CHAPTER 113

MORISON EQUATION COEFFICIENTS AND DATA CONDITION

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Abstract

Identification of the Morison equation force coefficients C_m & C_d depends on the condition of the wave force/kinematic data. For simple harmonic kinematics, the data are equally conditioned for identifying C_m & C_d at the critical value of the period parameter ($K = U_m T/D$) defined by Keulegan and Carpenter (1958). Critical values for K are = 11.4 from geometric interpretations and = 13.16 from numerical interpretations. Stable transverse lift forces may also be shown to correlate with the critical values of K from the physical interpretation of the period parameter given by Keulegan and Carpenter (1958). Data from three different types of physical experiments are used to demonstrate the importance of the condition of the data.

Introduction

The Reynolds parameter ($R = U_m D/\nu$) and the Keulegan-Carpenter parameter ($K = U_m T/D$) are the two parameters that are most often used to parameterize the two empirical force coefficients C_m and C_d used in the Morison equation (Dean and Dalrymple, 1981 & 1984). Dean (1976) recognized the importance of the condition of the data when identifying these two empirical force coefficients from wave force data; but he did not correlate the condition of the data with these two parameters. The Dean error ellipse demonstrates geometrically the condition of data for identifying C_m and C_d by the alignment of the axes of the error ellipse as illustrated in Fig. 1.

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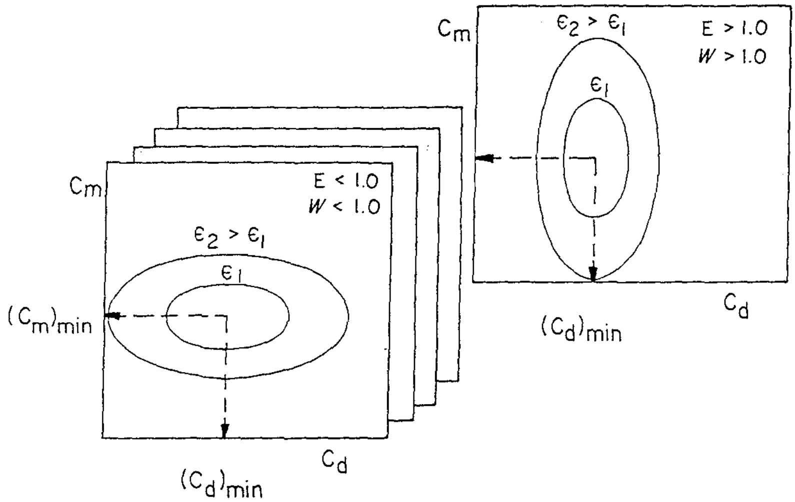

Fig. 1 Dependency of Dean Error Ellipses on $W = |f_d|/|f_m|$

The data are relatively better-conditioned for identifying the empirical force coefficient C_m or C_d on the axis that is parallel to the semi-minor axis of the error ellipse. Dean (1976) correlated the alignment of the axes of these error ellipses with the dimensionless O'Brien force ratio (1952) $W = |f_d|/|f_m|$; but not with either the Reynolds or Keulegan-Carpenter parameters. Hudspeth and Nath (1990) demonstrated that the Dean eccentricity parameter E is proportional to the Keulegan-Carpenter parameter K for kinematic data that are simple harmonic. Consequently, the Dean eccentricity parameter E connects the alignment of the semi-minor axis of the error ellipse to the parametric dependency of C_m and C_d on K. When $E = 1.0$, then $K = 11.40$ and the error ellipse is a circle with zero eccentricity.

The Dean error ellipse (1976) may be extended to include errors in the amplitudes/phases of the kinematics. This extension to include errors in amplitudes/phases demonstrates geometrically the parametric dependency of C_m and C_d on the parameter K (or E) by the magnitude of the slope of the contours of the dimensionless O'Brien force ratio

$W = |f_d|/|f_m|$ passing through the origin for zero error in phase. The advantage of the amplitude/phase error method is that the separate plots required by the Dean error ellipse method for each constant value of $W = |f_d|/|f_m|$ shown in Fig. 1 may be replaced by a single graph with contours of constant values of W. Comparisons of the amplitude/phase analysis with synthetically phase-shifted laboratory data for $E < 1.0$ or $K < 11.40$ and for $E > 1.0$ or $K > 11.40$ are excellent for phase shifts in the range of $|\omega\tau| < \pi/8$.

