CHAPTER 161

A QUASI-3D MODEL FOR SUSPENDED SEDIMENT TRANSPORT BY CURRENTS AND WAVES.

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ABSTRACT

A QUASI-3D model for suspended sediment transport based on an asymptotic solution of the convection-diffusion equation is developed for currents and waves. The influence of waves on the sediment concentration is included through the diffusion analogy and the bed boundary condition. Validity analysis provides conditions that permit the check of the model applicability. The presence of waves enhances the validity area of the model considerably. The model compares favourably with a 3D model and experimental data.

1. INTRODUCTION

In compound mathematical models for coastal morphological problems the suspended sediment transport modul forms an essential submodel. For the suspended sediment transport description a wide range of models has been presented in the past (e.g. transport formulas, 3D or 2D horizontal convection-diffusion models).

In this study a quasi-3D formulation is developed based on the analysis of Galappatti and Vreugdenhil (1985) for 2D vertical problems. This quasi-3D modelling technique combines the reduced computational effort of 2DH models with the generality of the 3D models (ie variations in vertical structure of sediment concentration are reproduced and the bed boundary condition is applied as in 3D models). The QUASI-3D model has been recently used for morphological problems of tidal rivers and estuaries (Wang, 1989). The present study is aimed at coastal morphology where wave influence is of importance.

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2. BASIC EQUATIONS

The mass balance equation for the suspended sediment when integrated over the wave period (after concentration and velocity components have been split into a wave averaged and an oscillating part) gives rise to wave averaged plus correlation terms. It is assumed that for the correlation terms the Boussinesq assumption holds so they can be connected to the concentration gradients by means of a proper mixing coefficient (diffusion analogy). Then the suspended sediment transport due to current and waves is governed by the wave-averaged convection-diffusion equation:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} + (w-w_s)\frac{\partial c}{\partial z} = \frac{\partial}{\partial x}(\varepsilon_x\frac{\partial c}{\partial x}) + \frac{\partial}{\partial y}(\varepsilon_y\frac{\partial c}{\partial y}) + \frac{\partial}{\partial z}(\varepsilon_z\frac{\partial c}{\partial z}) \quad (1)$$

where c,u,v,w are concentration, longitudinal, lateral and vertical velocity components respectively, all averaged over the wave period, sediment fall velocity is w_s and ε_x, ε_y and ε_z are mixing coefficients for the combined action of current and waves.

The convection-diffusion equation has been used to describe the current-waves induced suspended sediment transport before (e.g. van Rijn 1986, van Rijn and Meijer 1988).

Van Rijn (1986), based on measurements of equilibrium concentration profiles, proposed an empirical vertical mixing coefficient for current and waves. According to this empirical approach the influence of waves on the mixing coefficient comes through the following wave quantities: Wave period T, particle size diameter $D_* = D_{50}\{(\rho_s-\rho)/g(\rho\nu^2)\}^{1/3}$, significant wave height H_s, peak value of orbital velocity near the bed $\hat{u}_{orb}$, a_{br} a breaking coefficient, water depth h and thickness of the near bed mixing layer δ ($\simeq$ 3 ripple heights). The influence of the current comes through the current bed shear velocity u_*, the von Karman constant κ, the water depth h and the relative strength of the wave orbital velocity and mean current velocity (van Rijn 1986, van Rijn and Meijer 1988).

The horizontal mixing coefficients are taken constant over the depth. Their magnitude is of the order of 1 m^2/s (van Rijn and Meijer, 1988).

To solve equation (1) boundary conditions are needed not only at the vertical boundaries but also at the water surface and at the bottom.

At the water surface the vertical flux is taken to be zero.

The bed boundary is taken at a specified height z_a above the mean bed level (reference level) that here is specified as a small fraction of the depth. At the reference level the concentration is assumed to adapt immediately to equilibrium conditions and is given as function of the local hydraulic and sediment parameters ("concentration" bed boundary condition). Altervatively the concentration gradient at the reference level can be assumed to adapt immediately to equilibrium conditions ("gradient" bed boundary condition). Here the equilibrium concentration (and equilibrium gradient) at the reference level proposed by van Rijn (1986) is used.

3. THE QUASI-3D MODEL

3.1 Derivation

Galappatti and Vreugdenhil (1985) constructed an asymptotic solution of the convection-diffusion equation in 2DV space for current alone under certain assumptions concerning the scales of domain dimensions, time, flow velocities, and horizontal and vertical diffusion.

