of transport rate only, excluding Eulerian component. Shibayama et al. (1989) formulated visco-elastic model and calculated both Lagrangean and Eulerian components of mud mass transport velocity. Then they compared the model result of mass transport with the laboratory results and got good agreements.

3. THEORETICAL FORMULATION OF MUD BEHAVIOR

Dalrymple and Liu (1978) formulated two-layer viscous fluid model. In both water layer and mud layer, the Navier-Stokes equation with neglecting non-linear terms was given. For horizontal direction,

$$\frac{\partial u_j}{\partial t}=-\frac{1}{\rho_j}\frac{\partial p_j}{\partial x}+\nu_{ej}\left(\frac{\partial^2 u_j}{\partial x^2}+\frac{\partial^2 u_j}{\partial z^2}\right) \tag{1}$$

and for vertical direction,

$$\frac{\partial w_j}{\partial t}=-\frac{1}{\rho_j}\frac{\partial p_j}{\partial z}+\nu_{ej}\left(\frac{\partial^2 w_j}{\partial x^2}+\frac{\partial^2 w_j}{\partial z^2}\right) \tag{2}$$

The mass continuity equation is

$$\frac{\partial u_j}{\partial x}+\frac{\partial w_j}{\partial z}=0 \tag{3}$$

Tsuruya et al. (1986) divided mud layer into n layers in order to incorporate the effect on non-Newtonian viscous fluid (multi-layer viscous fluid model). In each layer, the governing equations are given by (1), (2), and (3). Here we assume the solution as (in j-th layer)

$$u_j=\hat{u}_j e^{i(kx-\sigma t)},\quad w_j=\hat{w}_j e^{i(kx-\sigma t)},\quad p_j=\hat{p}_j e^{i(kx-\sigma t)},\quad \eta_j=\hat{\eta}_j e^{i(kx-\sigma t)} \tag{4}$$

In mud layer, Mcpherson (1980) treated ν_{ej} as complex function. In j-th layer,

$$\nu_{ej}=\nu_j+i\frac{G_j}{\rho_j\sigma} \tag{5}$$

where G_j is elasticity, ρ_j is density, σ is radian frequency. Here the real part gives viscous effect and the imaginary part gives elastic effect. Shibayama et al. (1989a) incorporated the function (5) into equations (1) to (3), and determine the unsolved unknowns by using the following boundary conditions (which is the same as Tsuruya et al., 1986, for viscous fluid),

In water surface $(z=\eta_1+h_1)$

$$\frac{\partial \eta_1}{\partial t}=w_1 \tag{6}$$

$$p_1-2\rho_1\nu_1\frac{\partial w_1}{\partial z}=0 \tag{7}$$

$$\rho_1\nu_1\left(\frac{\partial u_1}{\partial z}+\frac{\partial w_1}{\partial x}\right)=0 \tag{8}$$

In inter-surface $(z=-\sum_{l=2}^{j}h_l,\ z=0)$

$$\frac{\partial \eta_{j+1}}{\partial t}=w_j \tag{9}$$

$$u_j=u_{j+1} \tag{10}$$

$$w_j=w_{j+1} \tag{11}$$

$$p_j-2\rho_j\nu_{ej}\frac{\partial w_j}{\partial z}-\rho_j g\eta_j=p_{j+1}-2\rho_{j+1}\nu_{e(j+1)}\frac{\partial w_{j+1}}{\partial z}-\rho_{j+1}g\eta_{j+1} \tag{12}$$

$$\rho_j\nu_{ej}\left(\frac{\partial u_j}{\partial z}+\frac{\partial w_j}{\partial x}\right)=\rho_{j+1}\nu_{e(j+1)}\left(\frac{\partial u_{j+1}}{\partial z}+\frac{\partial w_{j+1}}{\partial x}\right) \tag{13}$$

In the bottom $(z=-\sum_{l=2}^{n}h_l)$

$$u_n=0 \tag{14}$$

$$w_n=0 \tag{15}$$

By using above conditions, simultaneous equations are obtained. Then we can calculate unknowns numerically by using computer.

In order to calculate, we have to give the values of elasticity and plasticity. A diagram to give viscosity is given by Tsuruya et al. (1987) as a function of water content ratio and velocity gradient. The way to give the value G, the elasticity is not yet established. However Shibayama et al. (1989b) used oscillating type viscous meter which is shown in Figure 2 and got the value of elasticity and viscosity simultaneously. Figure 3 shows some of the results of measured elasticity.

