Wave forces on cylindrical members at offshore structure

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1. Preface

Until now, offshore areas have rarely been utilized because of restrictions caused by waves. To utilize offshore areas more fully, it is necessary to develop technology for construction of offshore structures able to withstand large waves. To achieve this, C.E.R.I. of Hokkaido Development Bureau has carried out in situ offshore experiments to investigate structures in an area with large waves as part of studies funded by Funds for Science and Engineering Development: research on utilization of maritime zones by constructing offshore structures. This project has analysed data of wave forces against circular cylindrical members of offshore structures in a large wave area. This type of data has rarely been reported in the past.

2. Experimental structure and observation

The offshore experimental structure is 10mX5m and 18m high, and was constructed in 7.0m deep water off the Ohgon Cape at Rumoi in Hokkaido on the Japan Sea coast in 1984¹. Phot.1 shows the offshore experimental structure in winter, and Fig.1 shows the location of the omnidirectional wave force transducer, the ultrasonic wave gauge, the current meter, and the strain gauge.

The wave force transducer is shown in Phot.2, it was developed especially for this project and can measure the magnitude and direction of forces on cylindrical members (0.508m in diameter)². Fig.2 shows a sectional view of the wave force transducer.

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Phot. 1 Offshore experimental structure

Fig. 1 Sensor location
Phot. 2 Omnidirectional wave force transducer

Fig. 2 Sectional view of wave force transducer
Wave induced forces were determined from the difference between the highest and the lowest shear forces obtained by measuring the strain along the axes of special alloy members at both the upper and lower ends of the wave force transducer.

Fig. 3 shows the distribution of heights and periods of significant waves by the zero-down-cross method using data gathered in 1984-86. It shows some waves higher than 4 meters, and so the data can be used for wave force characteristics in the high Reynolds number range. The subject of the analyses are the largest 14 data sets (2,487 waves) of 48 sets from the winter of 1986: the waves are 2.47m - 4.52m and the periods 6.49sec - 9.64sec. Both total and local wave forces acting on the members were examined. Total wave forces were measured by the strain gauge set at the foot of the experimental structure in Fig.1. Local wave forces were measured by an omnidirectional wave force transducer set on a vertical circular cylindrical member at the center of the structure between 1.0m to 3.0m below the water.

Current velocities were measured by the current meter 2.0m under the water surface 2m from the wave force transducer perpendicular to the wave direction.

Fig. 3 Distribution of heights and periods of significant waves by the zero-down-cross method
3. Drag coefficient and inertial force coefficient of vertical circular cylindrical members.

In general, wave forces acting on submerged cylindrical members can be calculated with Morrison's equation\(^3\).

\[
\frac{dF}{dz} = C_d \frac{\omega_0 D u}{2 g} + C_m \frac{\omega_0 x D^2}{4 g} \frac{u}{t}
\]  

(1)

where, \(dF\) is the normal force action upon a length \(dz\) along the axis of member (tf); \(u\) is the water particle velocity acting normal to the member axis (m/s); \(C_d\) is the drag coefficient; \(C_m\) is the inertial force coefficient; \(D\) is the diameter of the member (m); \(\omega_0\) is the weight of a unit volume of sea water (tf/m\(^3\)); and \(g\) is the acceleration of gravity (m/s\(^2\)).

The first term on the right side of Eq. (1) represents the drag force and the second the inertial force. The drag and inertial force coefficients are believed to be influenced by Reynolds number \(uD/v\), KC number \(uT/D\), and others. Widely deviating experimental data of these wave force coefficients have been reported, and data using Reynolds numbers in the range to \(10^6\) are not adequate. There is also no standardized system for measuring wave force coefficients. This paper follows the procedure:

1. Define each wave with the zero-up-cross time of the water surface-wave profile.
2. The direction of the resultant maximum wave force computed by combining the horizontal wave force components is to be considered the direction of each wave.
3. The direction component of water particle velocity and its acceleration are derived for each wave.
4. The \(C_d\) and \(C_m\) of each wave are derived by the least squares method.
5. The \(C_d\) and \(C_m\) are corrected to restore peak values.