The same steps are followed here for the solution of the 3D equation (1). The first order solution can be written (see Katopodi and Ribberink, 1988):

$$c(\zeta) = a_{11}(\zeta)\bar{c} + a_{21}(\zeta)\frac{h}{w_s}\frac{\partial\bar{c}}{\partial t} + a_{22}(\zeta)\frac{h\bar{u}}{w_s}\frac{\partial\bar{c}}{\partial x} + a_{22}(\zeta)\frac{h\bar{v}}{w_s}\frac{\partial\bar{c}}{\partial y}$$

$$-a_{21}(\zeta)\frac{h}{w_s}\frac{\partial}{\partial x}\left(\varepsilon_x\frac{\partial\bar{c}}{\partial x}\right) - a_{21}(\zeta)\frac{h}{w_s}\frac{\partial}{\partial y}\left(\varepsilon_y\frac{\partial\bar{c}}{\partial y}\right) \qquad (2)$$

where $\bar{c}$ is the depth averaged concentration, $\bar{u}$ and $\bar{v}$ depth averaged velocity components, $\zeta = z/h$, h the water depth and $a_{ij}(\zeta)$ profile functions.The profile functions depend only on the explicit knowledge of the vertical mixing coefficient, fall velocity and normalized velocity profile and can be computed in advance.

In the applications that follow the velocity profile is assumed to have the logarithmic shape -an assumption that can hold for boundary layer flows in combination with small wind waves (van Rijn and Meijer, 1988). Nevertheless the method can handle 3D velocity profiles as soon as they are given as similarity series.

The solution (2) and the equations that give the profile functions have been derived with the use of the surface boundary condition and the assumptions that only the zero order concentration terms contribute to the depth averaged concentration and that shape variations of the equilibrium concentration profile can be neglected.

If further the bed boundary condition is used, then from (2) follows ("concentration" condition):

$$\bar{c}_e = \bar{c} + \frac{\gamma_{21}}{\gamma_{11}} \frac{h}{w_s} \frac{\partial \bar{c}}{\partial t} + \frac{\gamma_{22}}{\gamma_{11}} \frac{h\bar{u}}{w_s} \frac{\partial \bar{c}}{\partial x} + \frac{\gamma_{22}}{\gamma_{11}} \frac{h\bar{v}}{w_s} \frac{\partial \bar{c}}{\partial y} -$$

$$- \frac{\gamma_{21}}{\gamma_{11}} \frac{h}{w_s} \frac{\partial}{\partial x} (\varepsilon_x \frac{\partial \bar{c}}{\partial x}) - \frac{\gamma_{21}}{\gamma_{11}} \frac{h}{w_s} \frac{\partial}{\partial y} (\varepsilon_y \frac{\partial \bar{c}}{\partial y}) \quad (3)$$

where $\gamma_{ij} = a_{ij}(0)$ and $\bar{c}_e$ the depth averaged equilibriun concentration.

Equation (3) is a partial differential equation with constant coefficients and $\bar{c}$ as the unknown. The coefficients γ_{11}, γ_{21}, γ_{22} are known in advance. After solving (3) for $\bar{c}$ (with proper initial and boundary conditions), the vertical concentration profile $c(\zeta)$ can be computed from (2). The sediment transport is calculated afterwards.

An equation analogous to (3) can be derived if "gradient" bed boundary condition is used.

The QUASI-3D model (equation 3) was implemented in the SUSTRA 3D/2DV system following the same numerical (finite volume) method (van Rijn and Meijer, 1988).

3.2 Adjustment effects

If horizontal diffusion terms are omitted equation (3) can be written as:

$$\bar{c}_e = \bar{c} + T_A \frac{\partial \bar{c}}{\partial t} + L_x \frac{\partial \bar{c}}{\partial x} + L_y \frac{\partial \bar{c}}{\partial y} \quad (4)$$

with

$$T_A = \frac{\gamma_{21}}{\gamma_{11}} \frac{h}{w_s}, \quad L_x = \frac{\gamma_{22}}{\gamma_{11}} \frac{\bar{u}h}{w_s}, \quad L_y = \frac{\gamma_{22}}{\gamma_{11}} \frac{\bar{v}h}{w_s} \quad (5)$$

Equation (4) describes the adjustment of the depth-averaged concentration to its equilibrium value. The parameters T_A, L_x and L_y represent the characteristic scales in time and space of this adjustment process (adaptation time, adaptation length). The ratios γ_{21}/γ_{11} and γ_{22}/γ_{11} in (5) can be considered as dimensionless adaptation times and lengths.