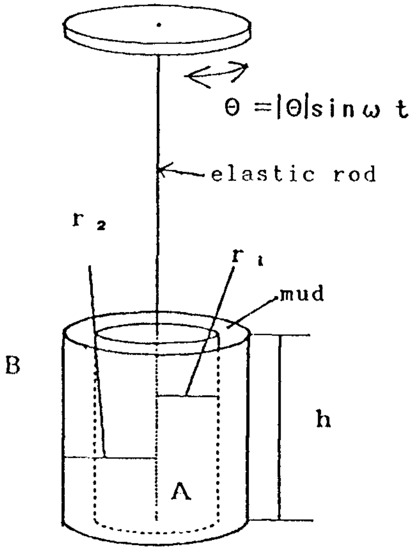


Figure 2: Apparatus of oscillating type visco-elastic meter.

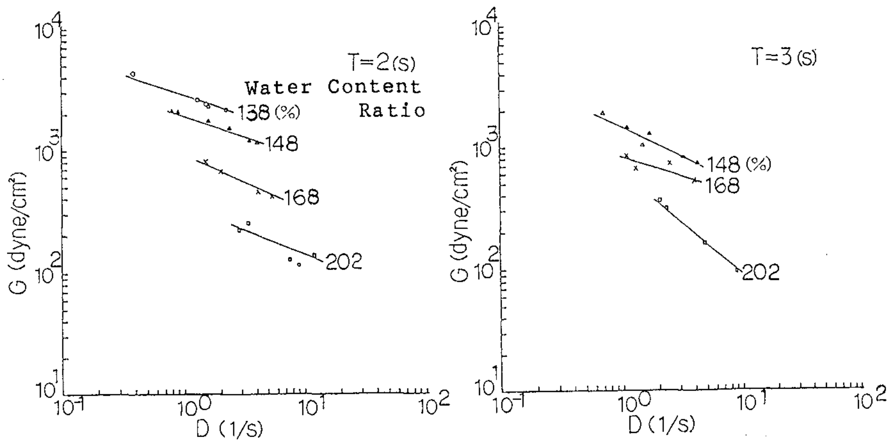


Figure 3: Measured results of mud elasticity.

Here we formulate a model called visco-elastic-plastic model. In the model, we calculate magnitude of shear stress in mud layer by using the visco-elastic multi-layer model (Shibayama et al., 1989a). If the magnitude of stress exceeds the yield value of mud, the layer (in multi-layered mud layer) is assumed to be visco-plastic layer. In the visco-plastic layer, the value of elasticity, G, is given as 0 and instead of elastic term, a fixed value of yield stress is incorporated into governing equations and boundary conditions. In the real calculations, the following false elasticity G_y is used in order to left the effect of yield stress to the momentum conservation equation (Equation 1 and 2) in the visco-plastic assumption. The G_y is given as follows.

$$G_y = \frac{\tau_{y2} \quad \sigma}{\left(\frac{\partial u_j}{\partial z} + \frac{\partial w_j}{\partial x}\right) i} \tag{16}$$

Figure 4 shows the flow curve of Kaolinite mud which was given by Otsubo and Muraoka (1986) based on their measured results by using rotating type viscosity meter. In the figure, we will use τ_{y2} as the yield stress of the mud, that is the boundary between visco-elastic model and visco-plastic model.

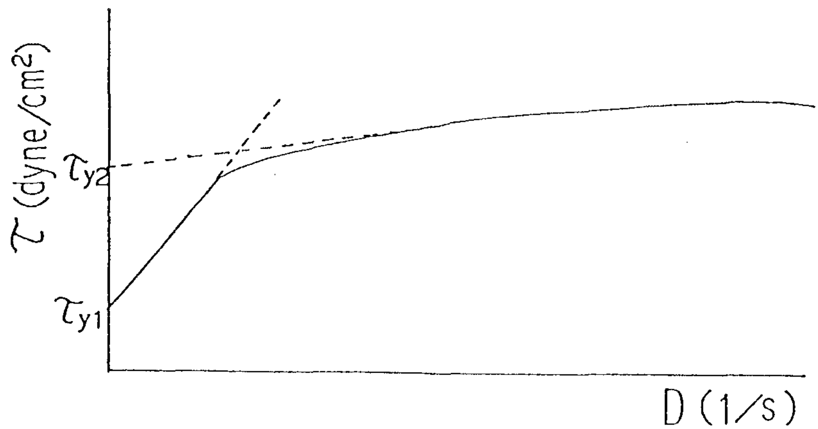


Figure 4: Flow curve for Kaolinite mud. (Otsubo and Muraoka, 1985)

