

CHAPTER 122

CONCRETE ARMOR UNIT STRUCTURAL DESIGN CRITERIA

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ABSTRACT

A comprehensive examination of the structural response of breakwater armor units to static and wave-induced loads has been completed using a dolos armor unit load cell in a physical breakwater model. Three extensive test series were completed to assess the influence of a wide range of breakwater and sea state parameters, and data reduction and analyses techniques were developed to estimate the structural response as a function of the critical independent parameters.

Using these data, improved design criteria, which consider both the hydraulic and structural aspects of breakwater design, have been developed for the dolos unit. Perhaps even more significant is the fact that the basic instrumentation scheme, model testing program, and data reduction and analysis procedures could be utilized to develop improved design procedures for other armor units, or to assist in the development of a new armor unit.

INTRODUCTION

Concrete armor units are used to protect rubblemound breakwaters when quarried stone of sufficient size to resist the incident wave climate is not available. The design of these breakwaters considers the hydraulic stability of the armor layer, but does not generally consider the structural integrity of the armor units themselves. The resultant limitations in these design approaches were clearly demonstrated by a number of damages and failures of rubblemound breakwaters in the late 1970's and early 1980's.

A co-operative effort between W.F. Baird and Associates, the Department of Civil Engineering at Queen's University, and the Hydraulics Laboratory of the National Research Council of Canada was initiated in 1980 to investigate the nature and magnitude of the forces acting on breakwater armor units. This effort led to the development of an armor unit "load cell" (Scott, 1986) which could accurately measure static and wave-induced (quasi-static) loads in a hydraulic breakwater model. This instrumentation was used in several extensive test

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series to provide the structural response data required to develop structural design criteria for the dolos armor unit, as discussed in this paper.

INSTRUMENTATION - THE DOLOS LOAD CELL

In this study, the structural response of the dolos armor unit to static and wave-induced (quasi-static) loads was measured in a breakwater model using the armor unit load cell developed by Scott (1986). The load cell utilizes the concept of geometric distortion, and consists of a thin-walled aluminum tube instrumented with strain gauges and inserted into the shank of a hollowed out dolos unit, as shown in Figure 1. Three full strain gauge bridges measure two orthogonal bending moments and the torque at the mid-shank section; instrumentation to measure other load components, such as axial and shear forces, was not provided, as a review of prototype damages and previous analyses for the dolos unit indicated that bending and torsional stresses were dominant. The accuracy and sensitivity of the load cell instrumentation were investigated during calibration by comparing the measured moments and torques to theoretical values.

The original load cell design utilized in the first parametric study (wave-induced loads) was instrumented at the mid-shank section. For the subsequent investigations of static forces and the combined effects of static and wave-induced forces, four improved load cells were constructed, two instrumented at the mid-shank (MS) section and two at the fluke-shank (FS) section. These improvements are discussed in detail in W.F. Baird and Associates (1989b). Details of the two designs were identical except for the location of the split in the dolos shank. In general, all of the load cells utilized in the various testing programs performed well in the harsh environment of a hydraulic breakwater model.

TESTING PROGRAMS

Approximately 2000 tests were undertaken in three separate test series using the dolos load cell in a physical breakwater model in order to measure static and wave-induced (quasi-static) forces in the armor units, and to provide structural response data from which preliminary design criteria could be developed. All tests were undertaken using a "conventional" breakwater cross-section, with a primary armor layer of dolos units (mass of 482 g and density of 2500 kg/m³) placed over a filter layer of crushed stone (mass of 30 to 100 g), as shown in Figure 2. The dolos units had a waist ratio (shank width/overall length) of 0.32 and were randomly placed in two layers with a packing density, ϕ (as defined by Zwamborn, 1978), of 0.8 to 0.9.

The first parametric study of quasi-static (wave-induced) loads was completed using a two-dimensional wave flume at the Hydraulics Laboratory of the National Research Council of Canada (Baird and Associates, 1989a). Over 1000 tests were run with regular waves and approximately 140 tests with irregular waves to assess the effects of breakwater slope, armor unit location, wave period and wave height on armor unit structural response.

After placing two load cells (mid-shank units only) at selected locations in the armor layer, the breakwater was subjected to a short of waves, and the wave gauge and amplified load cell responses (M_1 , M_2 , T) were sampled at 500 Hz for 30 s (regular waves) or 60 seconds (irregular waves). This sample length covered from 10 to 34 wave periods depending on the incident wave period.