If the "gradient" bed boundary condition is used only the expressions for dimensionless adaptation time and length change (see Katopodi and Ribberik, 1988).

The adaptation time and lengths as well as the depth averaged equilibriun concentration are determined by the local, instantaneous hydrodynamic conditions and sediment characteristics and can thus be computed in advance before the calculation of $\bar{c}$.

If adaptation length (time) is smaller than the maximum allowable grid size (time step) of the numerical calculation, the sediment redistribution is not resolved by the grid. Then the concentration can be assumed close to its (local, instantaneous) equilibrium value and a suspended sediment transport formula can be used for the calculation of the bed level changes. If this is not the case, the adaptation phenomena should be taken into account and the concentration can be computed from (3).

The influence of the wave and current parameters that determine the coefficients of equation (4) and consequently the dimensionless adaptation time and length was studied by computing the coefficients for a wide range of these parameters.

The dimensionless adaptation time and adaptation length appeared to be affected most strongly by the parameters w_s/u_* (current mixing) and H_s/h or H_s/w_sT (wave mixing). The combined influence of the two parameters is shown in figure (1). It is shown that the dimensionless adaptation time (and length) show a considerable increase when waves are superimposed to a current. (Compare the curves of $\gamma_{21}/\gamma_{11} = f(w_s/u_*)$ for $H_s/h = 0$ and for $H_s/h = .2$ etc).This increase is more significant when current alone causes negligible suspension adjustement effects ($w_s/u_* > .5$).

The influence of the other parameters $\hat{u}_{orb}/w_s$, D_*, δ/h, $\bar{u}/u_*$ and z_a/h was found to be considerably smaller.

3.3 Validity conditions

The validity of the model was studied for the case of waves superimposed to a current through comparison with an analytical solution of the convection-diffusion equation, based on an earlier analysis of Wang and Ribberink (1986) for current alone. The results of this analysis are:

The presence of waves leads to a considerable extension of the validity area of the quasi-3D approach. Generally the validity area increases as the suspended load increases (i.e large current velocities, large waves and fine sediment) or for small values of a suspension parameter "modified for the waves", indicated by the analysis:

$$Z_{c,w} = \frac{w_s}{\varkappa u_* + .14\ \alpha_{br} H_s/T} < .75 \qquad (6)$$

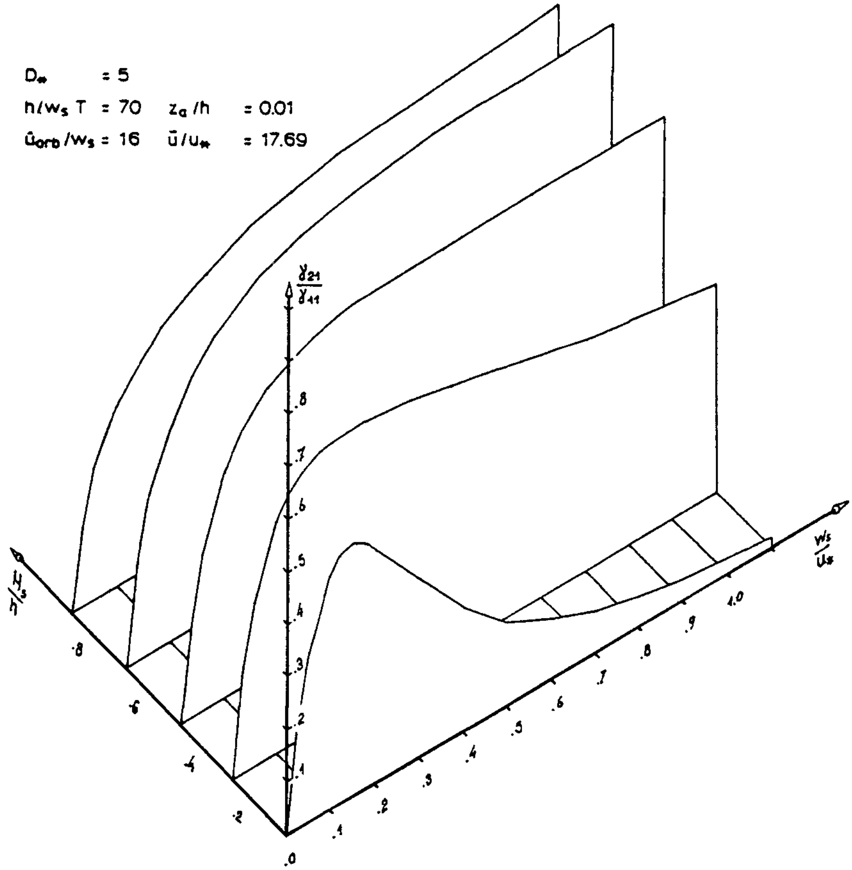


Fig.1 Dimensionless adaptation time as function of (w_s/u_*) and (H_s/h).

