CHAPTER 111

Overtopping of Solitary Waves at Model Sea Dikes

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Abstract

This paper discusses an analytical model for solitary wave overtopping at sea dikes. The model is based on the "weir flow analogy" of Kikkawa et al.(1968). Results of laboratory test on smooth and impermeable model dikes with angles of 45°, 60° and 90° are utilized in constructing the model. Results for solitary waves are compared with those for regular oscillatory waves for the case of vertical dike.

Introduction

Coastal structures are being constructed often low-crested due to economical and/or aesthetic considerations. An important design criterion for such structures is the allowable discharge of wave overtopping. The problem of wave overtopping by oscillatory waves has been studied by various researchers since 1960's. The initial investigations were basically based on laboratory experiments. An analytical model was developed by Kikkawa at al(1968), by following an analogy with the flow over sharp crested weirs. In recent years, following advances on mathematical treatment on wave propagation on steep slopes, some researchers concentrated on numerical modelling of wave deformation on the dike slope and the subsequent overtopping (i.e. Kobayashi and Wurjanto,1989).

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To the knowledge of authors, the problem of solitary wave overtopping at coastal structures has not been studied before. Investigation of this problem is useful for at least two reasons. Firstly findings may contribute to the understanding of the basic mechanism of wave overtopping. Secondly, the results obtained may be used for practical purposes in situations where solitary-like waves interfere with coastal structures.

In this paper, the analogy proposed by Kikkawa et al(1968) is extended and applied to solitary wave overtopping to derive a closed form analytical model. Laboratory experiments are carried out to provide the empirical information required by the model. Solitary overtopping rates at vertical dikes are compared with those of regular oscillatory waves.

Theory

By considering analogy with steady flow over a sharp crested weir, wave overtopping rate at a sea dike may be equated to :

$$q(t) = \frac{2}{3} m (2g)^{\frac{1}{2}} \ \{z(t) - z_0\}^{\frac{3}{2}} \tag{1}$$

where $q(t)$ = unsteady overtopping rate per unit dike width; $z(t)$ = changing water level elevation measured from still water level; z_0 = crown elevation of the dike; g = gravitational acceleration; m = the weir coefficient which is equal to 0.611 in steady flow. The change of water level elevation during overtopping is written as:

$$z(t) = z_{max} \ F(t) \tag{2}$$

where z_{max} = maximum rise of the water level; and $F(t)$ = a function having the range of 0 and 1. It is assumed that the maximum water level rise is related to the incident solitary wave height (Fig. 1) as:

$$z_{max} = K \ H \tag{3}$$

where the maximum rise coefficient K may be a function of wave height-to-water depth ratio $(\frac{H}{d})$, dike angle (α), and wave height-to-crown elevation ratio $(\frac{H}{z_0})$. The water level elevation prior to overtopping is assumed to change in the same way as a deformed solitary wave profile (symmetrical) and the function $F(t)$ is taken as:

$$F(t) = sech^2(\varepsilon \ \lambda \ C \ t) \tag{4}$$

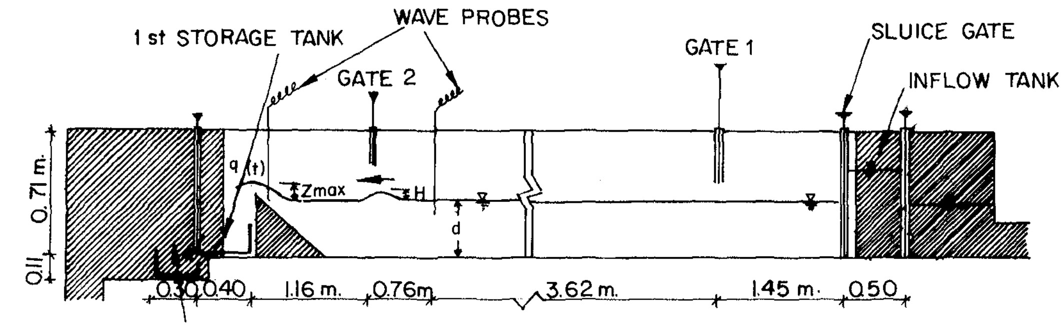

Figure 1: Experimental set-up and parameters

where; $\lambda = (\frac{3H}{4d^3})^{\frac{1}{2}}$, pulse width (phase) parameter; $C = (gd)^{\frac{1}{2}}(1 + \frac{H}{2d})$, celerity of the incident solitary wave; ε = an empirical coefficient (called the profile coefficient) which may account for deformation of the wave profile prior to overtopping and nonlinearity in the superposition of incident and reflected waves.