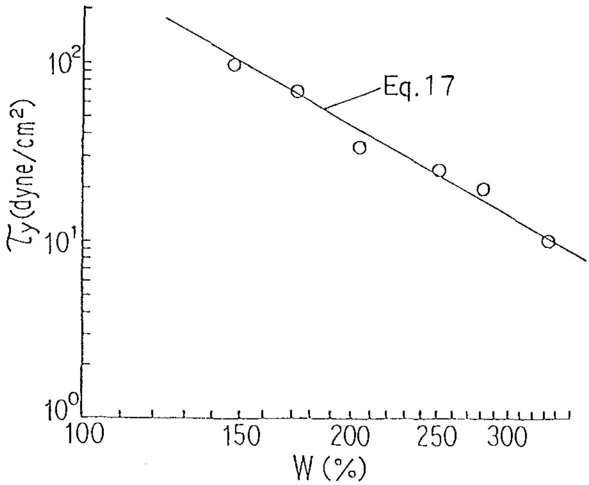


Figure 5: The relationship between the yield stress and the water content ratio. (Data from Otsubo and Muraoka, 1986)

Figure 5 shows the relation between water content ratio and the yield stress. The data for the yield stress is taken from the measured results of Otsubo and Muraoka (1986). From the figure, we will use the following formula for the yield stress based on the regression analysis.

$$\tau_y = 1.47 \times 10^8 \times W^{-2.83} \quad (\mathrm{dyne/cm^2}) \tag{17}$$

Figure 6 shows some of the representative results of numerical model. In the figures, the following natures of mud layer are indicated. (1)All layers are modeled as visco-plastic, (2)upper layers are modeled as visco-plastic and lower layers are modeled as visco-elastic, (3)middle layers are visco-plastic and lower and upper layers are visco-elastic, (4)all layers are modeled as visco-elastic.

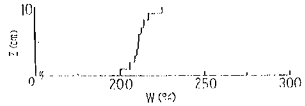
4.MUD MASS TRANSPORT RATE IN MUD LAYER

By using the above described method, we can evaluate the velocity field in both water layer and mud layer. The Lagrangean component of mass transport rate is given by ($\overline{}$ gives time average over one wave period)

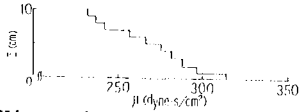
$$U_L = \overline{\frac{\partial u}{\partial x}\int_0^t u\,dt} + \overline{\frac{\partial u}{\partial z}\int_0^t w\,dt} \tag{18}$$

The Eulerian component of mass transport velocity is given by the

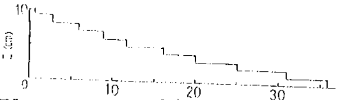
Period: T (s) Wave Height: H (cm)
Water Depth: h (cm) Mud Depth: d (cm)



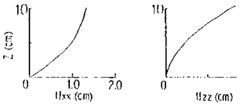
(a) Water content.



(b) Viscosity.

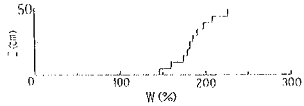


(c) Elasticity. G (dyne/cm²)



(d) Amplitude of mud particle.

(1)T = 1.0,H = 10.0,h = 35.0,d = 10.0



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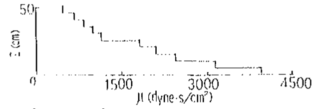
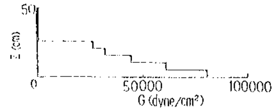
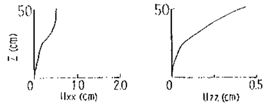


(b) Viscosity.

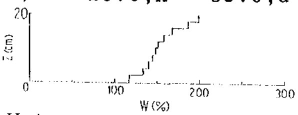


(c) Elasticity.

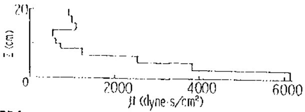


(d) Amplitude of mud particle.

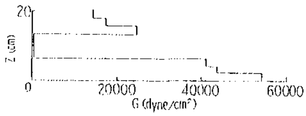
(2)T = 1.0,H = 3.5,h = 35.0,d = 50.0



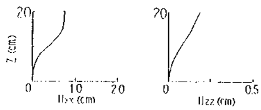
(a) Water content.



(b) Viscosity.

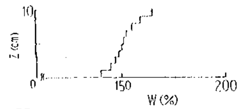


(c) Elasticity.

