CHAPTER 195

Movable Bed Modeling Criteria For Beach Profile Response

Hsiang Wang,¹ Takao Toue,² and Hans H. Dette³

Introduction

Modeling coastal phenomena using movable bed means is a complicated problem with no general solution at present. Most modeling laws that are realizable in the laboratory usually only apply to certain restricted conditions. This paper, like many others, elects to deal with a restricted case, here, the two dimensional beach profile changes under the influence of wave action.

Numerous papers have been written on this subject. A general review can be found in Hudson et al. (1979), LeMenhaute (1990) and Wang (1985). Those of particular relevance to the present study are briefly reviewed here. Noda (1978) examined several theoretical scaling laws but recommended a completely empirical model law based on similarity of equilibrium profiles in the breaker zone. His model involves three scaling parameters, the sediment diameter ratio, the vertical and horizontal length scales. Vellinga (1982) and Graaff (1977) conducted a comprehensive laboratory study by using different scales in attempting to duplicate the beach and dune erosion of the Dutch's coast. Vellinga also settled on a pair of empirical relationships. The geometrical scale relationship bears certain resemblance to the Noda's with an additional fall velocity scale incorporated into it. A morphological time scale is also added. Both Noda's and Vellinga's laws require that the wave steepness and the Froude number be preserved. Hughes (1983) presented a model based upon preserving the Froude number and a non-dimensional fall velocity parameter. Hughes' model allows for wave distortion. There are similarities as well as very settled differences among the three modeling laws. All these modeling laws compared well with its own data set. Inter comparisons in most cases were not successful.

Kamphis (1974) using an entirely different approach listed four different non-dimensional parameters as requirement for complete similarity. Realizing that preserving all of them in the model is impractical, he proposed a set of four different modeling laws requiring preservation one or more non-dimensional parameters but not all of them. Each modeling law is suitable for a specific range of environmental conditions. There was no comparison with Laboratory data reported. Table 1 summarized the modeling laws by these authors.

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Table 1 Existing Beach Profile Modeling Laws

<table>
<thead>
<tr>
<th>Author</th>
<th>Requirement</th>
<th>Modeling Law</th>
</tr>
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<td>Noda (1978)</td>
<td>( \left( \frac{H_s}{L_0} \right)_p = \left( \frac{H_s}{L_0} \right)_m )</td>
<td>( N_D = (N_s)^{0.55} )</td>
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<tr>
<td></td>
<td>( F_p = F_m )</td>
<td>( N_s = (N_\lambda)^{0.76} )</td>
</tr>
<tr>
<td>Vellinga (1982)</td>
<td>( \left( \frac{H_s}{L_0} \right)_p = \left( \frac{H_s}{L_0} \right)_m )</td>
<td>( N_T = N_t = (N_s)^{\frac{1}{3}} )</td>
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<tr>
<td>Hughes (1983)</td>
<td>( F_p = F_m )</td>
<td>( N_T = N_t = N_\lambda/(N_s)^{\frac{1}{3}} )</td>
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<tr>
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<td>( \left( \frac{H}{T^W} \right)_p = \left( \frac{H}{T^W} \right)_m )</td>
<td>( N_s = (N_W N_\lambda)^{\frac{1}{3}} )</td>
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<tr>
<td>Kamphis (1974)</td>
<td>( (N_{R*}) = 1 )</td>
<td>( (N_{F*})_p = 1 )</td>
</tr>
<tr>
<td></td>
<td>( \frac{(N_{fl})}{N_p} = 1 )</td>
<td>( N_s/N_D = 1 )</td>
</tr>
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</table>

\( \varepsilon \): Vertical Scale; \( \lambda \): Horizontal Scale; \( D \): Sand Size; 
\( T \): Fluid Motion Time Scale; \( t \): Morphological Time Scale; 
\( W \): Fall Velocity.

Judging from the argument used by the authors and the evidence provided by them, the modeling laws by the first three authors are clearly based on suspended-load dominated case. Kamphis' model did not specify the mode of transport. From the selection of the parameters, it appeared that the resulting modeling laws should be more pertinent to bed-load dominated cases.