Substitution of Eqns. (2), (3) and (4) into (1) leads to:

$$q(t) = \frac{2}{3}m(2g)^{\frac{1}{2}}K^{\frac{3}{2}}H^{\frac{3}{2}}[Sech^2(\varepsilon\lambda Ct) - (\frac{z_0}{KH})^{\frac{3}{2}}] \quad for \quad |t| \leq \frac{1}{\varepsilon\lambda C}Sech^{-1}(\frac{z_0}{KH})^{\frac{1}{2}}$$

$$q(t) = 0 \qquad\qquad otherwise \qquad (5)$$

Integration of Eq.(5) with respect to time gives the overtopping volume of a solitary wave as:

$$Q = \frac{4}{3}m(2g)^{\frac{1}{2}}K^{\frac{3}{2}}H^{\frac{3}{2}}\frac{I}{\varepsilon\lambda C} \qquad (6)$$

where Q = overtopping volume of a solitary wave per unit dike width;

$$I = \int_0^{\tau^*}(Sech^2\tau - \frac{z_0}{KH})^{\frac{3}{2}}d\tau$$

and $\tau^* = Sech^{-1}(\frac{z_0}{KH})^{\frac{1}{2}}$. It has been shown that the integral I is approximately equal to (Özhan, 1975):

$$I = \frac{1}{2}sech^{-1}(\frac{z}{KH})^{\frac{1}{2}}(1 - \frac{z_0}{KH})^{\frac{3}{2}} \qquad (7)$$

Then, the final result giving the overtopping volume of a solitary wave per unit dike width is obtained from (6) and (7) as:

$$\frac{Q(1+\frac{H}{2d})}{Hd} = \frac{0.6652}{\varepsilon}(K)^{\frac{3}{2}}(1 - \frac{z_0}{KH})^{\frac{3}{2}}sech^{-1}(\frac{z_0}{KH})^{\frac{1}{2}} \qquad (8)$$

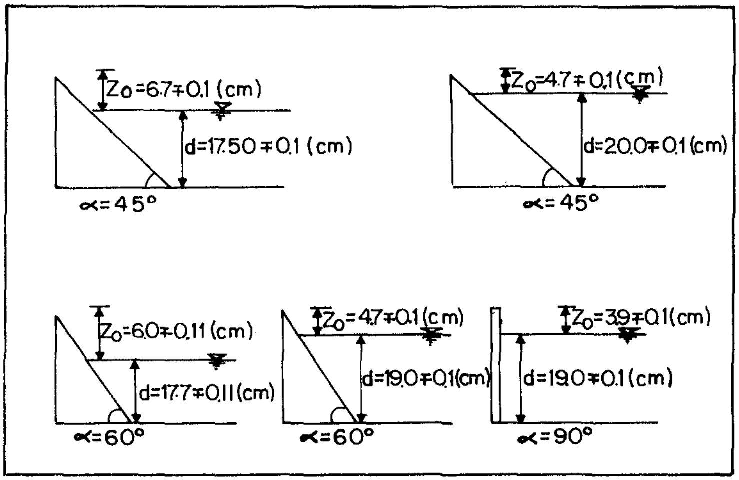

Figure 2: The geometries of model dikes and the water depths used in each experimental group

where $m = 0.611$ is used. This equation includes two empirical coefficients ε and K. Laboratory experiments were designed to investigate the values of these coefficients together with their dependence on various parameters.