When the residual sum of squares of measured and calculated wave forces is minimized in step 4, wave force coefficients can be given as:

\[
C_d = \frac{2g}{\omega_0 D} \frac{S_5 S_4 - S_6 S_2}{S_1 S_3 - S_2^2} \]  

(2)

\[
C_m = \frac{4g}{\omega_0 x D^2} \frac{S_5 S_4 - S_6 S_2}{S_6 S_1 - S_2 S_4} \]  

(3)

\[
S_1 = \sum u_1^4, \quad S_2 = \sum (\delta u_1 \delta t), u_1 u_1, \quad S_3 = \sum (\delta u_1 \delta t)^2, \ldots
\]

\[
S_4 = \sum f_1 u_1 u_1, \quad S_5 = \sum f_1 (\delta u_1 \delta t), \ldots
\]  

(4)
where, $f_i$ and $u_i$ show the measured wave forces and water particle velocities at time $i$, and $\Sigma$ means the sum total of the data of a wave at a particular time.

Fig. 4 shows a time-series profile of representative wave forces calculated from measured wave forces and water particle velocities, and water particle accelerations and wave force coefficients calculated by the above equations. Despite its quasi-empirical nature, Morrison's equation reproduces the wave force profile fairly accurately. However, since the above method is designed to compute wave force coefficients that on average agree to measured values, there is a danger that the peak values of wave forces – the most important in actual designs – will not always correctly correspond to the measured wave forces.

![Fig. 4 Time-series profile of measured and calculated wave forces](image-url)

To observe this, a comparison of measured peak wave forces and computed peak wave forces of all analyzed data is made in Fig. 6 where the relation between these two values is given by the least squares method. The results show that the calculated values tend to be only 87% of the measured values, and in the following, wave force coefficients are corrected by this factor ($1/0.87$).

Using the previous procedure, Figs. 6 and 7 indicate the relation between the drag coefficient/inertia force coefficient of wave forces obtained by the omnidirectional wave force transducer, (effective measurement range above 0.07tf
(0.688 kN)) and Reynolds number, using parameters derived from the KC numbers. The Reynolds and KC numbers are computed with the maximum water particle velocity of each wave.

![Graph showing relation between peak values of measured and calculated wave forces.](image)

**Fig. 5** Relation between peak values of measured and calculated wave forces

![Graph showing relation between drag coefficient and Reynolds number.](image)

**Fig. 6** Relation between drag coefficient and Reynolds number
The omnidirectional wave force transducer in this experiment was painted to eliminate the growth of organisms, and none were found on the structure prior to the beginning of the measurements. Therefore, the data may be considered applicable for smooth circular cylindrical members.

As seen in Fig. 7, there are correlations between wave height and period, and the Reynolds and KC numbers are related, which makes it difficult to determine the exact relation between wave force coefficients and the numbers. However, it may be concluded that the drag coefficient will likely decrease with increases in Reynolds number, while the inertial force coefficient remains unaffected by Reynolds number. The dependence of the two wave force coefficients on the KC number is not shown explicitly. The drag coefficient becomes almost independent of Reynolds numbers above 7.0 x 10^6. The average values of wave force coefficients of data in this range are Cd=0.60 ± 0.17 and Cm=1.23± 0.34 considerably less than the Cd=1.0 and Cm=2.0 commonly used.

Reported wave force coefficient measurements are given in Table 1 [14, 18, 36, 77]. The results cannot simply be compared to these values because of the different Reynolds numbers involved, and the values in this study scatters less than other results. The data are relatively close to the results of Kim et al., but extend to higher Reynolds numbers.
Another point is that the drag coefficient in Kim et al. becomes constant at Re > 2.0 x 10^5.4).

Table 1 Reported wave force coefficient measurements

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Re</th>
<th>KC</th>
<th>Cd</th>
<th>Cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kim</td>
<td>1.5 x 10^5~</td>
<td>15~</td>
<td>0.61±</td>
<td>1.20±</td>
</tr>
<tr>
<td>Bishop</td>
<td>1.0 x 10^5</td>
<td></td>
<td>0.73</td>
<td></td>
</tr>
<tr>
<td>Yoshida</td>
<td>2.5 x 10^5~</td>
<td>3~</td>
<td>0.6 ±</td>
<td>1.2</td>
</tr>
<tr>
<td>Dean</td>
<td>1.0 x 10^7</td>
<td>10~</td>
<td>0.5 ±</td>
<td>1.33</td>
</tr>
<tr>
<td>Evan</td>
<td>1.0 x 10^6</td>
<td>10~</td>
<td>0.58±</td>
<td>1.76±</td>
</tr>
<tr>
<td>Reid</td>
<td>1.0 x 10^5</td>
<td>10~</td>
<td>0.53±</td>
<td>1.47±</td>
</tr>
</tbody>
</table>

4. Results of analyses of total wave force data

The values of the total wave force data measured by the strain gauges set at the foot of the experimental structure were compared with values computed by a three-dimensional static structural analysis program, and features of the wave induced forces were examined.

