CHAPTER 99

DYNAMIC ANALYSIS OF FLOATING BREAKWATER MOORING SYSTEMS

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Abstract

This paper presents a numerical model for computation of floating breakwater mooring forces. The model is based on time domain mooring analysis techniques which permit simulation of: (1) nonlinear mooring line load characteristics and (2) mooring line loads associated with second order wave drift forces. Numerical model results are compared to physical model tests (Torum, 1989) and prototype measurements (Nelson and Broderick, 1986). These comparisons demonstrate that the numerical model provides good estimates of floating breakwater mooring line forces. Accordingly, it is concluded that the numerical model serves as a useful engineering tool for analysis and design of floating breakwater mooring systems.

Introduction

Floating breakwaters have been the subject of research for many years. Much of this research, however, has focused on breakwater wave transmission characteristics. Although wave transmission is an important functional design consideration, floating breakwaters and their moorings must also withstand survival environmental loadings. This paper presents a systematic and practical procedure for analysis of floating breakwater mooring systems.

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Previous Investigations

Floating breakwater performance can be evaluated using numerical and physical models. Physical models provide the most reliable estimates of floating breakwater performance, but are generally expensive. Numerical models provide a relatively inexpensive means of assessing floating breakwater performance and are particularly useful in feasibility level analyses which can be used to refine designs prior to physical model testing. Numerical modeling of moored floating breakwaters has most often been performed using frequency domain techniques. Examples of frequency domain numerical models may be found in Adee et al (1976) and Tekmarine Inc. (1986) which can be used to estimate floating breakwater wave transmission characteristics and mooring line forces. Georgiadis and Hartz (1982) developed a numerical model to evaluate the internal loads of a floating breakwater module using both frequency and time domain techniques.

Laboratory and field studies of various floating breakwater installations (Adee et al, 1976) have shown that frequency domain analysis provides a good engineering estimate of floating breakwater wave transmission characteristics. However, as will be discussed below, frequency domain analyses often do not adequately predict breakwater motions and attendant mooring line forces.

Miller et al (1984) found that the frequency domain approach adequately predicted floating breakwater motions in heave and roll, but could not predict the low frequency sway motions which generally dominated mooring forces. Similarly, Adee et al (1976) concluded that mooring forces developed from frequency domain analysis must be increased substantially in order to provide an estimate of actual mooring forces. Figure 1 presents an example force time history reported by Adee et al (1976) for floating breakwater installation at Tenakee, Alaska. Incident waves had periods ranging from 1 to 2 seconds, however, mooring line forces were dominated by long period oscillations with periods of approximately 55-60 seconds. A review of the dynamic characteristics of the floating breakwater mooring system indicated that the recorded sway motion of the floating breakwater was very close to the sway natural period. Similar prototype measurements were reported by the Japanese Ministry of Transport at site near Kumamoto, Japan (see Tekmarine (1986)). Clearly, any numerical analysis of floating breakwater mooring line forces must account for both high and low frequency motions.

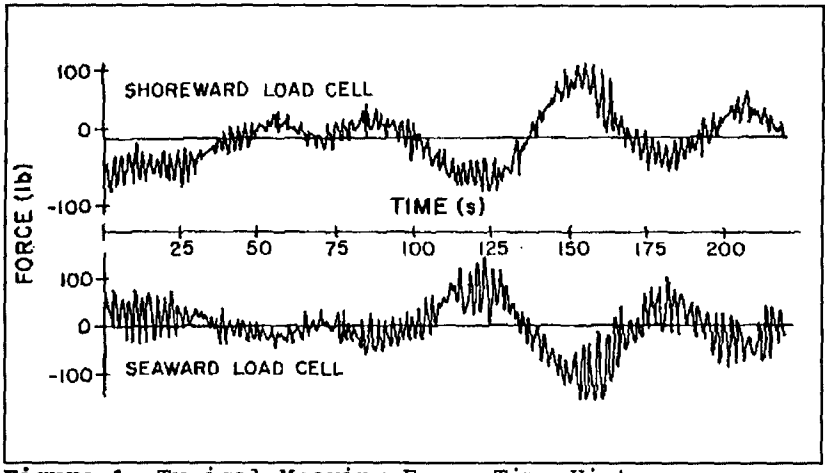

Figure 1. Typical Mooring Force Time History

Low frequency motions of moored structures result from second order wave drift forces (Pinkster, 1980). Such forces are neglected in a first order frequency domain analysis such as that reported in Adee et al (1976) and Tekmarine Inc. (1986) which assume applied hydrodynamic loading and attendant breakwater response are sinusoidal at a frequency equal to the incident wave frequency. As a result, such frequency domain analyses do not account for the "second order" wave drift forces which can dominate mooring line forces. The following paragraphs outline a time domain numerical model capable of estimating first and second order floating breakwater motions and mooring line forces.