For current only an upper limit holds for the suspension (Rouse) parameter Z_c:

$$Z_c = w_s/(\varkappa u_*) < .75 \tag{7}$$

It is obvious from (6) and (7) that the presence of the waves enhances the model validity area considerably. Of course, the smaller the suspension parameter the better the asymptotic solution.

The assumptions made in order to get the asymptotic solution impose restrictions on the length scale L and time scale T of the particular problem. L and T can be considered as the length scale of changes in hydraulic conditions (e.g bottom shear stress) and time scale of changes in hydraulic conditions (e.g tidal period).

The analysis shows that these changes should be so gradual that the following conditions are fulfilled:

$$L \gg \frac{1}{Z}\frac{u\bar{h}}{w_s} \quad \text{and} \quad T \gg \frac{1}{Z}\frac{h}{w_s} \tag{8}$$

In (8) the suspension parameter Z refers to current or to current and waves(ie Z_c or $Z_{c,w}$)

Comparison of length (and time) scales allowable by (8) for the case of current and current and waves shows that superposition of waves on a current allows the application of the model for less gradual changes in hydraulic conditions than in case of current only and thus enlarges the validity area.

4. RESULTS

4.1 Comparison with a 2DV model

In 2DV plane the problem of suspended sediment transport adjustment in steady uniform flow conditions and zero sediment input at the upstream boundary (similar to fig. 5A)is solved with the asymptotic model and a 2DV numerical model (SUTRENCH, van Rijn, 1986). Several cases with varying current and wave parameters are compared. It should be noted that the chosen problem with a sudden change in bed boundary condition at the upstream boundary (reference concentration goes from zero to equilibrium) is a hard test for the asymptotic model that requires gradually varying conditions.

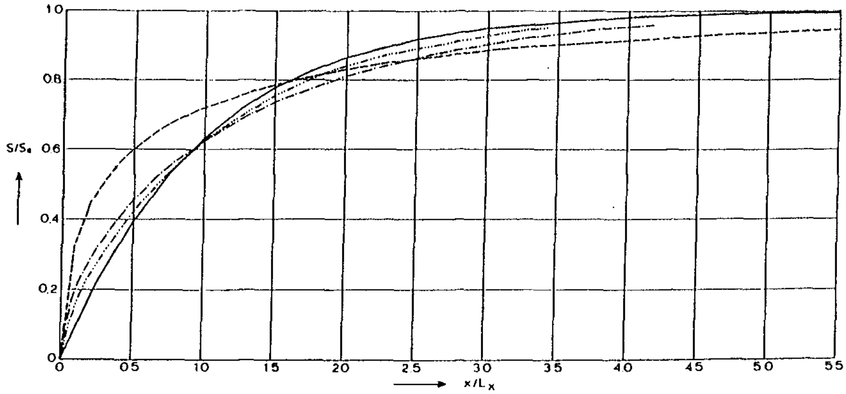
In figure (2) the transport computed with the asymptotic model and SUTRENCH is shown. The x coordinate is divided by the adaptation length so that the QUASI-3D model is represented in all cases by the same line.

For current only (2A) it is evident that the smaller w_s/u_* (or Z_c), the closer the asymptotic model and SUTRENCH are. The agreement is rather good for the last two cases. This confirms the validity analysis (eqn. 7).

In figure (2B), the worse case of fig 2A (w_s/u_* =.5) improves more and more as the wave contribution increases.

4.2 Comparison with a 3D numerical model

The QUASI-3D model is tested by comparison with a numerical 3D solution (SUSTRA 3D). In figure (3) the non-equilibrium suspended sediment transport computed with the two models is presented for the case of flow around a local construction. The agreement is satisfactory except in the zone behind the dam where the two models use different bed boundary conditions.