In the first three modeling laws, Noda's and Vellinga's are undistorted but Hughes' is distorted. This needs some clarification. In the modeling law itself, the vertical and horizontal scales are all distorted as can be seen from Table 1. The distortion or non-distortion originates from the requirement. Noda and Vellinga both require the wave steepness to be preserved, thus, the wave form should be undistorted. Hughes' model, on the other hand, preserves non-dimensional fall velocity, thus, permits wave form distortion. If one insists upon a consistent undistorted model (wave form treated as a geometrical scale) by letting \( N_s = N_\lambda = 1 \), then Noda's model reduces to \( N_D = N_t^{0.55} \) and Vellinga's and Hughes' models yield identical results in both morphological scale and fall velocity scale:

\[
N_T = N_t = N_s^{\frac{1}{3}}
\]
\[ N_W = N_s^{\frac{1}{3}} \]

This is of great comfort that all the model laws are somewhat consistent at undistorted scale except none of the authors actually intended to propose \( N_s = N_\lambda = 1 \) as quite evident from the formulas they presented.

**Modeling Law By Inspectional Method**

So far all the above modeling laws were derived from dimensional analysis of physical quantities. A slightly different approach is taken here by the inspection of the basic governing equation. The two-dimensional sediment conservation equation states that

\[ \frac{\partial h}{\partial t} = \frac{\partial q}{\partial x} \]  

where \( h \) is the bottom elevation, \( q \) is the volumetric sediment transport rate in the \( x \) direction; \( t \) is time and \( x \) is on-off shore direction. This equation can be non-dimensionalized as follows:

\[ \frac{\partial \bar{h}}{\partial \bar{t}} = \frac{q_{n} t_{n} \, \partial \bar{q}}{\delta \lambda \, \partial \bar{x}} \]  

where the overbar refers to non-dimensional quantities and \( q_{n}, t_{n}, \delta \) and \( \lambda \) represent, respectively, the reference values of sediment transport rate, the morphological time scale, vertical geometrical scale and horizontal geometrical scale. To maintain similitude between the model and prototypes requires

\[ \frac{N_q N_t}{N_s N_\lambda} = 1 \]

where \( N \) refers to the ratio of prototype to model.

**Suspended Load Transport**

The suspended load transport rate can be expressed by depth averaged properties:

\[ q_s = h V c \]

where \( h \) is equal to depth; \( V \) is equal to mean transport velocity and \( c \) is equal to depth-averaged mean sediment concentration. The suspended sediment concentration is directly proportional to the ratio of stirring power due to turbulence and the settling power due to gravity and can be expressed as (Hattori and Karvamata, 1980):

\[ c \propto \frac{\rho u'}{(\rho_s - \rho) W} \propto \frac{u'}{SW} \]

where \( u' \) is the turbulent intensity, \( W \) is the particle settling velocity and \( S \) is the submerged specific weight. Thorntorn (1978) suggested that the ratio of turbulent velocity and wave induced velocity is a function of surf zone parameter, \( \xi \), i.e.,

\[ \frac{u'}{u} = f(\xi) \]
The surf zone parameter is defined as $\tan \beta / \sqrt{H_o / L_o}$ with $\tan \beta$ the beach slope, $H_o$ and $L_o$, the deepwater wave height and length, respectively. Physically, this equation states that if the surf zone property is similar, the turbulence intensity should be proportional to the mean velocity scale, a plausible assumption. Since in a wave field $u$ is proportional to $H/T$, substituting Eqs. (5) and (6) into Eq. (4) and then combining it with Eq. (3) leads to

$$\frac{N_v N_f(\xi) N_t N_H}{N_\lambda N_w N_T N_s} = 1$$

(7)

If we let $N_f(\xi) = 1$, or $\xi_p = \xi_m$, we require

$$[g^{\frac{1}{2}} T \cdot Tan \beta/H^\frac{1}{2}] \cdot [g^{\frac{1}{2}} T \cdot Tan \beta/H^\frac{1}{2}] = [g^{\frac{1}{2}} T \cdot Tan \beta/H^\frac{1}{2}]$$

(8)

which when expressed in basic scaling quantities becomes:

$$N_T = N_\lambda / N_{\xi}^\frac{1}{2}$$

(9)