Data condition is defined as the ability of a least-squares algorithm to locate a global minimum on an error surface for given wave kinematic/force data (cf. Marquardt, 1963). This geometric definition is related to the numerical matrix condition number (cf. Atkinson, 1989). The matrix condition number is computed by four standard measures for the Morison equation. The matrix condition number is identically equal to unity when $K = 13.16$ and $E = 1.15$.

Geometric Interpretations

1) Dean error ellipses: The mean squared error ϵ^2 between the "true" force per unit length (denoted by upper case unprimed letters) $F(\theta)$ and the "computed" force per unit length (denoted by lower case primed letters) $f'(\theta)$ may be estimated from

$$\epsilon^2 = \langle [F(\theta) - f'(\theta)]^2 \rangle \tag{1}$$

where $\theta = 2\pi t/T$; T = wave period; and the temporal averaging operator $\langle(\cdot)\rangle$ is

$$\langle(\cdot)\rangle = \frac{1}{2\pi}\int_o^{2\pi}(\cdot)d\theta$$

If the "computed" force per unit length is given by the Morison equation

$$f'(\theta) = f'_m(\theta) + f'_d(\theta)$$

$$= k'_m\dot{u}(\theta) + k'_d u(\theta)|u(\theta)| \tag{2a}$$

then the "computed" inertia and drag constants are, respectively

$$k'_m = C'_m[\rho\pi D^2/4] \; ; \quad k'_d = C'_d[\rho D/2] \tag{2b}$$

When simple harmonic kinematics are used in Eq.(2a), Eq.(1) becomes a conic section equation for an ellipse whose origin has been translated and axes rotated (Hudspeth and Nath, 1990; and Thomas, 1965)

$$(\alpha X)^2 + 2HXY + (\beta Y)^2 + 2GX + 2JY + C = 0 \quad (3)$$

where $X = C_d'$ and $Y = C_m'$.

Equation (3) may be written in a form that is more familiar for an ellipse according to (Dean and Dalrymple, 1984)

$$\frac{(X - X_o)^2}{(R/\alpha)^2} + \frac{(Y - Y_o)^2}{(R/\beta)^2} = 1 \quad (4)$$

The eccentricity of the error ellipse may be expressed in terms of the Dean eccentricity parameter E or the Keulegan-Carpenter parameter K or the O'Brien force ratio W by (Hudspeth and Nath, 1990)

$$E = \frac{\alpha}{\beta} = \frac{\sqrt{3}}{2\pi^2}K = \frac{C_m}{C_d}\frac{\sqrt{3}}{2}W \quad (5)$$

When $K = 2\pi^2/\sqrt{3} \approx 11.40$ then $E = 1.0$ and the error ellipse is a circle and the data are equally conditioned for identifying both C_m & C_d. The eccentricity e^2 of the error ellipse determines geometrically the condition of the data for identifying C_m & C_d. The eccentricity e^2 may be expressed by

$$e^2 = 1.0 - E^{(\pm)2} \quad (6a,b)$$

where +2 is to be used when $E < 1.0$ and -2 is to be used when $E > 1.0$. Equations (6) imply that a separate error ellipse plot will be required for each constant value of the O'Brien force ratio W (vide Fig. 1 and Dean, 1976).

Specifically, when $E < 1.0$, then $K < 11.40$ and the semi-minor axis of the error ellipse is parallel to the C_m axis. Conversely, when $E > 1.0$, then $K > 11.40$ and the semi-minor axis is parallel to the C_d axis. Note that a value of $K \approx 11.40$ is approximately the value of K at which the peak in C_d and the trough in C_m occur in the replotted Keulegan-Carpenter data shown in Fig. 2 (cf. Sarpkaya and Isaacson, 1981 and Chakrabarti, 1987).