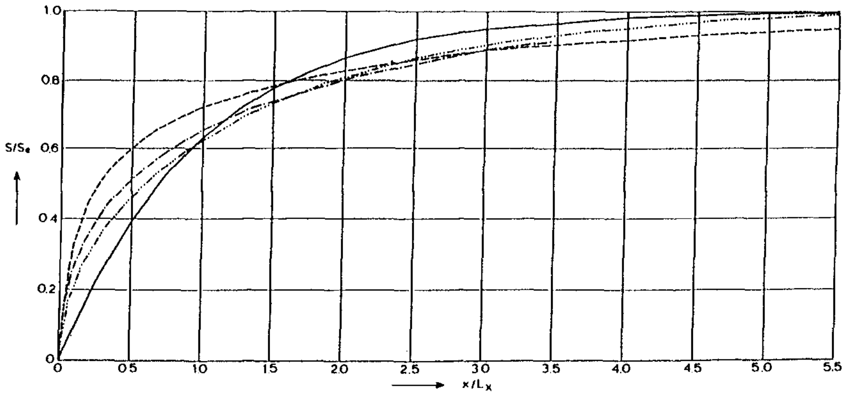
SUTRENCH	H_s/h	$\hat{u}_{orb}/w_s$	w_s/u_*	γ_{22}/γ_{11}	L_X/h
– – – – – –	0	0	0.5	0.1465	5.184
—·—·—·	0	0	0.2	0.4345	38.443
–··–··–··	0	0	0.1	0.4482	76.296
———	asymptotic model (transport zero order)				

$L_X/h = (\gamma_{22}/\gamma_{11})\,(\bar{u}/u_*)\,(u_*/w_s)$

z_a/h = 0.01

$\bar{u}/u_*$ = 17.69

A: CURRENT



SUTRENCH	H_s/h	$\hat{u}_{orb}/w_s$	w_s/u_*	γ_{22}/γ_{11}	L_X/h
– – – – – –	0	0	0.5	0.1465	5.184
—·—·—·	0.2	34	0.5	0.3276	11.594
–··–··–··	0.4	34	0.5	0.4544	16.081
———	asymptotic model (transport zero order)				

$L_X/h = (\gamma_{22}/\gamma_{11})\,(\bar{u}/u_*)\,(u_*/w_s)$

D_* = 5

h/w_sT = 30 z_a/h = 0.01

δ/h = 0.02 $\bar{u}/u_*$ = 17.69

B: CURRENT AND WAVES

Fig.2 Adaptation of suspended sediment transport.
A: Current only, B: Current and waves

The agreement in the main flow area is depicted more detailed in figure (4 left) where the equilibrium and the non-equilibrium transport along grid line j=5 are plotted for the two models. The equilibrium transport, as expected, is exactly the same for both SUSTRA 3D and the QUASI-3D model. The non-equilibrium transport (substantially different than the equilibrium one) as computed with the two models show a very good agreement.

In figure (4 right) the computed non-equilibrium transport along line j=18 (much stronger variations) is shown for the two models. In the recirculation zone the difference between the two models becomes considerable.

The necessary computation times of the two models were compared. The QUASI 3D model proved to be 8-10 times faster than SUSTRA 3D.

4.3 EXPERIMENTAL VERIFICATION (CURRENT AND WAVES)

The first order asymptotic model was compared with a laboratory experiment carried out in a mobile bed flume with current and wave conditions and zero sediment transport at the upstream boundary. The experiment was carried out by Galappatti and van Rijn (1984).

Fine sediment particles were entrained by a sediment free flow over a sand bed in the presence of monochromatic waves. The flow and wave conditions were uniform. The velocity profile was logarithmic.The concentration profiles (eight depth points) were measured at six stations along the flume. Upstream the mobile bed, the bed was fixed (experimental set-up in figure 5A).

In figure (5.B) the zero order suspended sediment transport computed with the asymptotic model is plotted against the dimensionless distance along the flume. The results of SUTRENCH (van Rijn,1986) are also presented. The computed sediment transport is always somewhat smaller than the measured one. The largest deviation is about 20% which can be considered as rather good result.

In figure (6) the concentration profiles computed and measured at the six stations indicated in (5.A) are shown. The agreement again is reasonably good.

In total, the above comparison shows that the QUASI-3D model is able to predict the suspended sediment concentration due to current and waves with a reasonable degree of accuracy against reasonable costs.

Moreover the choice to treat wave mixing in a similar manner to turbulence mixing (diffusion analogy) seems to be justified.