Here the wave height is treated as a vertical scale parameter. In Froude-number similitude, the horizontal velocity is scaled in accordance with $N_{\xi}^\frac{1}{2}$, i.e.,

$$N_v = \frac{N_v \text{(horizontal)}}{(N_{\xi}/N_{\xi}^\frac{1}{2})} = 1$$

(10)

or

$$N_v \text{(horizontal)} = N_{\xi}^\frac{1}{2}$$

(11)

Thus, Eq. (9), in essence, is the scale ratio of horizontal distance to horizontal velocity in a distorted model. Now, substituting Eq. (9) into Eqs. (7) and (4) and letting $N_f(\xi) = 1$, we obtain a pair of model laws as follows:

$$N_q = \frac{(N_{\xi})^3}{N_v N_w N_\lambda}$$

(12)

$$N_t = \frac{N_v N_w N_\lambda^2}{(N_{\xi})^2}$$

(13)

Morphological Time Scale and Field Evidence

Equations (12) and (13) essentially form the proposed modeling laws with the basic modeling requirement that the surf zone parameter be matched. One of the important but also more difficult aspects of beach profile modeling law is the determination of morphological time scale. Two different hypotheses are tested here. First, if we assume that the number of incoming waves per unit time is preserved for the similitude of erosion rate, then,

$$N_t = N_T = \frac{N_\lambda}{N_{\xi}^\frac{1}{2}}$$

(14)

Substituting the above equation into (13), one obtains

$$N_{\xi} = (N_v N_w)^\frac{3}{2} N_\lambda^\frac{3}{2}$$

(15)
This equation when expressed in dimensional form becomes:

$$ h = \alpha_1 \left( \frac{SW}{g^\frac{1}{2}} \right)^\frac{2}{3} x^\frac{2}{3} $$

(16)

with $\alpha_1$, a constant of proportionality. This equation is similar to Dean's empirical equilibrium profile which was deduced from field data provided the scale parameter “A” in Dean's equation equals to $\alpha_1 \left( \frac{SW}{g^\frac{1}{2}} \right)^\frac{2}{3}$.

Second, if we assume that instead of preserving the number of waves, the trajectory of a fallen particle be preserved in the erosion rate, then the morphological time scale becomes

$$ N_t = \frac{N_\delta}{N_\varepsilon(h\text{orizontal})} = N_\delta^\frac{1}{2} $$

(17)

Again substituting the above equation into Eq. (13) results in

$$ N_\delta = (N_s N_w)^\frac{1}{3} (N_\lambda)^\frac{1}{8} $$

(18)

The dimensional counterpart is:

$$ h = \alpha_2 \left( \frac{SW}{g^\frac{1}{2}} \right)^0.4 \lambda^{0.8} $$

(19)

This is very close to the empirical equilibrium profile form proposed by Vellinga (1982) which was derived from field data along the Dutch coast.

Laboratory Data

As was shown, the proposed modeling laws are consistent with field data even though the morphological time scale remains somewhat unclear. In this section, laboratory experiments were compared. We put our emphasis on inter-comparisons among data sets from different laboratories and on the morphological time scale. We shall refer to Eqs. (14) and (15) as model “A” and Eqs. (17) and (18) as model “B”.

The selected data sets used in the inter-comparisons are listed in Table 2, with the key parameters associated with each experiment. Here, Saville's experiment is treated as the reference and the vertical scales of the other tests are all referred to it. Figure 1 shows the comparison between Saville and Kriebel. According to the proposed modeling law, the model has no geometrical distortion by either Law “A” or Law “B”, i.e., $N_\delta = N_\lambda = 9.6$ from both “A” and “B”. The comparison is good except in the foreshore where the mode of transport in the small scale (may be even in the large scale) can no longer be considered as suspended-load dominated. Since there is no geometrical distortion, Law “A” and Law “B” yield the same morphological time scale as they should. Hughes and Fowler (1990) recently presented the results of a mid-scale experiment intended to duplicate the large scale experiment of Dette and Uliczka (1986). The model was also geometrically undistorted and the comparisons were judged to be excellent. Both sets of data support the modeling law but are not able to differentiate Law “A” from Law “B”.
Table 2: Selected Data Sets