Numerical Model Theory

Equations of Motion. Research on the behavior of moored vessels has shown that mooring systems are best modeled in the time domain using the impulse-response method (Van Oortmerssen, 1976). The time domain approach can be used to simulate nonlinear mooring line behavior as well as arbitrary applied loadings. Because an arbitrary applied loading can be specified, the time domain approach can be used to assess first and second order wave forces. The governing equations of motion, which account for time domain motion in six degrees of freedom, are as follows:

$$\sum_{j=1}^{6} \left[(M_{kj}+m_{kj})\ddot{x}_j + \int_{-\infty}^{t} K_{kj}(t-\tau)\dot{x}_j(\tau)\,d\tau + C_{kj}x_j \right] - F_k(t) \qquad (1)$$

$$k = 1,2,\ldots.6$$

Where:

x_j = motion in the j-th mode
M_{kj} = inertia matrix
C_{kj} = hydrostatic restoring force matrix
K_{kj} = impulse response function matrix
m_{kj} = constant added mass matrix
$F_k(t)$ = arbitrary external force in k-th mode due to waves, mooring line reactions, viscous damping, winds, currents,etc.
k = denotes mode of motion, (i.e. 1(surge), 2 (sway), 3 (heave), 4 (roll), 5(pitch), 6 (yaw))

The left hand side of equation (1) contains linear hydrodynamic reaction forces while the right hand side represents an arbitrary forcing function which may include linear terms (e.g. first order wave forces) and nonlinear terms (e.g. second order wave drift forces, mooring line forces, viscous damping,etc.). The primary advantage of the impulse-response function approach is that arbitrary motion over a range of motion frequencies can be simulated correctly. Other time domain methods, such as that used by Georgiadis and Hartz (1982), assume constant values of added mass and damping at a single frequency even though breakwater motion may occur over a range of frequencies.

Hydrodynamic Coefficients. The inertia matrix, M_{kj}, and hydrostatic restoring force matrix, C_{kj}, are computed using standard methods of naval architecture. The impulse-response function matrix, K_{kj}, and the constant added mass coefficient, m_{kj}, are computed as follows (Van Oortmerssen, 1962):

$$K_{kj} = \frac{2}{\pi}\int_0^{\infty} b_{kj}(\omega)\cos\omega t\,d\omega \qquad (2)$$

$$m_{kj} - a_{kj}(\omega^*) + \frac{1}{\omega^*}\int_0^\infty K_{kj}(t)\sin\omega^*\tau d\tau \qquad (3)$$

Where:

a_{kj} = frequency-dependent added mass
b_{kj} = frequency-dependent damping coefficient

The above frequency dependent added mass and damping coefficients may computed using either strip-theory or diffraction theory hydrodynamic analysis. The Frank Close-Fit strip theory approach, as documented by Kaplan (1989), was used in the present analysis.

Wave Forces. The applied force resulting from waves, $F_{kw}(t)$, can be written as:

$$F_{kw}(t) - \sum_0^n f_k^{(1)}(\omega_n)\, a_n \cos(\omega_n t + \epsilon_n + \epsilon_k)$$

$$+\sum_0^n \sum_0^m f_k^{(2)}(\omega_n)\, a_n a_m \cos((\omega_n t + \epsilon_n) - (\omega_m t + \epsilon_m)) \qquad (4)$$

Where:

$f_k^{(1)}(\omega_n)$ = first order wave transfer function in k-th mode for n-th wave component

$f_k^{(2)}(\omega_n)$ = second order wave transfer function in k-th mode for n-th wave component

ϵ_k = phase of first order wave transfer function in k-th mode for n-th wave component

a_n = wave amplitude of n-th wave component

ϵ_n = phase of n-th wave component

ω_n = frequency of n-th wave component

The above formula is used to simulate first and second order wave force time-histories resulting from an incident wave spectrum composed of n-waves. The incident wave spectrum is represented in the above formulation by

the wave amplitudes and phases (i.e. a_n and ϵ_n) which correspond to wave frequencies, ω_n . Wave amplitudes and frequencies are determined from an incident wave spectrum using the "equal area" method described by Borgman (1969).