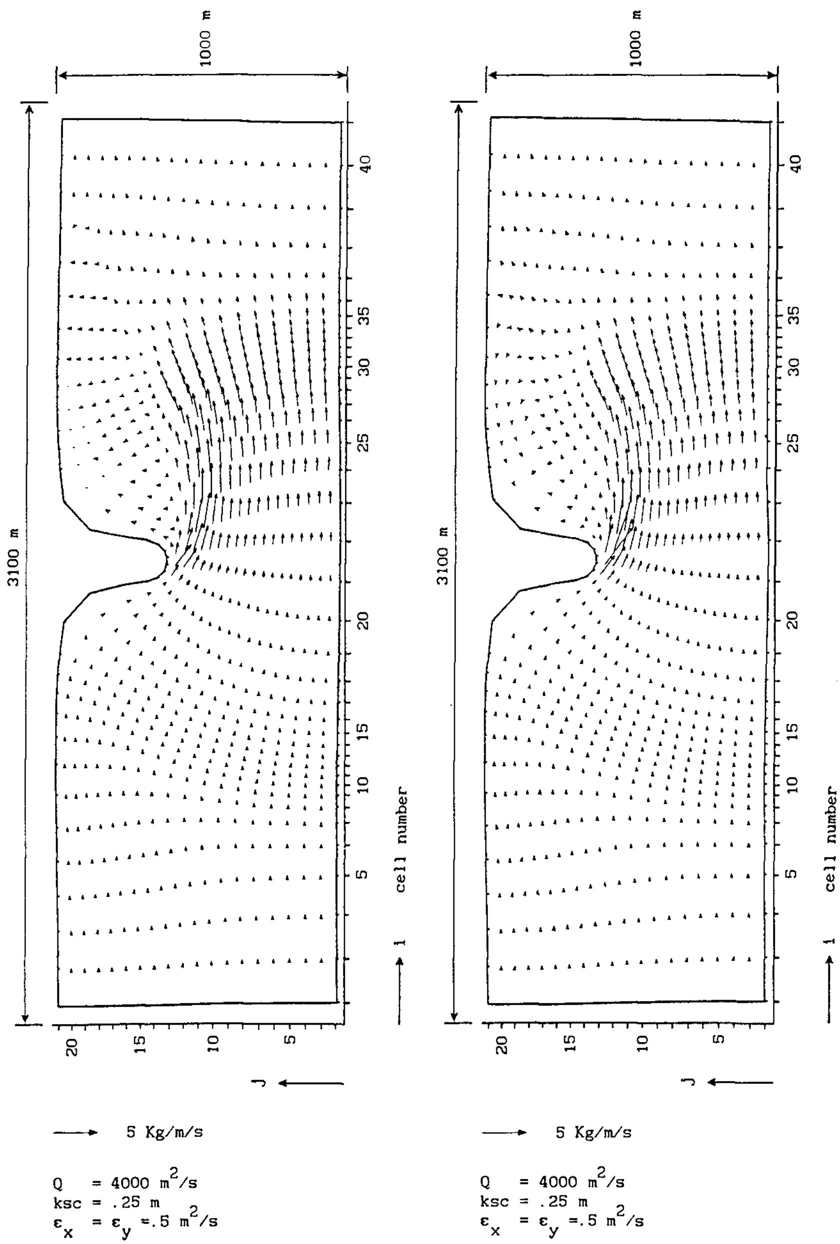


Fig.3 Non-equilibrium transport
Left: QUASI-3D, Right: 3D

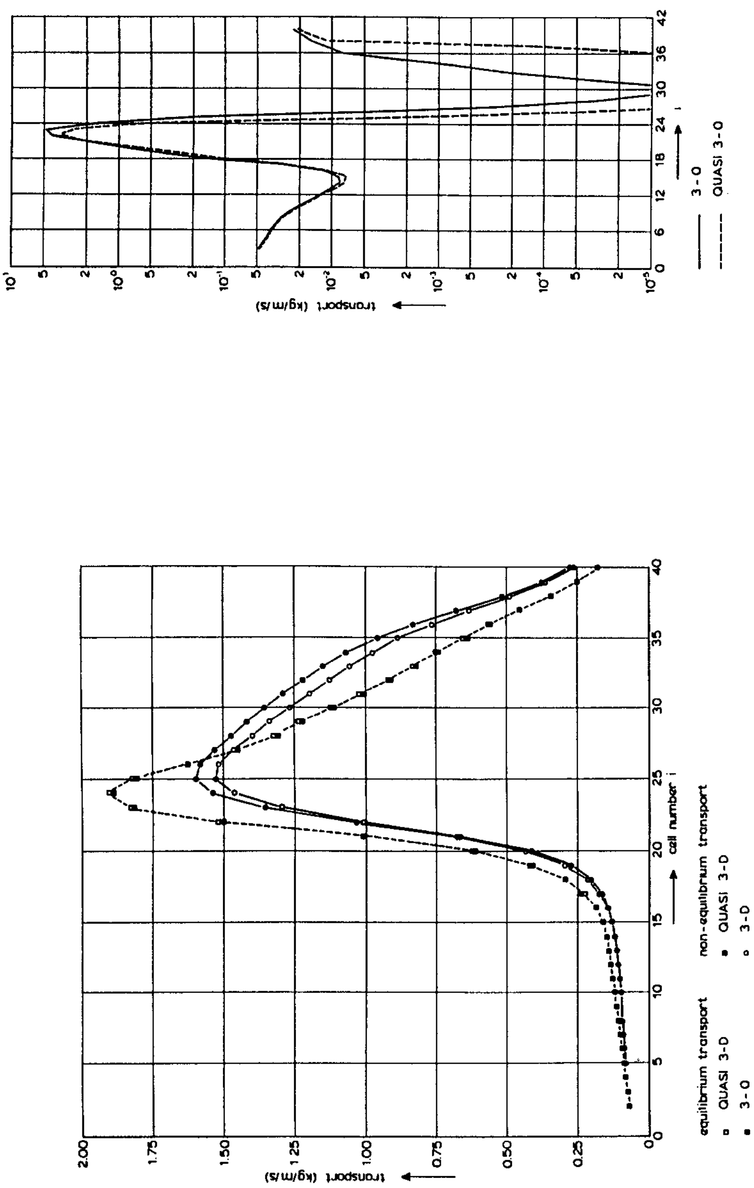
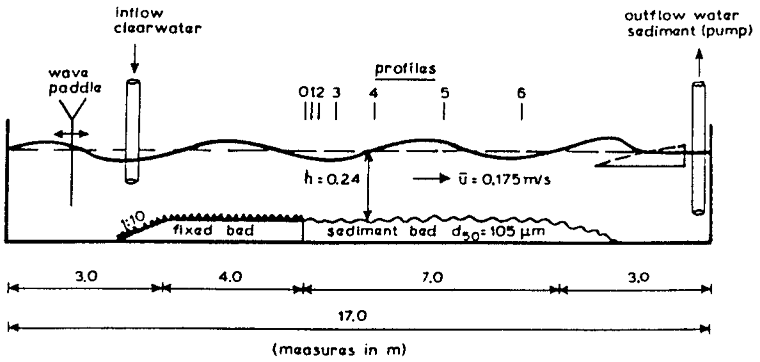
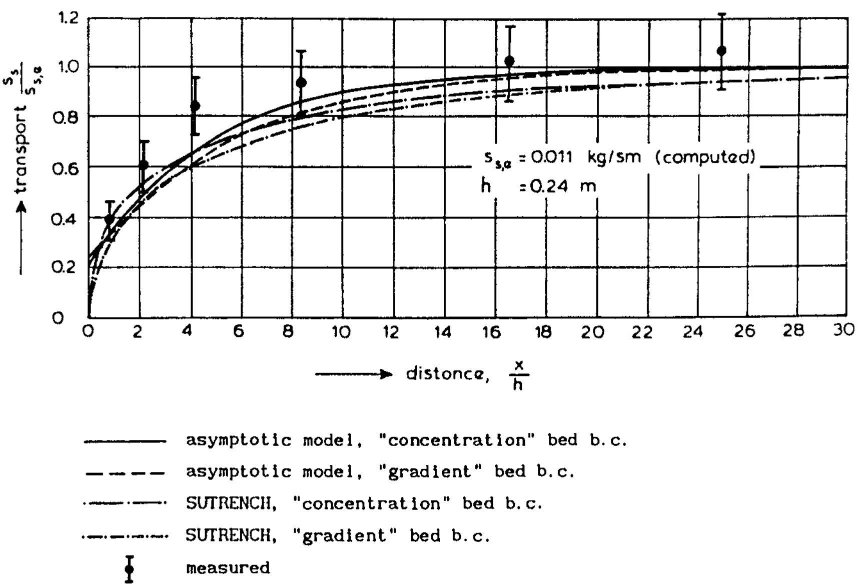


Fig.4 Left: Equilibrium and non-equilibrium transport along grid line j=5
Right: Non non-equilibrium transport along grid line j=18



A. EXPERIMENTAL SET-UP



B. SUSPENDED SEDIMENT TRANSPORT

Fig.5 Adaptation of sediment transport in a flume (current and waves).