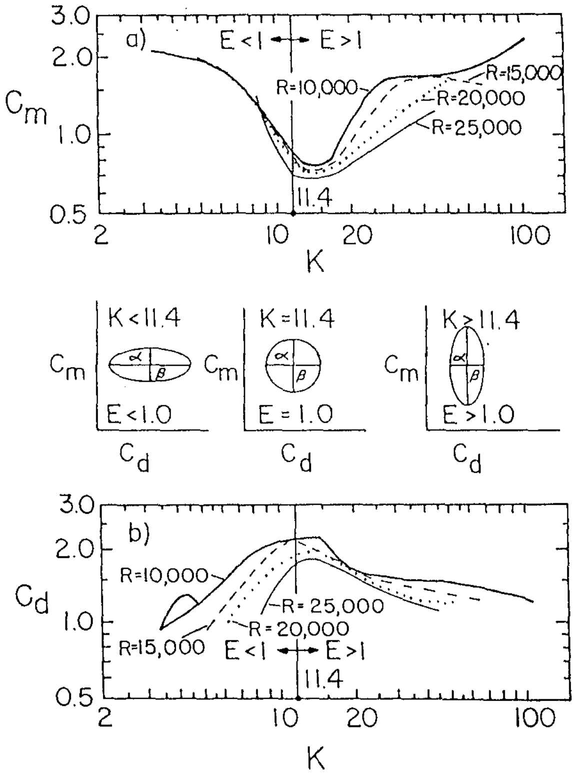

Fig. 2 Correlation of Replotted Keulegan-Carpenter Data with Dean Error Ellipses

2) Amplitude/phase analysis: Hudspeth, et.al. (1988) extended the Dean error ellipse method to include the effects of errors in amplitudes and phases of the wave force/kinematic data. This extension eliminated the need for separate error ellipses for each constant value of the O'Brien force ratio W.

Minimizing Eq.(1) with respect to the computed coefficients C_m' and C_d' yields 2 equations which may be rearranged to give the following dimensionless inertia coefficient ϵ_m and drag coefficient ϵ_d ratios for small values of the phase shifts $|\omega\tau| \approx 0$:

$$\epsilon_m \approx V[1-(8W/3\pi)\omega\tau] \tag{7a}$$

$$\epsilon_d \approx V^2[1+(32/9\pi)(\omega\tau/W)] \tag{7b}$$

with slopes near the origin given by (approximately)

$$S_m = \frac{\partial\epsilon_m}{\partial(\omega\tau)} \approx -\left(\frac{8WV}{3\pi}\right) \tag{8a}$$

$$S_d = \frac{\partial\epsilon_d}{\partial(\omega\tau)} \approx \left(\frac{32}{9\pi}\right)\left(\frac{V^2}{W}\right) \tag{8b}$$

where V = dimensionless velocity amplitude ratio. Note that it is not necessary to construct a separate error ellipse for each constant value of the O'Brien force ratio W.

Figure 3 compares Eqs. (7&8) with synthetically phase-shifted experimental data for W = 0.49 (or E < 1.0) and for W = 2.32 (or E > 1.0).

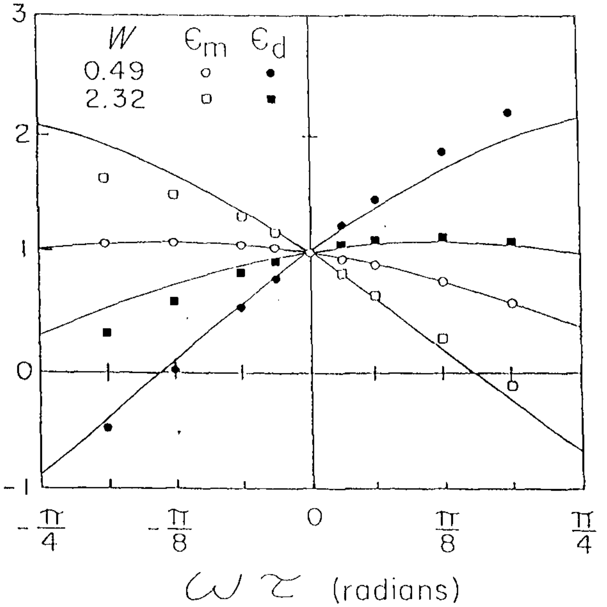

Fig. 3 Comparisons of ϵ_m & ϵ_d with Synthetically Phase-Shifted Laboratory Data (Hudspeth and Nath, 1990)