Figure 1. The Dolos Armor Unit Load Cell

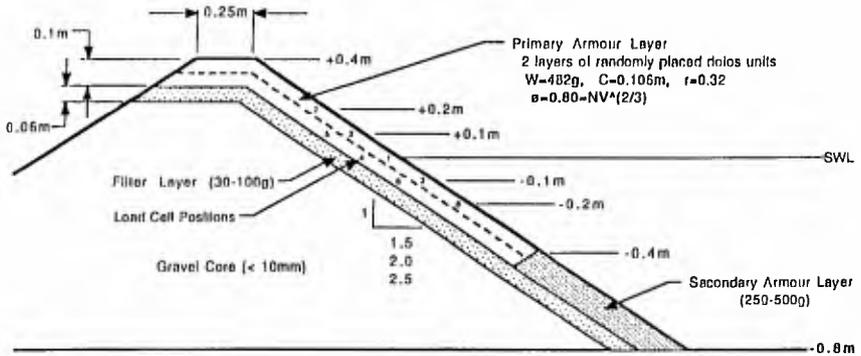


Figure 2. Model Breakwater Cross-Section

The next set of tests, a parametric study of static loads, was completed in a dry environment, with the armor and filter layers constructed on a 0.7 m wide by 1.3 m long plywood ramp. Over 400 tests were run to assess the effects of breakwater slope, load cell location (relative to the "crest" of the breakwater), vibration, and the location of the force measurements (mid-shank section or fluke-shank section).

An additional series of flume tests was then undertaken, using the improved mid-shank and fluke-shank armor unit load cells, in order to measure the combined effects of both static and quasi-static (wave-induced) forces. The primary objective of this test series was to obtain data to allow an assessment of the interaction of the two force components (static and quasi-static). Consequently, a procedure could be developed to combine the results of the two independent parametric studies of these force components to estimate the overall response of the dolos unit to the combined loading conditions. The breakwater cross-section was the same as that used in the earlier flume study (see Figure 2), but the range of parameters tested was reduced as there was no need to repeat all the previous tests to assess the interaction of these forces.

DATA REDUCTION

An initial review of the structural response of the armor unit load cell to wave-induced forces was undertaken using the time series and spectral density plots. An example of these presentations for a selected regular wave test are shown in Figures 3 and 4. The time series plot (Figure 3) shows the repeatability of the response with the wave period, but shows considerable variation between the magnitude and shape of the "response profile" for the three measured load components (M_1 , M_2 and T). The spectral density plot (Figure 4), which shows the energy in the signal as a function of frequency, reveals the presence of an energy peak in the response at the incident wave frequency (f), and also at second and higher harmonic frequencies (i.e. nf , where $n = 1, 2, 3 \dots$). The largest peak in the response was normally located at the incident wave frequency or at the second harmonic frequency. The presence of second and higher harmonics in the response indicates a non-linear interaction between the response and the incident waves. Harmonic peaks in the response signals were observed to occur under the full range of wave conditions and load cell locations tested, although not in every test.

Similar observations were made in reviewing the response of the dolos load cell to irregular waves. The time series plots demonstrated the grouping effect of the incident wave train, with groups of large waves generating a large response in the armor units. The spectral density plots showed that the armor unit response typically had a peak frequency equal to that of the incident wave train (f_p); however, as with the regular wave tests, higher order harmonic peaks were observed in many of the tests.

In order to undertake the various analyses on the load cell response data necessary to develop design criteria, it was desirable to combine the effects of the three measured load components into a single quantity that could be related to a relevant failure criterion for concrete armor units. For unreinforced concrete structures, the exceedance of the tensile cracking strength at any point in the structure is a well accepted failure criterion. Consequently, the measured moment and torque time series (or simply values for the static tests) were used to calculate the time series (or value) of the maximum principal stress (σ) at the measurement location (mid-shank or fluke-shank). This calculation was undertaken using standard structural engineering equations, as summarized below:

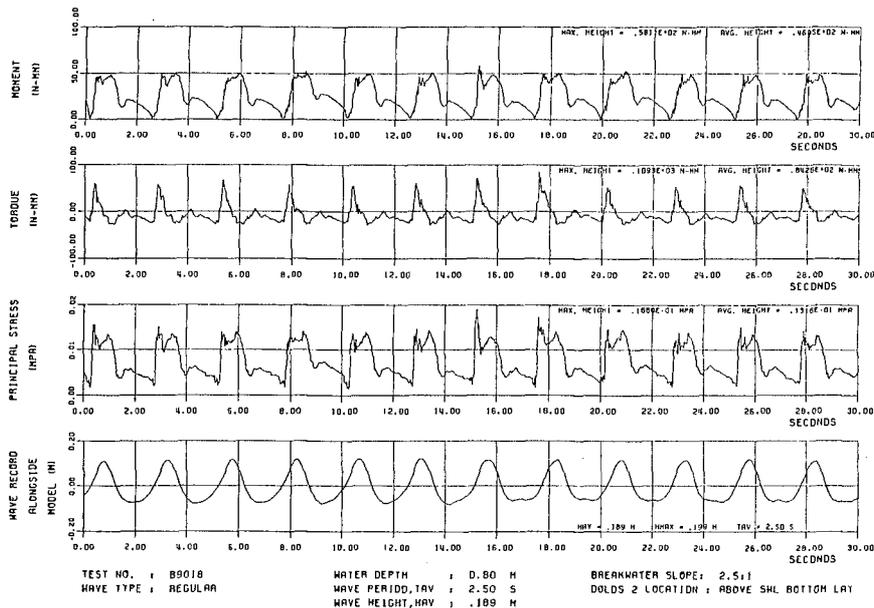


Figure 3. Load Cell Response Time Series

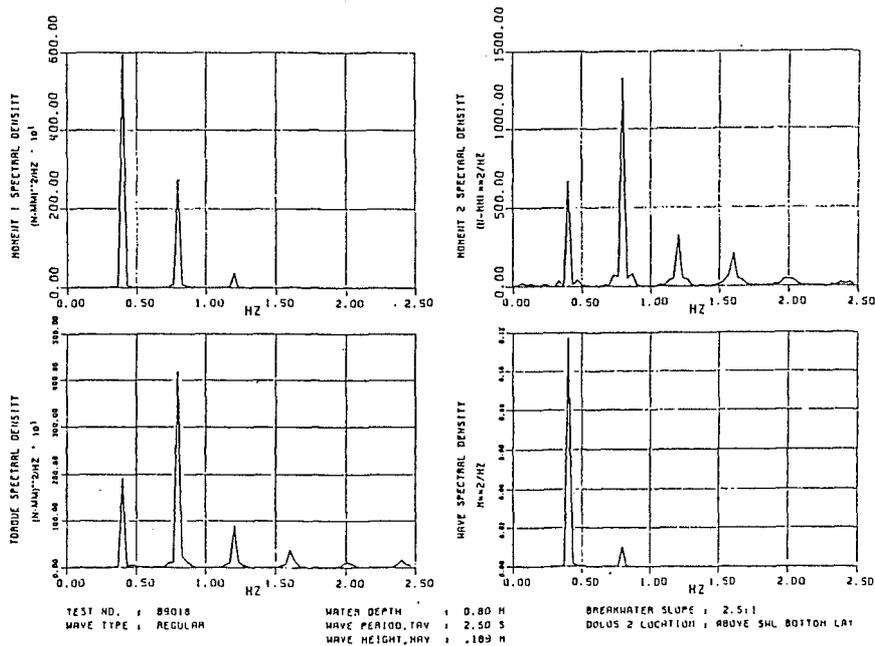


Figure 4. Energy Spectra for Load Cell Response

$$\sigma = 0.5(\sigma_x) + \sqrt{(0.5\sigma_x)^2 + \tau^2} \quad (1)$$

where $\sigma_x = Mc/I$ (longitudinal stress due to resultant bending moment, M) (2)

$\tau = Tc/J$ (shear stress due to torque, T) (3)

with $M = \sqrt{M_1^2 + M_2^2}$ (resultant bending moment calculated from measured moments M_1 and M_2) (4)

T = measured torque

c = distance from neutral axis to surface of dolos shank

I = moment of inertia of dolos shank section

J = polar moment of inertia of dolos shank section

For the static tests, the calculated maximum principal stress value, after scaling to prototype, could simply be compared to the tensile strength of the concrete (ft) to assess the structural integrity of the unit under specified conditions. However, in the case of the quasi-static tests, where the wave-induced principal stress consists of a 30 or 60 s time series rather than a single value, it was desirable to derive a characteristic value of the principal stress time series. Preliminary data analyses were undertaken using the "average stress height" ($\sigma_{avg ht}$) for regular waves and the "one third stress height" ($\sigma_{1/3 ht}$) for irregular waves.