Laboratory Experiments

The experimental set up is shown in Fig.(1). The solitary waves were generated by a sluice gate operation through introducing suddenly a volume of water into the wave flume. As this deformation evolved into a solitary wave, the trailing disturbances were cut off by closing the vertical gate 1, right after the passage of the solitary wave. The solitary wave moved over a constant water depth up to the dike. Incident solitary wave profiles and water level changes near the dike crown (3 cm in front) just prior to overtopping were measured. The overtopped volume of water was collected successively by two storage tanks. In order to eliminate the possibility of overtopping for a second time by the reflected wave, the gate 2 was closed after the reflected wave moved away from the dike.

The geometries of model dikes and the water depths used in each experimental group are shown in Fig. (2).

Results

In the evaluation of the experimental results, the first consideration was given to the incident solitary wave profile. The measured wave profile was compared with the theoretical Boussinesq profile. The runs which did not show close agreement were disregarded. The experimental data were first used to derive the values of two empirical coefficients, ε and K, which appeared in the theoretical model developed in section 2, and then to check the validity of the theoretical model to predict the overtopped wave volume. Finally, the solitary wave overtopping rates were compared with the data available for regular oscillatory waves.

The shape coefficient, ε

The shape coefficient, can be written as:

$$\varepsilon = \frac{Sech^{-1}\left(\frac{z(t)}{z_{max}}\right)^{\frac{1}{2}}}{\left(\frac{3H}{4d^3}\right)^{\frac{1}{2}} Ct}$$

The measured values of z(t) at the location 3 cm in front of the dike crown were used to determine the value of the shape coefficient which provides the best fit to the measured profile. For this purpose, only the water level elevations in excess of the crown elevation (i.e. $z(t) > z_0$) were utilized. The procedure followed minimized the square error in the value of $sech^{-1}\left(\frac{z(t)}{z_{max}}\right)^{\frac{1}{2}}$.

The values of ε for all three dike slopes are plotted in Fig. (3) against $\frac{H}{z_0}$ ratio. Also shown are the least square lines for each slope. The values of profile coefficient is seen to decrease with increasing $\frac{H}{z_0}$ value for all dike slopes. The least square lines for 60^o and 90^o dike angles are comparable. However, the slope for the dike with the angle of 45^o is somewhat steeper.

The correlation coefficients between ε and $\frac{H}{z_0}$ ratio are computed as -0.84, -0.49 and -0.54 respectively for the dike angles of 45^o, 60^o and 90^o. These rather low correlation coefficients indicate the importance of other parameters as well (such as $\frac{H}{d}$ ratio). In the present study however, the least square relationships shown in Fig.(3) were used for the computation of volume of overtopping as it will be discussed later. The intercept and slope (a_1 and b_1) of the linear relationship $\varepsilon = a_1 + b_1(\frac{H}{z_0})$ for three dike slopes tested are given in Table 1.

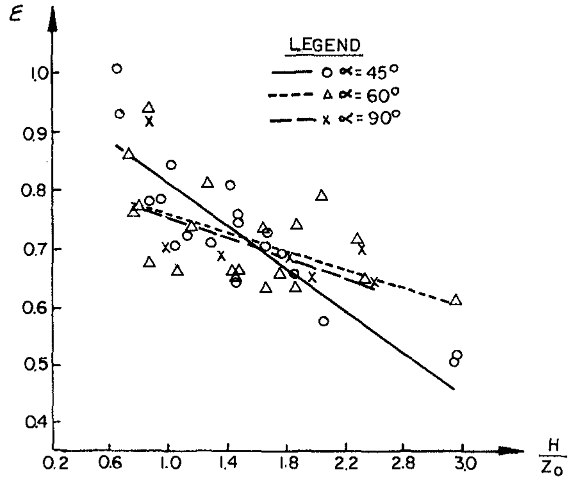

Figure 3: The values of shape coefficient for all three dike slopes

Table 1. The intercept and slope of the linear relationship between ε and $\frac{H}{z_0}$

Dike Angle	Intercept, a_1	Slope, b_1
45^o	.994	-0.179
60^o	.834	-0.076
90^o	.839	-0.080