Numerical Interpretations

Scaling the variables in Eq. (1) by the following:

$$\mathrm{F} = \frac{F}{\rho D a^2} \qquad \mathrm{u} = \frac{u}{a} \qquad \dot{\mathrm{u}} = \frac{\dot{u}}{a\omega}$$

where a = amplitude of the "computed" velocity; $\omega = 2\pi/T$; and minimizing Eq. (1) with respect to C_m' and C_d' gives the following scaled (or nondimensional) matrix equation :

$$\mathbf{AX} = \mathbf{B} \tag{9}$$

where the scaled matrices are

$$\mathbf{A} = \begin{bmatrix} \frac{4\pi^2}{3K} & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{E\sqrt{3}} & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathrm{X} = \begin{bmatrix} C_m' \\ C_d' \end{bmatrix} \qquad \mathbf{B} = \frac{16}{3}\begin{bmatrix} <\mathrm{F}\dot{\mathrm{u}}> \\ <\mathrm{F}\mathrm{u}|\mathrm{u}|> \end{bmatrix}$$

Matrix A is Hermitian and unitary. It becomes a unit matrix with matrix condition numbers identically equal to unity when $K = 4\pi^2/3 \approx 13.16$ and $E = 2/\sqrt{3} \approx 1.15$.

Table 1. Summary of Matrix Condition Numbers.

Matrix Condition Number (1)	$K < 13.16$ $E < 1.15$ (2)	$K = 13.16$ $E = 1.15$ (3)	$K > 13.16$ $E > 1.15$ (4)	$K = 11.40$ $E = 1.0$ (5)
$\mathrm{Cond}(A)_1$ $\mathrm{Cond}(A)_\infty$ $\mathrm{Cond}(A)_2$ $\mathrm{Cond}(A)_*$	$\frac{4\pi^2}{3K} = \frac{2}{E\sqrt{3}}$	1.0	$\frac{3K}{4\pi^2} = \frac{E\sqrt{3}}{2}$	1.15

The four standard matrix condition numbers listed in column 1 of Table 1 are defined as follows (Atkinson, 1989):

$$\mathrm{Cond}(A)_1 = \mathrm{Cond}(A)_\infty = ||A||\cdot||A^{-1}||$$

$$\mathrm{Cond}(A)_2 = \left[\frac{\underset{\lambda\in\sigma[A*A]}{\mathrm{Max}|\lambda|}}{\underset{\lambda\in\sigma[A*A]}{\mathrm{Min}|\lambda|}}\right]^{1/2} \quad ; \quad \mathrm{Cond}(A)_* = \left[\frac{\underset{\lambda\in\sigma[A]}{\mathrm{Max}|\lambda|}}{\underset{\lambda\in\sigma[A]}{\mathrm{Min}|\lambda|}}\right]$$

where $||\cdot||$ = a matrix norm; A^{-1} = matrix inverse; λ = eigenvalue of the matrix A; $\sigma[\cdot]$ = spectral radius of the matrix $[\cdot]$; and A^* = complex conjugate transpose.

<u>Stable Transverse Lift Forces</u>

Keulegan and Carpenter (p. 439, 1958) gave a physical interpretation of their period parameter in terms of the distance traveled in one direction by a fluid particle in the absence of the cylinder. Accordingly, this definition lead to the ratio between the wave period T and the eddy shedding period T_S being equal to the product between the Strouhal parameter S and the Keulegan-Carpenter parameter K; or

$$\frac{T}{T_s} = S\cdot K \tag{10}$$

When this ratio is exactly equal to two, then exactly two vortices will be shed during one wave period and the transverse lift force will be stable (Keulegan and Carpenter, 1958). For normal wave conditions, $0.18 < S < 0.20$ and $K \approx 11$ when the ratio $T/T_S \approx 2$.

Data are available that correlate with the critical values of the Keulegan-Carpenter parameter K computed from both the geometric and the numerical interpretations. The only stable transverse lift force seen in the Hayashi and Takenouchi data (1979) shown in Fig. 4 occurs when $K = 11.8$ which is approximately the critical value of K determined from geometric considerations.