<table>
<thead>
<tr>
<th>Origin</th>
<th>Profile (Slope)</th>
<th>$H$ (m)</th>
<th>$T$ (sec)</th>
<th>$W_0$ (cm/s)</th>
<th>$N_2$ (est.)</th>
<th>$c_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEBC (Saville)</td>
<td>$\frac{1}{4}$</td>
<td>1.28-1.72</td>
<td>5.8-11.3</td>
<td>5.57</td>
<td>1</td>
<td>0.37</td>
</tr>
<tr>
<td>GoR (Graaff, Vellinga)</td>
<td>Composite</td>
<td>0.05-2</td>
<td>0.98-7.6</td>
<td>0.6-2.3</td>
<td>0.86 to 34</td>
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<td>Germany</td>
<td></td>
<td>1.50</td>
<td>6</td>
<td>4.0</td>
<td>1.1</td>
<td>0.30</td>
</tr>
<tr>
<td>G W K</td>
<td>Composite</td>
<td>1.50</td>
<td>6</td>
<td>4.0</td>
<td>1.1</td>
<td>1.53</td>
</tr>
<tr>
<td>Flume</td>
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<td>0.15</td>
<td>1.9</td>
<td>4.0</td>
<td>11</td>
<td>1.53</td>
</tr>
<tr>
<td>U. Florida</td>
<td></td>
<td>$\frac{1}{4}$ and $a = AX_{1}^{1}$</td>
<td>0.085-0.175</td>
<td>1.6-3.87</td>
<td>1.3</td>
<td>9.8</td>
</tr>
<tr>
<td>Kriebel et al.</td>
<td></td>
<td>$a = AX_{1}^{1}$</td>
<td>0.64-0.12</td>
<td>1.3-1.8</td>
<td>1.7</td>
<td>14-43</td>
</tr>
<tr>
<td>Barnett</td>
<td></td>
<td>$a = AX_{1}^{1}$</td>
<td>0.63-0.11</td>
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<td>1.7</td>
<td>16-57</td>
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<td>Tough</td>
<td>Composite</td>
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<tr>
<td>Hughes</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Field data</td>
<td>Dean Vellinga</td>
<td>$y = AX_{1}^{1}$</td>
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</tr>
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<td></td>
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<td>$y = 20.25W^{0.44}X^{0.78}$</td>
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</tr>
</tbody>
</table>

1 3D test
2 Estimate base on average slope between shoreline and breaker bar
The second comparison is between Saville’s and Barnett’s experimental results. Barnett’s experiments were not intended for verification of modeling laws; rather, they were designed for testing the effects of seawalls on the fronting beaches. However, his benchmark cases of beaches without seawalls could be used for the present purpose. As stated earlier, the initial condition in Saville’s tests was a beach of uniform slope of 1 to 15. The initial condition in Barnett’s tests, on the other hand, was a beach of equilibrium shape in accordance with $h = AX^2$, with $A = 0.075m^3$. Since the surf zone parameter is defined with a single beach slope value, we define here an equivalent slope as the mean slope value within the surf zone:

$$m = \frac{\int_0^{X_b} m\,dx}{X_b}$$  \hspace{1cm} (20)

where $m$ is local slope and $X_b$ is the surf zone width. Assuming $H_b = 0.8h_b$, 

**Figure 2**: Comparisons between Saville’s and Barnett’s results.
and $h_b = 0.075x_0^{\frac{2}{3}}$, the $\bar{m}$ value is found to be equal to 0.06 or 1 to 16.7, which is very close to 1 to 15 used in Saville's tests. Barnett tested three wave conditions; the one suitable for comparison was with $H_o = 8.8\text{cm}$ and $T = 1.3\text{ sec}$. For this case, the corresponding $\xi$ is 0.33, very close to Saville's value of 0.37. The comparisons are shown in Figure 2. It should be noted here that the morphological time scale could only be preserved approximately as profiles were measured at discrete time intervals selected at the convenience of the investigators in the respective experiments. This is also the case for other comparisons reported here after. To evaluate the goodness of the comparison, the RMS values of the profile elevation differences were computed from the shoreline to the crest of offshore bar (see Figure 2), by the following equation

$$\epsilon = \left[ \frac{1}{n} \sum_{i=1}^{n} (h_i^S - h_i^B)^2 \right]^{\frac{1}{2}}$$