The maximum rise coefficient, K

The maximum rise coefficient defined in Eq.(3) was determined for each experimental run from the water level elevation measured near the dike crown. However, due to the horizontal distance of 3 cm. between the probe location and the dike crown, this value K_m obtained from the maximum recorded water level would be somewhat lower then the actual value corresponding to the maximum level to be reached at the dike crown. Alternatively, the actual value of K could be determined from Eq.8 by using the measured values of overtopped volume and the respective ε

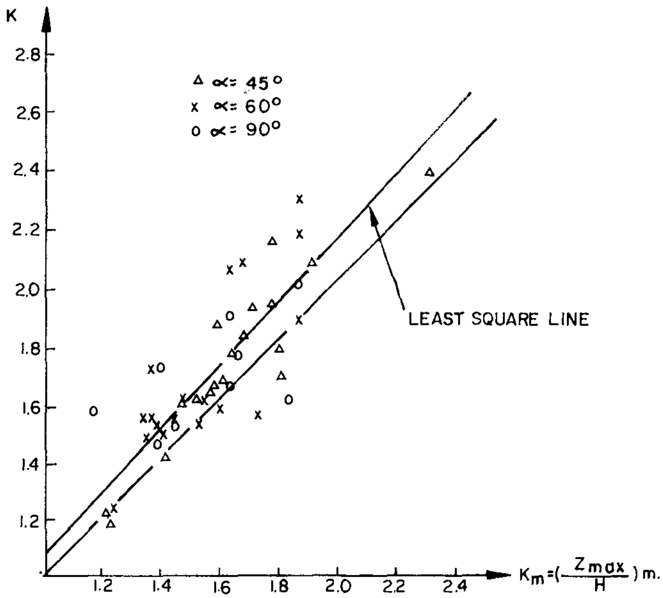

Figure 4: Comparison of theoretical (K) and measured (K_m) values of maximum rise coefficient

value corresponding to the $\frac{H}{z_0}$ ratio of the test. The implicit equation in terms of K which forms after substitution of Q and ε for a test, was solved numerically. The maximum rise coefficient, thus obtained are compared with the respective K_m values corresponding to the maximum recorded level in Fig. (4). It is observed that two values are correlated reasonably well. In line with expectations, the maximum rise coefficients computed from the theoretical model by using the measured overtopped volume are larger by 9 % on the average than the respective K_m values. As it was mentioned earlier, this is due to 3 cm. distance between the measurement location and the dike crown.

The maximum rise coefficient K correlates better with $\frac{H}{z_0}$ ratio then the shape coefficient ε (Fig. 5). The correlation coefficients were found as (-0.79) to (0.85) for three dike angles tested. For use in conjunction with the theoretical model of overtopping, the least square linear relationships between K and $\frac{H}{z_0}$ ratio were found. These are also shown in Fig. 5. The intercepts and slopes of these equations for 3 dike slopes are given in Table 2.

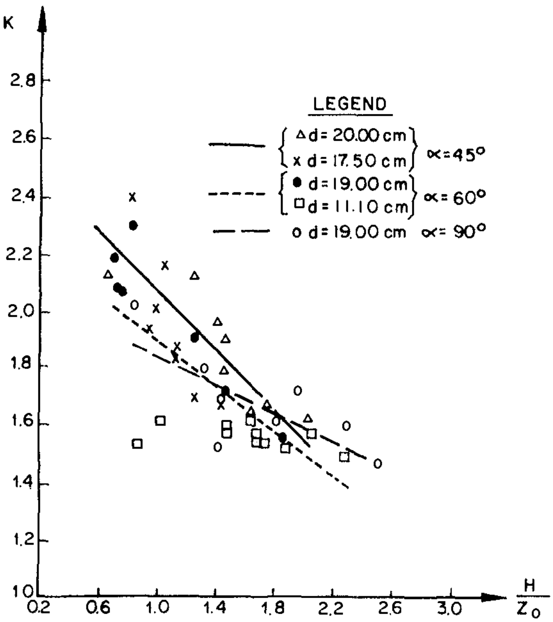

Figure 5: The values of maximum rise coefficient (K) for all three dike slopes

Table 2. The intercept and slope of the linear relationship between K and ($\frac{H}{z_0}$)

Dike Angle	Intercept, a_2	Slope, b_2
45^o	2.63	-0.534
60^o	2.27	-0.383
90^o	2.07	-0.243

The values of maximum rise coefficient K should approach the respective run up coefficients for small values of $\frac{H}{z_0}$.