It should be noted that the wave force formulation presented in equation (4) is only valid for long-crested, uni-directional wave conditions. The present version of the model cannot be used to assess mooring loads from directional spectra. Fortunately, the results of Torum et al (1989) indicate that directional spreading has little influence on mooring line forces.

The first order wave transfer functions were also computed using the Frank-Close Fit method documented by Kaplan (1989). The second order wave transfer functions were computed from the first order wave analysis using methods based on the work of Gerritsma and Beukelman (1972) and Newman (1967). As will be discussed later, mooring simulations using these computed drift forces over estimated the sway motions and attendant mooring line forces of scale model and prototype floating breakwaters. This overprediction was believed to be due to the fact that both the physical model and prototype breakwaters were heavily overtopped by waves. In an effort to account for overtopping, wave drift forces were computed using the following expression derived by Longuet-Higgins (1977):

$$f_2^{(2)} = \frac{1}{4}\rho g(a^2+a_R^2-a_T^2)\left(1+\frac{2kh}{\sinh(2kh)}\right) \qquad (5)$$

Where:

a = incident wave amplitude
a_R = reflected wave amplitude
a_T = transmitted wave amplitude
k = wave number
h = water depth

The reflected wave amplitude can be estimated from the incident and transmitted wave amplitudes as follows:

$$a^2 \approx a_R^2 + a_T^2 \qquad (6)$$

Given measured values of the incident, reflected and transmitted wave amplitudes, equation (5) can be used to estimate second wave drift forces. Wave overtopping conditions are taken into account implicitly through the measured incident, reflected and transmitted wave amplitudes.

Viscous Damping Coefficients. Numerous studies (e.g. Wichers, 1988) have shown that potential theory damping terms for surge, sway and yaw motions of moored structures are negligible in comparison to viscous damping terms at low frequency. Accordingly, an additional term is added to the time-varying applied force, $F_k(t)$, to account for viscous damping effects. Both nonlinear and linear viscous sway damping formulations were investigated. In accordance with the findings of Wichers (1988), initial efforts were directed towards evaluation of a quadratic damping term formulated as follows:

$$F_{2D}(t) = -\frac{1}{2}\rho C_{2D}|\dot{x}_2|\,\dot{x}_2 A_2 \qquad (7)$$

Where:

$F_{2D}(t)$ = nonlinear damping force in sway

ρ = water density

A_2 = lateral wetted area of breakwater

C_{2D} = viscous drag coefficient in sway

After considerable analyses, it was concluded that the above nonlinear damping formulation provided very little damping for typical floating breakwater sway motion velocities and was abandoned in favor of a linear damping formulation as follows:

$$F_{2D}(t) = -B_{2D}\dot{x}_2 \qquad (8)$$

Where:

B_{2D} = linear viscous damping coefficient

A similar formulation has been shown to successfully predict surge damping characteristics of vessels secured to single point moorings in still water (Wichers, 1988). Using the results of a prototype extinction tests, Seelig (1990) demonstrated that the above linear formulation could be used to estimate the sway motions of a spread-moored vessel in still water. Seelig's prototype measurements were used to estimate linear damping coefficient, B_{2D}.

Mooring Forces. Mooring line restoring forces in the numerical model are computed using static catenary theory. Mooring line loads are determined from the instantaneous position of mooring attachment point on the breakwater and from static load deflection curves for each line. The total mooring restoring forces and moments acting on the center of gravity of the breakwater in each mode of motion are computed by summing the force (and moment) contributions from each mooring line. These forces are then added to the $F_k(t)$ term on the right hand side of equation (1). Although Torum et al (1989) suggest that such effects may be important at high frequency, no attempt has been made to incorporate dynamic oscillations of the mooring lines in the numerical model.

MODEL APPLICATION

The U.S. Army Corps of Engineers initiated the Floating Breakwater Prototype Test Program in 1981. A concrete box or caisson-type floating breakwater was constructed, installed and monitored an exposed site in the Puget Sound as part of this program. Results of this study are presented in Nelson and Broderick (1986). The general arrangement of the floating breakwater installation is shown in Figure 2. The particulars of the floating breakwater unit are summarized in Table 1.