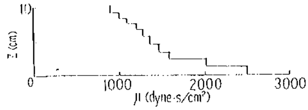


(d) Amplitude of mud particle.

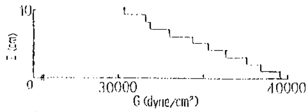
(3)T = 1.0,H = 3.0,h = 35.0,d = 20.0



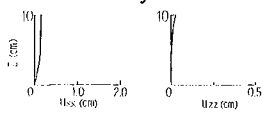
(a) Water content.



(b) Viscosity.



(c) Elasticity.



(d) Amplitude of mud particle.

(4)T = 1.0,H = 2.0,h = 35.0,d = 10.0

Figure 6: Results of numerical model.

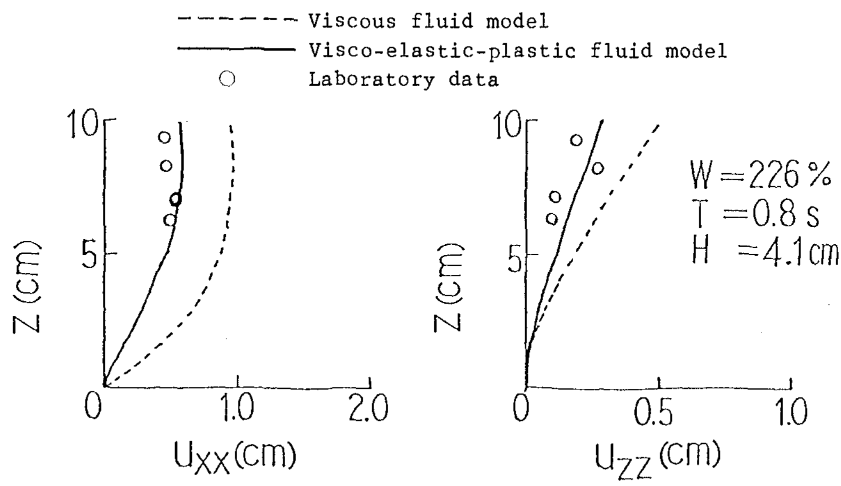


Figure 7: Comparison between laboratory data of mud particle excursion amplitude and visco-elastic-plastic model or viscous fluid model.

following momentum-conservation equation.

$$\overline{(uw)_j}-\overline{(uw)_\infty}=\nu_j\frac{dU_{Ej}}{dz} \tag{19}$$

where ∞ indicates the location of outer edge of boundary layer. The sum of Lagrangean and Eulerian components of mass transport velocity gives the total mass transport velocity.

Shibayama et al. (1989a) performed laboratory experiments in a wave flume. A number of tracers were inserted into the mud layer, and from the movement of tracers mud mass transport rate was quantitatively obtained. Figure 7 shows the comparison of laboratory results of mud particle excursion amplitude and calculated results by using visco-elastic-plastic model and viscous fluid model. It can be seen that the agreement of laboratory data and the visco-elastic-plastic model is better than that of viscous fluid model.

In Figure 8, the comparisons between the visco-elastic-plastic model and laboratory data are shown. The distributions of amplitude of vertical and horizontal excursion amplitude and mass transport velocity are shown in the figure. It can be observed that a good agreement is seen between laboratory results and model calculations.

5. CONCLUSIONS

A numerical model has been developed and used to predict mud mass transport rate in mud layer under the effect of water surface waves. In the model, the water layer was modeled as viscous fluid and the mud layer is modeled as visco-elastic or visco-plastic fluid. It was concluded that the present numerical model based on the assumption of visco-elastic or visco-plastic mud layer is useful to predict mud transport rate due to wave action.