Volume of Overtopping

The least square lines for ε and K were used in the analytical expression (Eq. 8.) to compute the dimensionless volume of overtopping as a function of $\frac{H}{z_0}$ ratio. The resulting curves for three dike slopes tested are compared with the experimental data in Fig.(6)- (8). The theoretical model developed together with the empirical

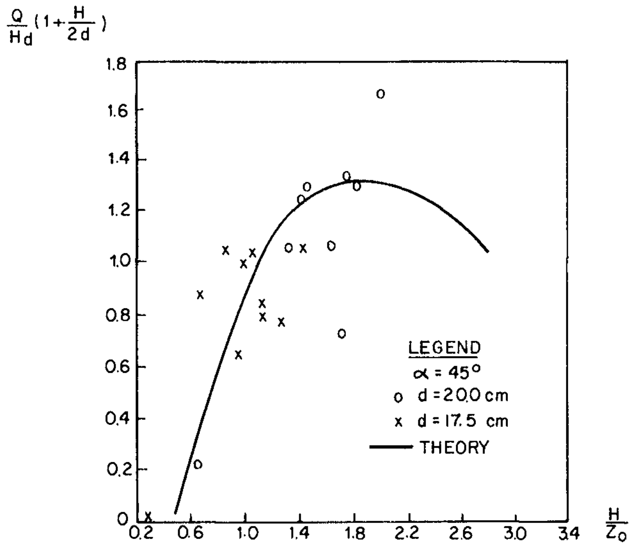

Figure 6: Comparison of measured and calculated overtopping volume $\alpha = 45^o$

relationships for ε and K are observed to provide reasonable agreement with experimental data for all three dike slopes. For the dikes with 45^o and 60^o slope angles, the dimensionless overtopping volume is seen to decrease with increasing $\frac{H}{z_0}$ ratio after a critical value of $\frac{H}{z_0}$ 2.0 to 2.1 is exceeded. Within the experimental range covered ($\frac{H}{z_0}$ ratio less than 2.5), such a change of trend is not observed for the vertical dike.

Comparison of the predictions of the presented theoretical model for different dike slopes reveals that the overtopping volume is greater for $\alpha = 45^o$ than the other two slopes for $\frac{H}{z_0} \leq 1.9$. The results for 60^o and 90^o dikes are very similar up to $\frac{H}{z_0} \leq 1.4$. For larger values, the overtopping volume for the vertical dike becomes greater than the respective value for the 60^o dike.

Comparison with regular wave overtopping

For comparison of solitary wave overtopping with that of regular waves, it is necessary to define a practical wave period for the solitary wave. This may be done as the time length over which a certain percentage of solitary wave volume passes a fixed point. The resulting expression reads as:

$$T_p = \frac{\sqrt{\frac{g}{d}}\tanh^{-1}(\rho)}{\frac{\sqrt{3}}{4}\left(\frac{H}{d}\right)^{\frac{1}{2}}\left(1 + \frac{H}{2d}\right)} \qquad (9)$$

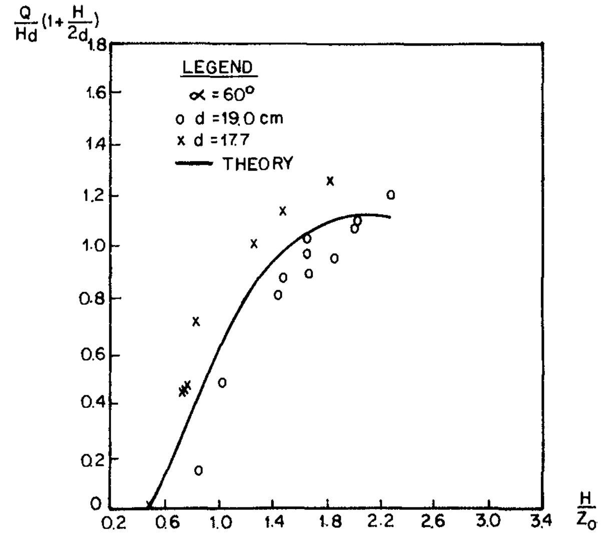

Figure 7: Comparison of measured and calculated overtopping volume $\alpha = 60^o$