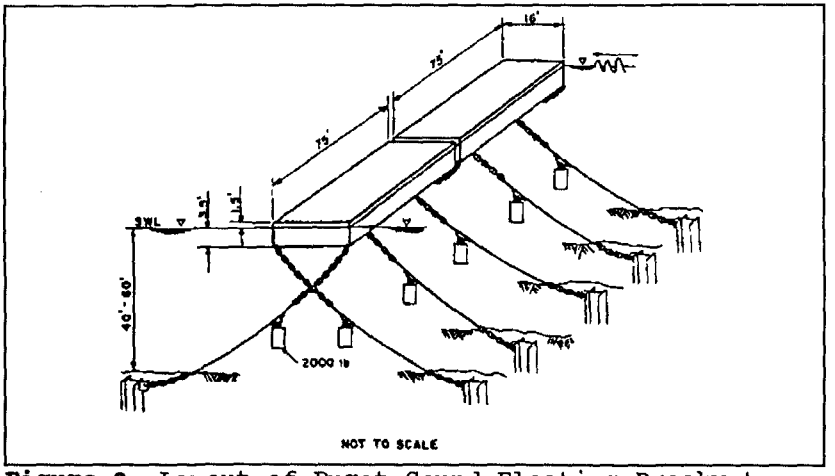

Figure 2. Layout of Puget Sound Floating Breakwater

Table 1
Characteristics of Puget Sound Floating Breakwater

Length=150 ft (46m)	Natural Period-Sway = 20 sec
Beam =16 ft (4.9m)	Natural Period-Heave = 3.2 sec
Draft=3.5 ft (1.1m)	Natural Period-Roll = 2.4 sec
Mass=16,696 slugs (244,022 kg)	Natural Period-Pitch = 3.4 sec
C.G = 1 ft (.31m)	Natural Period-Yaw = 14 sec
GM = 5.35 ft (1.6m)	

Table 1 also summarizes the natural period for each mode of motion of the breakwater. The natural periods were determined by performing extinction tests with the numerical model for mid-tide conditions. As can be seen in Table 1, the natural periods of motion in sway and yaw are much longer than: (1) heave, roll and pitch natural periods and (2) typical floating breakwater design waves periods.

The U.S. Army Corps of Engineers, Waterways Experiment Station, Coastal Engineering Research Center recently sponsored a series of 1:10 scale physical model tests to evaluate the performance of the prototype structure. The tests are given in Torum et al (1989).

In order to validate the numerical model presented in this paper, the predictions of the numerical model were compared to measurements obtained in the prototype experiment and physical model tests. Results of this comparison are described below.

Physical Model Tests. Torum et al (1989) present physical model test results for both a continuous breakwater structure (i.e. "stiff" model) and a discontinuous series of breakwater pontoons separated by fenders (i.e. "fendered") model. Only the "stiff" model results were used in the evaluation of the time domain numerical model. The linear hydrodynamic coefficients and wave transfer functions for the caisson-type floating breakwater were computed in the frequency domain and converted to time domain functions using the techniques described above. It would be highly desirable to measure these parameters directly with the results of physical model tests. Unfortunately, given the nature of physical modeling program described by Torum et al (1989), there is no way to validate these quantities directly. A systematic series of physical model tests would be

required to evaluate the added mass, damping and wave transfer functions separately. At this juncture, one can only compare the final output of the physical model (i.e. breakwater motions and mooring line forces) to that simulated by the numerical model.

Mooring analyses were prepared for a variety of the wave conditions tested in the physical model. As previously mentioned, second order wave transfer functions were initially computed without accounting for wave overtopping. The resulting mooring dynamic simulations significantly overpredicted breakwater motions and attendant mooring line forces. A systematic variation of the second order wave transfer functions and viscous damping coefficients demonstrated that the floating breakwater response was relatively sensitive to the second order wave transfer functions and less sensitive to the viscous damping coefficient as long as the linear damping formulation was used. Hence, the original method for computing the second order drift forces was abandoned in favor of the method of Longuet-Higgins (1977) presented in Equation (5). In order to apply equation (5) it was necessary to know the incident, reflected and transmitted wave amplitudes for each wave frequency. Fortunately, breakwater transmission characteristics were presented in Torum et al (1989). Hence, the second order wave transfer functions could be estimated directly from equations (5) and (6). Mooring line load deflection curves were estimated from plots presented in Torum et al (1989).