The experimental structure was analyzed by a structural analysis program for complex structures: ISAP-11 of TOSBAC 600. Wave forces were calculated by Morrison's equation - wave direction designated by an arrow in Fig.1 and water particle acceleration was determined by the small amplitude wave theory. Phase differences were not considered when computing wave forces because the structure is small compared with the wave length. Buoyancy acting against the members of the structure above the hydrostatic surface was also considered, buoyancy was applied to the entire cross section of the members on the assumption that the water level in the members was little affected by the waves. The range of wave forces and buoyancy was obtained from calculations of maximum wave heights.

Fig.8 shows the relation between the measured axial forces and wave heights in six samples (1043 data) where the wave directions were relatively close to the calculations; the axial forces are the peak axial forces of each wave defined from the zero-up-cross time. The solid line (Fig.8) indicates axial force values computed by the typically used wave force coefficients - Cd=1.0 and Cm=2.0 - and the broken line presents values computed using the coefficients Cd=0.6 and Cm=1.2 obtained in the preceding section. In both cases, the wave period was assumed to be 10 seconds.
The experimental structure has numerous horizontal members in addition to the vertical ones, and the calculated wave forces increase discontinuously with increases in wave height; however, the lines in Fig. 8 are continuous because specific wave heights were chosen for the computation and the results were smoothed to a continuous curved line. This curve is approximately in proportion to the cube of the wave height.

Although the measured values are relatively dispersed as indicated in Fig. 8, they tend to correspond well to the calculated values of the three-dimensional static structural analysis. However, even the values calculated with the coefficients $Cd=0.6$ and $Cm=1.2$ are greater than the measured values, especially at large wave heights. According to this figure, the present standard coefficient values ($Cd=1.0$ and $Cm=2.0$) are clearly in a safe range.

Furthermore, it was observed that the shock wave forces on structural members in this experiment were far larger than Morrison's wave forces above the hydrostatic plane\(^9\); however, no effect of these shock wave forces were found in the total wave force values.

Apparently, this resulted from the shock waves being so localized that there were phase differences among the members, and also because the duration of shock waves is so short that the structure as a whole, was unable to respond.
In the present project the experimental offshore structure was set in shallower water than actual offshore structures. However more members need not be considered when examining the overall stability of an actual offshore structure, except in particular cases.

5. Summary

This report examines features of wave induced forces on circular cylindrical structural members with data from an offshore experimental structure. The main results of the project are as follows:

① Data of wave induced forces on circular cylindrical members in an offshore area were obtained: some of the waves were large and in the high Reynolds number range. Wave induced forces on the members were examined for both total and local wave forces. Local wave forces were measured by an omnidirectional wave force transducer developed for this project.

② On the basis of wave forces (local wave forces) induced on the circular cylindrical members and the simultaneously measured water particle velocity data, the drag coefficient and inertial force coefficient were computed as \( Cd = 0.6 \pm 0.17 \) and \( Cm = 1.23 \pm 0.34 \), in the Reynolds numbers range above \( 7.0 \times 10^5 \). The drag coefficient decreases with increases in the Reynolds number, whereas the inertial force coefficient is unaffected by fluctuations in Reynolds number.

③ Measured values of total wave forces tended to agree with the three-dimensional static structural analysis calculations. However, the calculated values, even with wave force coefficients of \( Cd = 0.6 \) and \( Cm = 1.2 \), became larger than the measured values. There was no effect of shock wave forces in the measured values.

6. Acknowledgments

We wish to thank the committee for offshore structure research and experiments in offshore areas for its cooperation in the preparations for the present project - Director: Akira Ozaki; and Hideo Kondo; Takeo Hori; Shinichi Ishii; Yoshimi Goda; Takafumi Takaishi; Keizo Kuwahara; Tadaoki Itakura.

7. Bibliography

2) Kadono Takashi; Momose Osamu; Nagai Yutaka: "Development of the Omnidirectional Wave Force Transducer." Monthly report of the civil Engineering Research Institute of