For the development of design criteria, it seemed more appropriate to select an extreme value from the time series with a specific probability of exceedance. Statistical analyses of the regular wave results were undertaken which showed that the distribution of the response peaks within any given sample could be described by a normal distribution. Thus, by calculating the mean (\bar{x}) and standard deviation (s) of the principal stress peaks within a particular sample, a characteristic large value with a specified probability of exceedance could be calculated.

DATA ANALYSES

Initial parametric analyses of the static and quasi-static test data were completed by producing a series of scatter plots for the dependent variable (a characteristic value of the principal stress response) plotted as a function of selected independent variables. The effects of other independent variables was shown by sorting the data into specific ranges for a selected variable and calculating regression lines for the various data subsets.

For example, the principal stress measured in the static armor layer tests was plotted as a function of the vertical distance below the breakwater crest for sixteen data subsets, defined by mid-shank (MS) or fluke-shank (FS) instrumentation, top or bottom layer, before or after vibration, and 1:1.5 or 1:2.5 slope. Figure 5 shows a plot of the full data set for a 1:1.5 slope. In general, the measured stresses tended to be higher on the steeper slope, at locations further down the slope, and in the bottom layer. However, no consistent trend was observed in the results for MS or FS instrumentation location or vibration.

For the quasi-static (wave-induced) test results, the average stress height (for regular waves) or one-third stress height (for irregular waves) was plotted as a function of wave height (H), wave steepness (H/L) and surf similarity parameter ($\xi = \tan \alpha / \sqrt{H/L}$). These plots were produced for selected armor unit locations and periods, as well as for all the data sorted by these parameters, for each breakwater slope tested. A typical plot is shown in Figure 6, which shows $\sigma_{avg ht}$ vs H for all test locations plotted with the different wave periods identified.

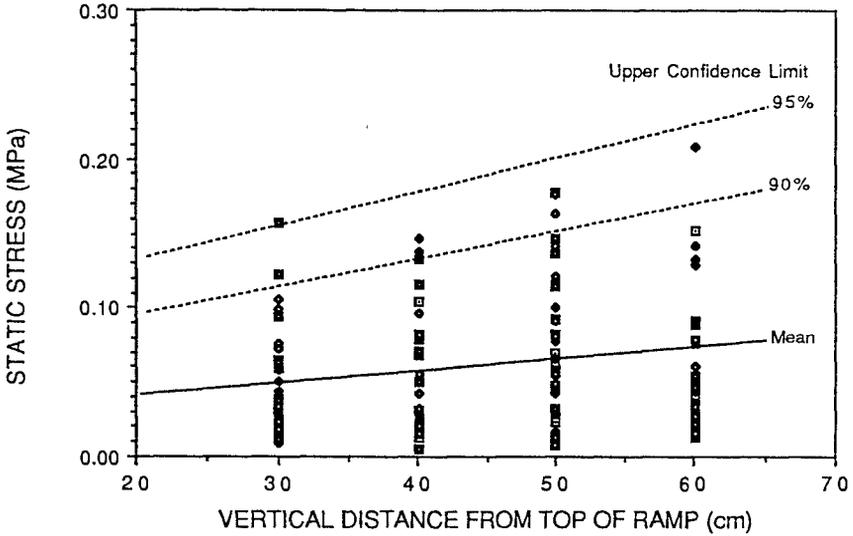


Figure 5. Static Stress as a Function of Vertical Distance (1:1.5 slope, bottom layer)