(21)

where $h^S$ and $h^B$ are profile elevations of Saville and Barnett, respectively. These values are also given in Figure 2. From these values, it can be seen that Law “B” performs better than Law “A”. Also, as time progresses, the modeling law becomes better as the influence of initial condition becomes less.
Dette and Uliczka (1986) performed small scale tests treating results from large wave channel (GWK) as prototype. The small scale tests were conducted using same size sand as the prototype at an undistorted geometrical scale of 1:10. Straight comparisons with $N_s = N_A = 10$ and $N_t = N_T$ or $N_t = 1$ were unsuccessful. Figure 3, for instance, shows their comparisons with $N_t = 1$ (which is the better of the two time scales according to the authors). The data are replotted in Figure 4 according to the proposed modeling laws. The comparisons are much more favorable, particularly with Law "B". Therefore, the experiment although has the appearance of an undistorted model is actu-
ally highly distorted according to the proposed modeling laws (a horizontal to vertical distortion of 3.2 or 1.8 depending on whether Law "A" or Law "B" is adopted). The reason for this high distortion is because the size of sand is the same in the model and prototype. Since the fall velocity ratio is now unity, the geometrical scale has to be highly distorted to compensate for the fall velocity effect.

The results of the last two cases seemed to suggest that preservation of wave form and/or initial beach geometry are not essential, though might be desirable. The following case further reinforces this observation.

The GWK test and Kriebel's test have very different initial geometry and different $H_0/L_0$ values. However, the $\xi$ value of GWK's composite slope test matches well with the Kriebel's and the fall velocity ratio of the two tests lead to small geometrical scaling distortion according to the proposed laws. All these are favorable conditions to test the proposed modeling laws. The various scale ratio between these two experiments are summarized as follows:

\begin{align*}
N_{(H_0/L_0)} & \quad N_\xi & \quad N_5 & \quad N_W & \quad N_{\lambda(A)} & \quad N_{\lambda(B)} \\
0.81 & \quad 0.86 & \quad 9 & \quad 2.22 & \quad 12 & \quad 10.5
\end{align*}

The ratio is defined as GWK/Kriebel; the initial geometry of GWK is $\frac{1}{10}$ followed by $\frac{1}{20}$ slopes whereas the initial geometry of Kriebel is $\frac{1}{15}$ uniform slope. The comparisons are given in Figure 5. Again like the previous cases Law "B" performs better than Law "A". When time progresses, the effects of the initial condition gradually disappear and the agreement becomes better.

![Figure 5: Comparisons of GWK and Kriebel.](image-url)
Graaff (1977) and Vellinga (1982) used different vertical scales and sediment sizes in their experiments. Realizing that the surf zone in the model will be compressed, they purposely distorted their initial beach profiles in the model in accordance with a hypothetical profile representing the prototype Dutch coast condition. This, however, makes it difficult to compare their results with others and even among themselves. Table 3 summarizes their test conditions and scaling requirement according to Law “A” and Law “B”. The $N_H$ values in the first column are in reference to prototype scale. For the convenience of inter-comparisons we designate the case $N_H = 26$, $D_{50} = 225$ as the reference, thus, all the corresponding scale values are unity for this case. Two major difficulties arose; the surf zone parameters are difficult to determine because of the distortion and the composite nature of the profiles; secondly, the $N_T$ values used in the tests do not match those required by the current modeling laws. Here, we simply selected cases (indicated by underline in Table 3) that the actual $N_T$ values approximately meet the requirement and ignored the surf zone parameter criterion completely. Thus, the comparisons could only be viewed as qualitative. Figure 6 shows the results. Law “B” seemed to yield acceptable results considering the crude nature of the comparisons.

Toue’s (1989) experiment is the only 3-dimensional laboratory test available. His results revealed a number of points worth mentioning. 1). He separated the suspended-load and bed-load dominated cases. 2). For suspended-load dominated cases, his results compared well with the 2-dimensional counterpart carried out by Barnett for natural beaches but less desirable for beaches with seawall backings. 3). For bed-load dominated cases, the modeling law given above failed to produce reasonable results.