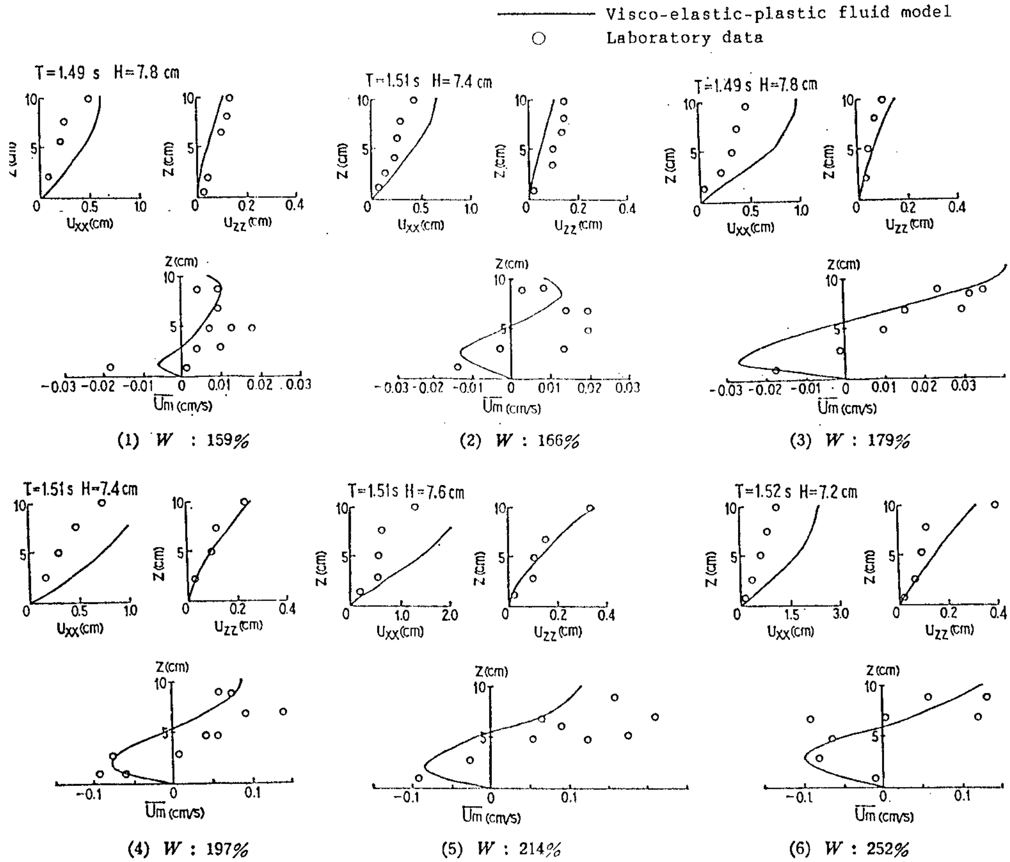


Figure 8: Comparison of Visco-elastic-plastic model and laboratory data.
W:water content ratio(%), $\overline{U_m}$:mass transport velocity
u_{xx} :amplitude of horizontal excursion
u_{zz} :amplitude of vertical excursion
water depth: 35cm, mud layer thickness: 10cm

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REFERENCES

(1)Dalrymple, A.R. and Liu, P.L.F., 1976. Waves over soft muds: a two-layer fluid model. Jour. of Physical Oceanography, 8: 1121-1131.

(2)Gade, H.G., 1958. Effect of nonrigid, impermeable bottom on plane surface waves in shallow water. Jour. of Marine Research, 16-2: 62-82.

(3)Hsiao, S.V. and Shemdin, O.H., 1980. Interaction of ocean waves with a soft bottom. Jour. of Physical Oceanography, 10: 1121-1131.

(4)Mehta, A.J. and Maa, P-Y., 1986. Waves over mud: Modeling erosion. Proc. of Third International Symposium on River Sedimentation, : 588-601.

(5)Mcpherson, H.,1980. The attenuation of waves over a non-rigid bed. JFM, 97-4: 721-742.

(6)Nagai, T., Yamamoto, T. and Figuerou, L., 1984. A laboratory experiment on the interactions between water waves and soft clay beds, Coastal Eng. in Japan, 27: 279-292.

(7)Otsubo, K. and Muraoka, K., 1985. Physical properties and critical shear stress of cohesive bottom sediments, Proc. JSCE, 363, 225-234.

(8)Shibayama, T., Takikawa, H. and Horikawa, K., 1986. Mud mass transport due to waves. Coastal Eng. in Japan, 29: 151-161.

(9)Shibayama, T., Aoki, T. and Sato, S., 1989. Mud mass transport rate due to waves: A visco-elastic model, Proc. of 23rd IAHR, B567-B574.

(10)Shibayama, T., Aoki, T. and Sato, S., 1989b. Mud mass transport rate due to waves: A visco-elastic-plastic model, Proc. of Japanese Coastal Eng. Conference, 36, 334-338 (in Japanese).

(11)Tsuruya, K., Nakano, S. and Takahama, J., 1986. Interaction between waves and mud. Proc. of Japanese Conf. on Coastal Engineering, 33, 317-321 (in Japanese).

(12)Tsuruya, K. and Nakano, S., 1987. Interactive effects between surface waves and a muddy bottom. Proc. of Coastal Sediments,1:50-62.