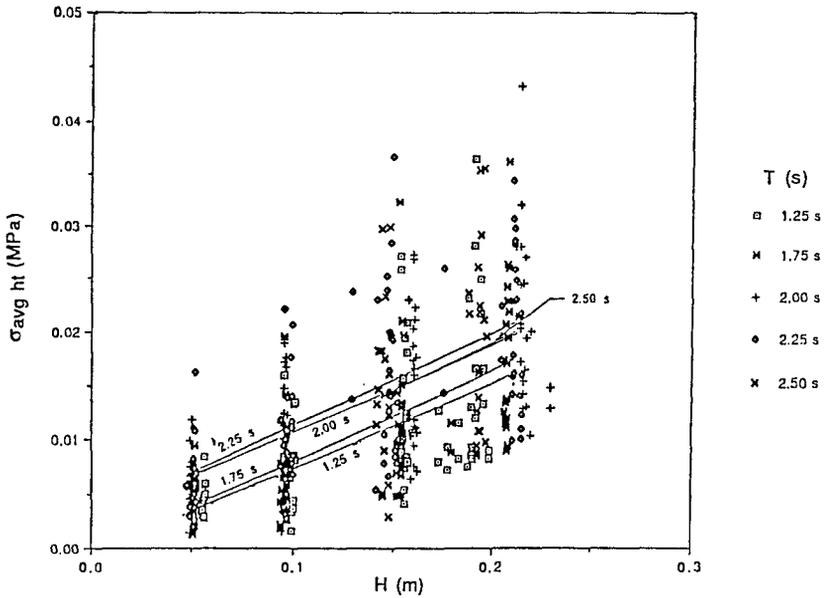


Figure 6. Average Stress vs Wave Height (1:1.5 slope)

In general, wave-induced stresses for both regular and irregular waves were observed to increase with wave height, wave period and wave steepness, while the effects of breakwater slope, armor unit location [top or bottom layer, and position relative to the still water level (swl)] were not as consistent. Wave-induced stresses under regular waves tended to be greatest on a slope of 1:2.0 and lowest on a slope of 1:2.5, with 1:1.5 falling in between; the effect of slope was not investigated in the irregular wave tests. Plots of the average stress height versus the surf similarity parameter (Figure 7) showed that the maximum stresses tend to occur for ξ between approximately 2 and 3.5, which is consistent with earlier literature concerning armor unit stability [for example, Van der Meer and Pilarczyk (1987)].

Multiple regression analyses were undertaken independently on the test results from the static and quasi-static (wave-induced) parametric studies in order to systematically assess the influence of the various independent parameters and to develop a mathematical expression defining the structural response (principal stress) of the dolos unit as a function of the critical parameters. A multiple regression analysis serves to identify three factors: a regression equation; standard error of estimate (s) and coefficient of multiple determination (R^2).

Linear combinations of the independent parameters were tested as an initial model for both the static and quasi-static test results. For the quasi-static test results, the regular and irregular tests were treated separately. In all three cases, it was found necessary to transform the dependent variable (principal stress) by taking its logarithm in order to satisfy the assumption of constant variance in the regression model. This transformation is consistent with the exponential relationship observed between the average stress height and the wave height in the regular wave tests, as mentioned earlier. The resulting regression equations for the three data bases, and their associated standard errors and coefficients of multiple determination, are summarized below:

Static Tests:

$$\log(\sigma_s)_{est} = -2.28 + 0.91(m) + 0.30(D_v - 0.45) + 0.34(layer) \quad (5)$$

$$s = 0.31, R^2 = 0.31$$

Note: the effects of instrumentation location (MS or FS) and vibration (before or after) were found to be insignificant.

Quasi-Static Regular Wave Tests:

$$\log(\sigma_{avg} h)_{est} = -2.70 + 0.18(m) + 0.31(D_{swl}) + 0.13(T) + 2.70(H) \quad (6)$$

$$s = 0.20, R^2 = 0.48$$

Note: the effect of layer (top or bottom) was found to be insignificant.

Quasi-Static Irregular Wave Tests:

$$\log(\sigma_{1/3} h)_{est} = -2.33 - 0.20(D_{swl}) + 0.07(T_p) + 1.91(H_s) \quad (7)$$

$$s = 0.20, R^2 = 0.48$$

Note: the effect of layer (top or bottom) was found to be insignificant.