Example numerical model results are presented in Figure 3 which presents time histories of water surface elevation, sway, heave, and roll motions and mooring line forces for beam-on waves with a significant wave height of 1.43 meters and a peak spectral wave period of 4.3 seconds. Figure 3 is actually a portion of the predicted time history as the numerical model was run for a total time of 960 seconds. With the exception of roll, the numerical model provided a good prediction of the breakwater motions. Specifically, the peak values of sway, heave, and roll measured in the physical model were 3.029 meters, .7022 meters, and 10.04 degrees, respectively, while the peak values of sway, heave, and roll predicted by the numerical model were 2.8 meters, 0.65 meters, and 25 degrees, respectively. The maximum mooring line load predicted by the numerical model (31.8 kilonewtons) was comparable to that measured in the physical model tests (32.97 kilonewtons). Figure 3 indicates several facets of the breakwater response, namely: (1) heave and roll response are at the incident wave frequency, (2) sway response and mooring line forces

are dominated by low frequency motion near the natural period of sway motion, and (3) roll motion is overpredicted by the numerical model. Similar results were obtained for other comparisons of the numerical model results with that measured in the physical model.

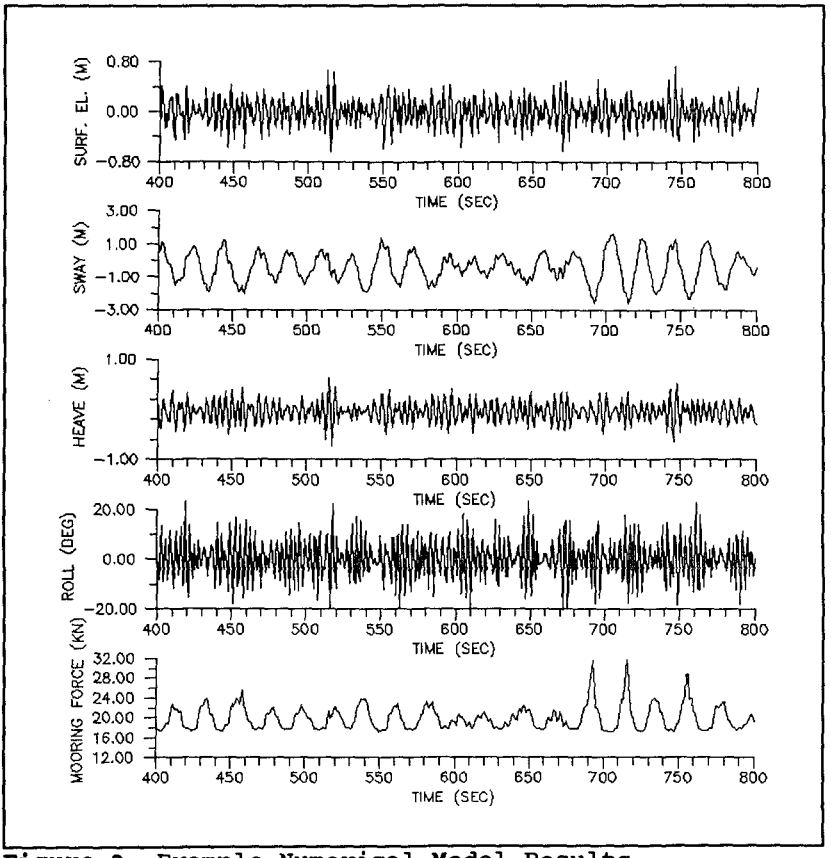

Figure 3. Example Numerical Model Results.

The numerical model results described above are generally consistent with the conclusions presented by Torum et al (1989) who concluded that: (1) floating breakwater sway response was dominated by low frequency motions and (2) mooring line forces were governed by low frequency sway motion and wave frequency roll and heave motions. However, spectral plots of mooring line forces presented in Torum et al (1989) indicate peak spectral energy at or near the wave frequency with a secondary

peak at low frequency. This led Stansberg et al (1990), in a summary of the Torum et al (1989) report, to conclude that mooring line forces were on the average governed by heave and roll motions with extreme events strongly influenced by low frequency say peaks. The numerical model presented in this paper indicates that mooring line loads are dominated by horizontal sway motions. Additional analyses will be required to evaluate the apparent discrepancies between the sway motion and mooring line force spectra presented in Torum et al (1989).