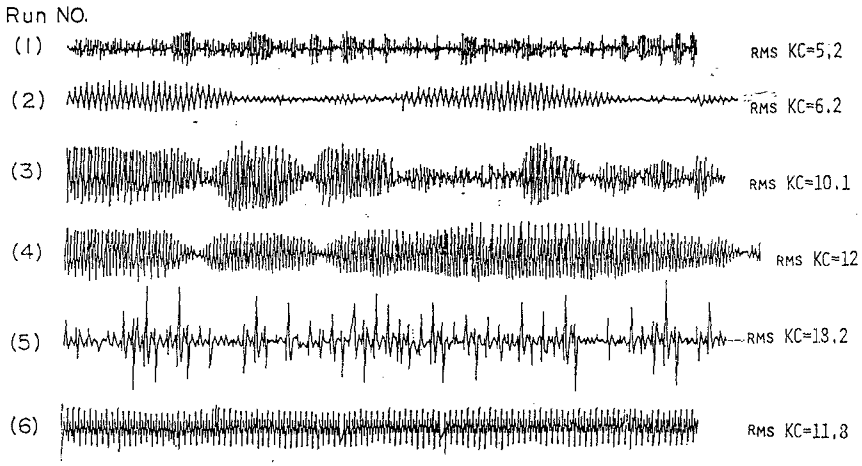

Fig. 4 Transverse Lift Forces on Vertical Circular Cylinders (Hayashi and Takenouchi, 1979)

The only stable transverse lift force seen in the Maull and Milliner data (1979) shown in Fig. 5 occurs in Run 145 when K = 13.02 which is approximately the critical value of K determined from numerical considerations.

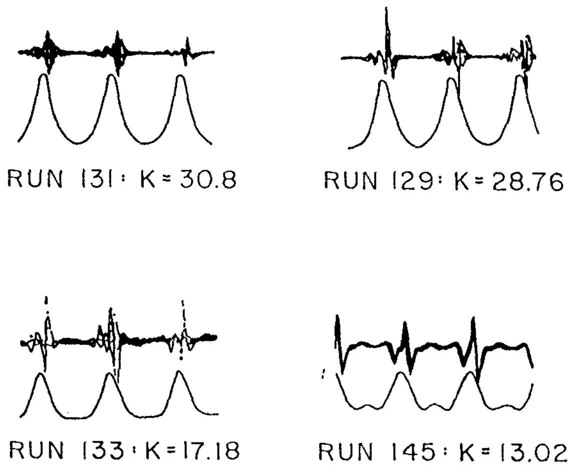

Fig. 5 Forced In-Line Motions and Transverse Lift Forces on a Vertical Circular Cylinder Oscillated in Still Water (Maull and Milliner, 1979)

Conclusions

The critical values of the period parameter $K = U_mT/D$ identified by Keulegan-Carpenter (1958) are shown to correlate with: 1) wave force/kinematic data that are equally conditioned for identifying C_m & C_d; and 2) stable transverse lift forces. The effects of the condition of the data on the inertia and drag coefficients C_m & C_d are evaluated from two geometric and one numerical interpretations. The two geometric interpretations of the condition of the data proposed by Dean demonstrate that when the Dean eccentricity parameter E equals unity, the data are equally conditioned for determining C_m & C_d. For simple harmonic data, the Dean eccentricity parameter may be shown to be proportional to the Keulegan-Carpenter parameter; i.e., $E = \sqrt{3}K/2\pi^2$. When $E = 1.0$, then $K = 11.40$ and the Dean error ellipse is a circle with zero eccentricity. The numerical interpretation of the condition of the data demonstrates that when the matrix condition number of the 2 X 2 matrix used to compute C_m & C_d in a best least-squares sense becomes identically equal to unity then $K \approx 13.16$ and $E \approx 1.15$. Stable transverse lift forces occur when the ratio of the wave period T to the period of eddy shedding T_s is exactly equal to two. For a Strouhal parameter $0.18 < S < 0.20$ and $T/T_s = 2$, $K \approx 11$. Three sets of experimental data are compared with the two geometric and one numerical analyses and with the stable transverse force hypothesis. These experimental data represent three very different physical conditions; viz., 1) a horizontal circular cylinder located at the node of standing surface gravity waves, 2) a vertical circular cylinder in propagating surface gravity waves, and 3) a vertical circular cylinder forced to oscillate in otherwise still water.

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