The parameters in these equations are defined below:

$(\sigma_s)_{est}$	= estimated principal stress from static tests (MPa)
$(\sigma_{avg ht})_{est}$	= estimated average principal stress height, regular wave tests (MPa)
$(\sigma_{1/3 ht})_{est}$	= estimated one-third principal stress height, irregular wave tests (MPa)
m	= tangent of breakwater slope angle (for a 1:2.0 slope, m = 0.5)
D_v	= vertical distance of load cell test location below breakwater crest (m)
D_{swl}	= vertical distance of load cell location from still water level (m) (positive is above the swl)
layer	= categorical variable for layer, top = 0, bottom = 1
T	= average wave period for regular wave tests (s)
H	= average wave height for regular wave tests (m)
T_p	= peak wave period for irregular wave tests (s)
H_s	= significant wave height for irregular wave tests (m)

The standard error of the estimate, which provides a measure of the scatter of the data, can be used to define confidence or exceedance limits for the data if one knows the distribution of the data. Preliminary statistical analyses of the regular wave test results demonstrated that the distribution of the $\log(\sigma_{avg ht})$ values about the regression hyperplane could be described by a normal distribution. Thus, an exceedance limit for the average stress height can be established at any specified probability level using the results of the multiple regression analysis, as shown below:

$$\log(\sigma_{avg ht})_{\alpha} = \log(\sigma_{avg ht})_{est} + \Phi^{-1}(\alpha)s \quad (8)$$

where $(\sigma_{avg ht})_{\alpha}$	= average stress height with a probability of exceedance of α
$(\sigma_{avg ht})_{est}$	= average stress height estimated by the regression equation
$\Phi^{-1}(\alpha)$	= standard normal variate evaluated at probability level α
s	= standard error of the regression equation estimate

This probabilistic approach was incorporated in the development of structural design criteria for the dolos armor unit, as described later in this paper. However, the design criterion for wave-induced loads was developed using a peak stress value with a specified probability of exceedance from each test time series, rather than the average stress height from the time series.

The coefficients of multiple determination were 0.3 for the static tests and approximately 0.5 for the quasi-static tests. This implies that the respective regression models account for only 30 and 50% of the observed variation in the logarithm of the stress value. Some of this variation may be attributed to experimental error; however, a significant portion of this scatter may be explained by the effects of random orientation and boundary conditions of the armor units in the complex armor layer. Quantification of these variables was not attempted in this study. More complex regression models incorporating non-linear terms of the independent variables were investigated, but did not provide any significant improvements over the log-linear models discussed above.

DEVELOPMENT OF DESIGN CRITERIA

Introduction

Design stress level relationships have been derived from the results of the extensive testing programs described earlier for the following loading conditions:

- Static Loads - stresses induced by armor unit self-weight and inter-unit contact
- Quasi-Static Loads - fluctuating stresses induced by wave action
- Total Loads - the total stress due to the combined effects of static and quasi-static (wave-induced) loading

These relationships were derived on a statistical basis such that the designer can select the exceedance probability for the design stress level estimated for a particular set of design conditions. The development of these structural design criteria is presented in detail in Baird and Associates (1989b). Due to space limitations, the following section of this paper summarizes the results for only the total load condition.

Design Equation for Total Loads

A design equation was developed for the total stress occurring in a dolos armor unit due to the combined effects of static and quasi-static loads. The design equation for the combined loading condition is based on a statistical combination of predictive equations for estimating the static and quasi-static stresses. The predictive equations for the static and quasi-static stresses were derived from detailed parametric model studies of the two independent loading conditions using multiple regression analyses, while the statistical combination of these two equations was based on a third model study which investigated the combined effects of the two loading conditions. The resulting design equation for the combined loading condition is presented below.

From a design perspective, we require that:

$$n(\sigma_{tsp})_{\alpha} \leq f_t \quad \text{or} \quad \frac{n(\sigma_{tsp})_{\alpha}}{f_t} \leq 1 \tag{9}$$

where:

- $(\sigma_{tsp})_{\alpha}$ = design total stress level in model armor unit (MPa) with a probability of exceedance of α
- n = model scale factor
- f_t = prototype tensile strength of concrete (MPa)

The design total stress level in the model (MPa) is given by:

$$(\sigma_{tsp})_{\alpha} = (\sigma_{tsp})_{est} + [\Phi^{-1}(\alpha)]s \tag{10}$$

where:

$$(\sigma_{tsp})_{est} = 0.905(\sigma_s)_{est} + 0.639(\sigma_{qsp})_{est} \tag{11}$$

$$(\sigma_s)_{est} = 10^{**}[-2.28 + 0.91(m) + 0.30(D_v/n - 0.45) + 0.34(\text{layer})] \tag{12}$$

$$(\sigma_{qsp})_{est} = 10^{**}[-2.36 + 0.15(m) + 0.10(T/\sqrt{n}) + 0.29(D_{swl}/n) + 2.20(H/n)] \tag{13}$$

s = 0.001 (standard error of the estimate)

$(\sigma_{tsp})_{est}$ is the estimated peak value (5% exceedance) of the principal stress time series for the total loading condition, while $(\sigma_s)_{est}$ is the multiple regression equation estimate of the principal stress due to static loads and $(\sigma_{qsp})_{est}$ is the estimate of the peak value (5% exceedance) of the principal stress time series due to wave-induced loads.