Prototype Measurements. The numerical model was used to estimate the mooring line loads recorded during the Floating Breakwater Prototype Test Program. Hydrodynamic coefficients and wave transfer functions were computed in the same manner as described above for the physical model. Wave transmission coefficients measured in the physical model tests of Torum et al (1989) were used to estimate the second order wave transfer functions for the prototype structure. Mooring line load-deflection characteristics were estimated on the basis of the field pull tests described in Nelson and Broderick (1986).

Example numerical model results are presented in Figure 4 for an incident significant wave height of 1.29 feet and a peak spectral period of 2.75 seconds. Numerical computations were performed for a total simulation time of 540 seconds to correspond with the results presented in Nelson and Broderick (1986). The results presented in Figure 4 for the prototype breakwater are similar to those presented in Figure 3 for the scale model breakwater. Specifically, breakwater sway response and attendant mooring line force is primarily at low frequency near the natural period of sway motion while the breakwater response in heave and roll is at a frequency corresponding to the incident wave. The maximum mooring line load measured for these wave conditions was 6,346 pounds (28.3 kilonewtons) while the numerical model prediction was 6070 pounds (27.0 kilonewtons). Similar results were found for other numerical model-prototype measurement comparisons.

CONCLUSIONS

A numerical model for dynamic analysis of floating breakwater mooring systems has been presented. Comparisons of numerical model results with physical model and prototype measurements indicate that the numerical model provides reasonable estimates of floating

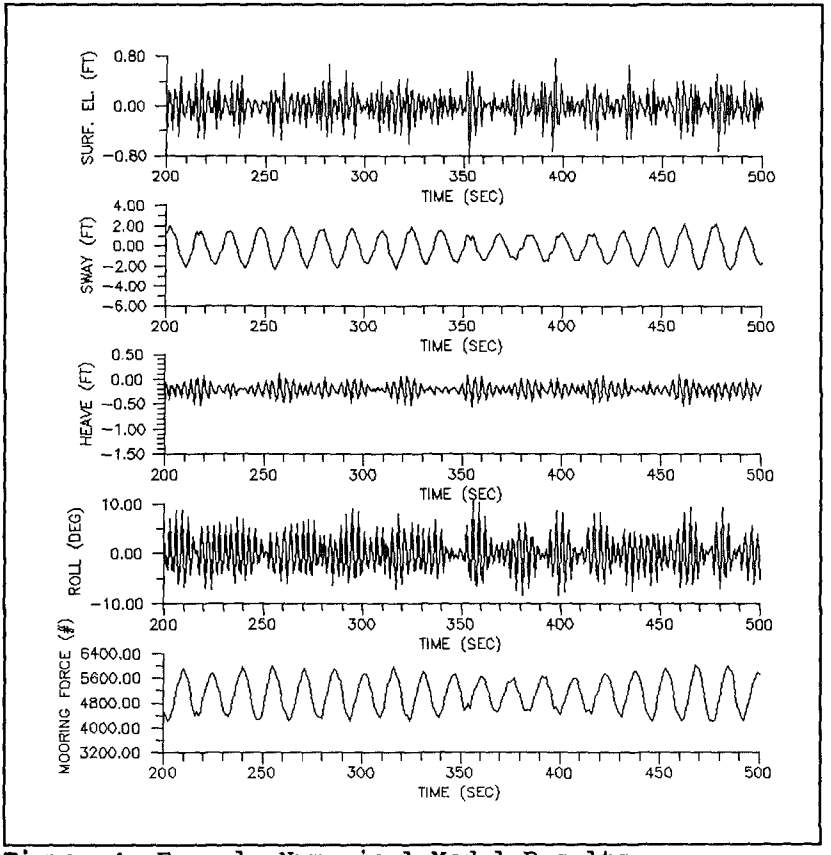

Figure 4. Example Numerical Model Results

breakwater mooring line forces for planning and design use. Moreover, the numerical model can be used as a planning tool prior to physical model studies of final floating breakwater mooring configurations. Additional studies, involving systematic physical model tests, will be necessary to reduce uncertainties associated with: (1) computation of the hydrodynamic coefficients, wave transfer functions, and applied wave force time histories, and (2) importance of roll and heave motions on mooring line loads.

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