### About the Scale Parameter in Equilibrium Profile


<table>
<thead>
<tr>
<th>$N_H$</th>
<th>$D_{50\mu m}$</th>
<th>Law A</th>
<th>Law B</th>
<th>$N_T$ (actual)</th>
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<td>26</td>
<td>225</td>
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### Initial Distortion

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<th>H-Seat</th>
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<td>3.0</td>
<td>3.0</td>
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</table>

#### Figure 6: Comparisons of Graaff Results at Different Model Scales.

(a) By Law "A"

(b) By Law "B"

### SEDIMENT FALL VELOCITY, \( w (\text{m/s}) \)

![Graph showing comparisons of Graaff results at different model scales.](image)

### SEDIMENT SIZE, \( D (\text{mm}) \)

![Graph showing profile scale parameter vs sediment fall velocity parameter.](image)
Vellinga (1982), through empirical data fitting, proposed a dimensional coefficient $A = 0.39 W^{0.44}$. Based upon the analysis presented in the paper, the correct form of $A$ should be $\alpha \left( \frac{W}{g^2} \right)^n$ where $\alpha$ is now a non-dimensional coefficient. The value of $n$ should be either equal to 0.67 or 0.4 depending upon which law is adopted. Dean (1990) re-plotted the $A$ values as a function of $W$ as shown in Figure 7. The results of GWK tests and all the Delft data are also added to the figure. The best fit is found to be $A = 0.51 W^{0.44}$. This relationship is very close to Vellinga's empirical formula as well as to the analytical form of law "B". This presents another favorable evidence that law "B" might be a better choice.

Bed Load Transport

Various formulas have been proposed for bed load transport; practically all of them relate bed load transport to the Shields parameter. Therefore, they will lead to the same modeling requirement by matching model Shields parameter to that of the prototype. The scaling of the transport rate and the relationship of morphological time are different for different transport equations. Table 4 summarizes the results based on different transport equations. We note here that the third row in Table 4 is consistent with the "best model" referred by Kamphuis (1974). By leaving $f$ as an independent parameter allows greater flexibility in selecting scales and material.

Table 4: Bed Load Transport Scaling Law

<table>
<thead>
<tr>
<th>Transport Equation</th>
<th>Modeling Requirement*</th>
<th>$N_q/D$</th>
<th>$N_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_b = wD^2(\varphi - \varphi_c)n$</td>
<td>$N_D = N_q N_f N_s$</td>
<td>$(N_q)^{1/2} N_s N_t$</td>
<td>$(N_q)^{1/2}$</td>
</tr>
<tr>
<td>$q_b = W D \varphi^3$</td>
<td>$N_D = N_q N_f N_s$</td>
<td>$N_a N_q N_f N_t$</td>
<td>$N_a N_q N_t$</td>
</tr>
<tr>
<td>$q_b = (s^2 D^2)^{1/2}(\varphi - \varphi_c)n$</td>
<td>$N_D = N_q N_f N_s$</td>
<td>$N_a (N_f)^{3/2} (N_t)^{1/2}$</td>
<td>$N_a (N_f)^{3/2} N_t^{1/2}$</td>
</tr>
</tbody>
</table>

Symbols
- $w$: Wave Frequency
- $s$: Specific Weight of Submerged Sand
- $\varphi$: Shields Parameter
- $\varphi_c$: Critical Shields Parameter
- $f$: Friction Factor
- *Assuming $\varphi >> \varphi_c$.

Conclusions

The advantage of the proposed modeling criteria over those presented by previous investigators is its ease to implement in ordinary laboratory facilities. It is restricted to cases that either bed load or suspended load dominates. Also, only the gross effect of transport and profile changes are modeled without concern whether the detailed flow pattern or the sediment motion is accurately portrayed.

For suspended load dominated case, the proposed modeling law is in consonance with the equilibrium beach profile concept. The surf zone parameter appears to be the most important similarity criterion that should be matched between the prototype and the model. In general, the modeling law leads to geometrical distortion. The degree of distortion is affected mainly by the sediment fall velocity ratio. Two different hypothesis of morphological time scale are tested. It appears that the preservation of particle trajectory lead to the correct morphological time scale. Laboratory results from various sources seem to support the proposed modeling law. For bed load dominated case, laboratory information is not available. Further investigation is suggested.

Finally, the scale limitations are such that the laboratory flow field should
maintain turbulent boundary layer (Jonsson’s flow regime criteria, for instance, 1966) and the mode of sediment should be correct (Shibayama and Horikawa’s sediment transport classification, for instance, 1980). It is always prudent to verify the flow and the mode of transport prior to data collection.

References