The breakwater geometry parameters in equation 11 have been defined earlier. The representative model scale factor, n , is calculated using the following expression:

$$n = 9.43 \sqrt[3]{\frac{W}{.1549 w_r}} \quad (14)$$

where: W = armor unit mass (tonnes)
 w_r = unit mass of concrete (tonnes/m³)

The design equation presented above can be used to determine the maximum dolos size that is structurally sound, given various breakwater and sea state parameters. For example, given a particular armor unit size (mass) in a specific location, the unit mass of concrete, the breakwater slope, the tensile strength of concrete and the wave height and period, the structural capacity of the unit may be directly computed for different levels of exceedance probability.

For example, Figure 8 presents the maximum permissible dolos mass as given by both the structural capacity criterion (described above) and the hydraulic stability criterion (Hudson's formula) for specified conditions. The design equation for structural capacity was solved for the location within the breakwater producing the highest total stress (i.e.: bottom layer, $D_v/n = 0.6$, $D_{swl}/n = 0.1$).

In a typical breakwater design, the dolos units may be sized initially by means of a hydraulic model study or through use of a stability formula such as Hudson's formula. The structural performance of the armor unit may then be checked by means of the structural design equation for the specified breakwater geometry and sea-state conditions. An appropriate minimum concrete strength may then be selected that ensures the structural integrity of the individual armor units. In addition, the capacity of the dolos unit could be increased by increasing the waist ratio, or by specifying reinforcement as necessary to resist the applied loads. It should be noted that although the design equation is based on dolos load cell data for a waist ratio of 0.32, it is expected that small changes in this variable would not significantly affect the loads acting on the units, but could significantly reduce the stresses in the units due to the increased capacity of the section.

Limitations in the Design Equation

It is important to recognize that in developing this design equation, there are, as in any design equation, a general set of limitations. The general limitations in the application of these results to the structural design of armor units include the following:

- All tests were undertaken in a two-dimensional wave flume; therefore, the results can not be used to assess the influence of three-dimensional effects such as oblique wave action and breakwater layout features (such as heads, corners and transitions).