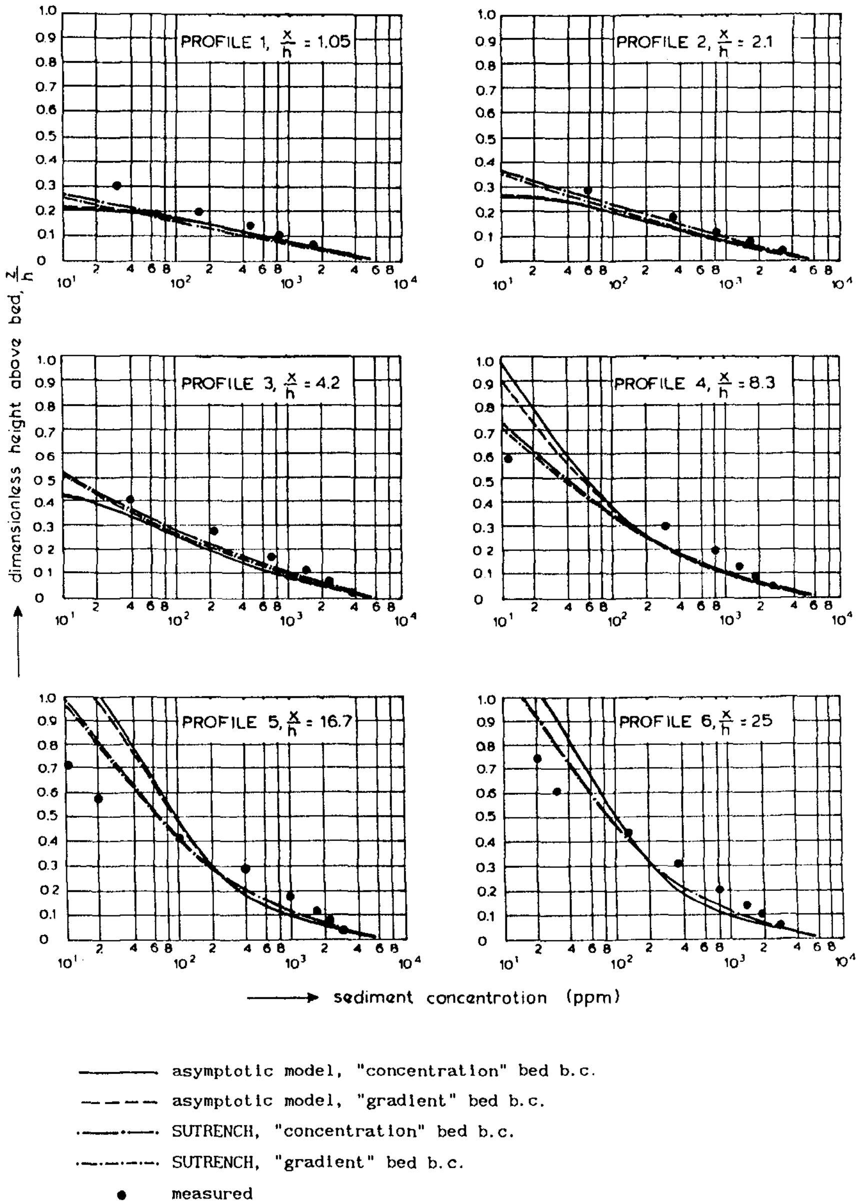


Fig.6 Adaptation of sand concentration profiles in a flume (current and waves).

5. CONCLUSIONS

The QUASI-3D model combines the reduced computational effort of 2DH models with the ability to describe vertical redistribution of suspended sediment concentration while the bed boundary condition is maintained.

The presence of waves leads to a considerable extension of the area of validity of the quasi-3D approach.

The applicability of the model can always be checked beforehand using the validity conditions and this is a big advantage of the model. The validity area indicated by the analysis is in the usual range of many practical (coastal) problems.

As concluded by the the sensitivity analysis superposition of waves on a current appeared to lead to a considerable increase of the importance of the delayed adjustment phenomena.

The numerical as well as the experimental comparison showed that the model behaviour is rather satisfactory.

Introduction of quasi-3D velocities is the next step for the model improvement.

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