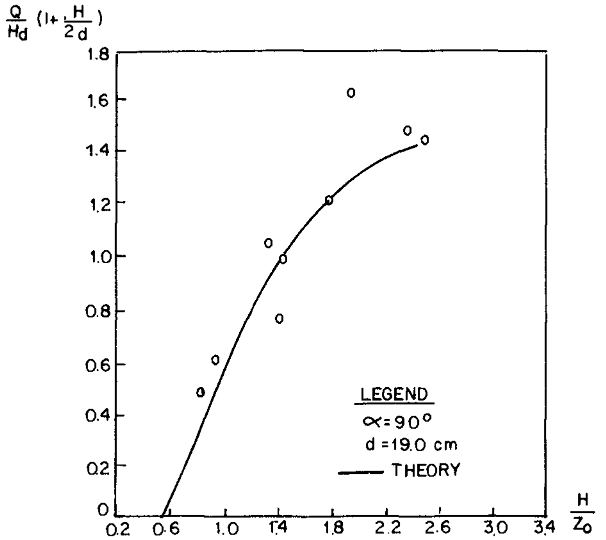

Figure 8: Comparison of measured and calculated overtopping volume $\alpha = 90^o$

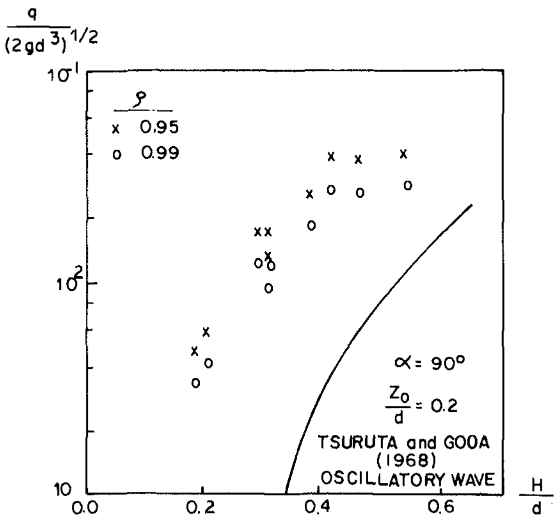

Figure 9: Comparison of solitary and oscillatory wave overtopping rates for a vertical dike

where T_p= the practical wave period; g = gravitational acceleration; and ρ= the percentage of wave volume. Furthermore, the series of solitary waves were treated as a cnoidal wave terrain, and the water depth of the experimental runs was increased by adding the trough amplitude of the respective cnoidal wave. The reanalized experimental data in this manner for the vertical dike is compared in Fig.(9) with the curve for regular oscillatory waves given by Tsuruta and Goda (1968). In this comparison, q is the average overtopping discharge over a wave period. The experimental data for solitary waves are plotted twice by using practical wave periods determined from two volume percentages, namely 95 % and 99 %. The presentation in Fig.(9) reveals that the solitary wave overtopping rates are significantly in excess of the respective oscillatory wave discharges.

Conclusions

The results of the present paper can be summarized as follows

1. An analytical expression is derived for the solitary wave overtopping, based on the analogy with sharp-crested weir flow.

2. Empirical data obtained from model dikes with $\alpha = 45^o, 60^o, 90^o$ slope angles is fed into the derived expression.

3. The analogy is shown to be successful for steep dike slopes ($\alpha \geq 45^o$).

4. Solitary wave overtopping rates are significantly greater than those for oscillatory waves.

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