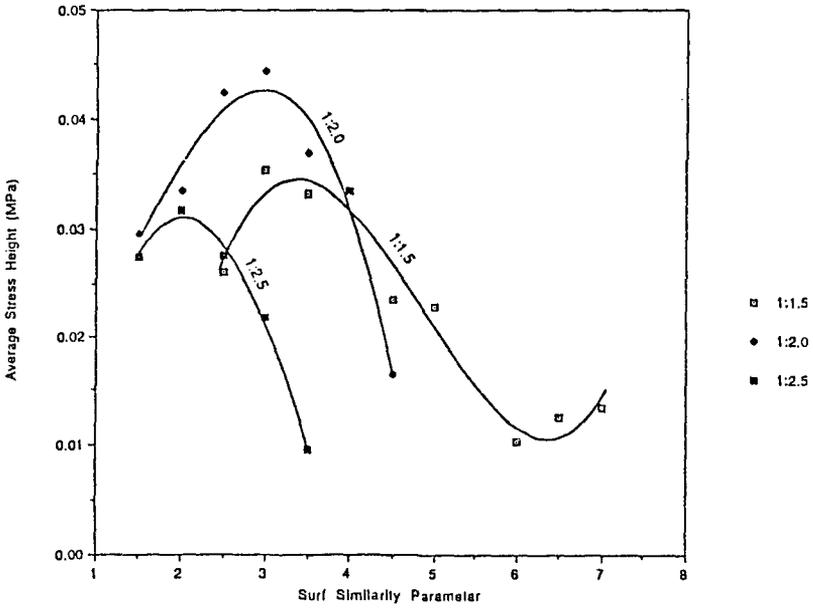


Figure 7. 5% Exceedance Average Stress Envelopes vs Surf Similarity Parameter

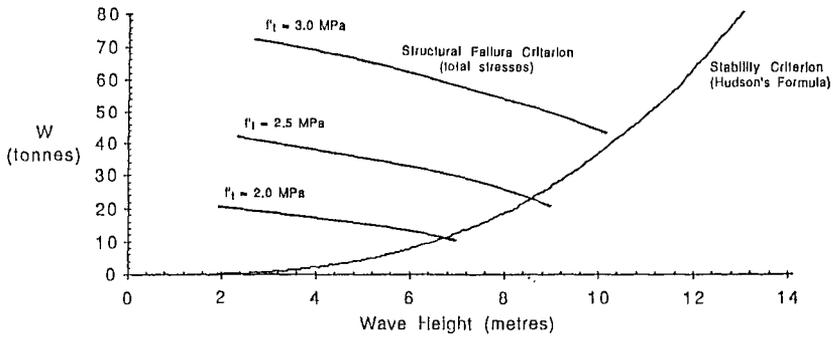


Figure 8. Dolos Hydraulic/Structural Failure Criteria
 ($w_r = 2.4 \text{ t/m}^3$, $K_D = 16$, 1:1.5 slope, $T = 12 \text{ s}$, 5% exceedance limit)

- All tests were completed using non-breaking waves ($H/d < 0.35$); therefore, shallow water effects, most importantly wave breaking, have not been considered.
- The design equation is based on regular waves tests; a limited test series was completed using irregular waves, but was insufficient to develop detailed design criteria. A detailed comparison between the response to regular and irregular waves should be undertaken and incorporated into the design equation.
- The tests were completed using a "no-overtopping" breakwater cross-section. Consequently, the effect of wave overtopping on the structural response of the armor units cannot be assessed.
- The tests were conducted at a single scale; thus, no quantitative assessment of scale effects can be obtained from the test results.
- Stresses derived from the model results are located at the surface of the mid-shank section of the dolos; the stress distribution throughout the dolos unit, in particular stress concentrations at the fluke-shank interface, have not been considered.
- Fatigue effects have not been considered.

Specific limitations on use of the design equations, based on the range of parameters tested, are summarized below:

Location	-with respect to the still water level:	$-0.1 \text{ m} < D_{sw}/n < +0.1 \text{ m}$
	-with respect to the breakwater crest:	$0.3 \text{ m} < D_v/n < 0.6 \text{ m}$
Wave Height:		$0.05 \text{ m} < H/n < 0.25 \text{ m}$
Wave Period:		$1.25 \text{ s} < T/\sqrt{n} < 2.50 \text{ s}$
Breakwater Slope:		$0.4 < m < 0.67$

If any of these quantities are outside these limits, the standard error of the predicted stress will increase, thus decreasing the reliability of the prediction.

So long as one is aware of these limitations, the design equation presented above can be used to provide an initial assessment of the structural integrity of the dolos unit under specified design conditions. However, these limitations must be addressed in future research work, which clearly represents a significant level of effort requiring global co-operation.

Application of the Design Procedure for Other Types of Armor Units

The load cell technology and analyses procedures developed in this study may be readily applied to other types of armor units, particularly those of a slender shape such as the tetrapod, quadripod, etc. As was described in this paper, general design equations may be developed, or it may be desirable to simply verify a preliminary design which has been based on hydraulic stability criteria.

The use of the load cell concept in a physical modelling environment also provides a tool for the development of new types of armor units. This procedure allows one to readily investigate the effects of changes to armor unit shape on both hydraulic stability and structural capacity.

CONCLUSIONS

A comprehensive examination of the structural response of armor units to static and quasi-static (wave-induced) loads has been completed using a dolos armor unit load cell in a physical breakwater model. Predictive equations have been derived which define the stress as a function of the critical breakwater geometry and sea state parameters, and a preliminary structural failure criterion for dolos

armor units has been developed for the first time. In addition, a design chart incorporating both the structural integrity and the hydraulic stability of the dolos unit has been presented. Limitations in the application of these results to the design of armor units have been extensively discussed to ensure that the results are not misused, and to provide guidance for future research. Finally, the procedures and methodologies developed in this study, including the instrumentation, model testing program, and data reduction and analyses techniques, can be readily applied to develop improved design procedures for other armor units, or to assist in the development of new armor units.

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