CHAPTER 78

Experimental and Numerical Study on Solitary Wave Breaking

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Abstract

Interior flow fields under drastically transforming solitary waves were visualized and recorded using a high-speed video system. Flow velocities were estimated by tracing fluid particles through spatial correlation analysis of video images. A Lagrangian equation system was successfully employed to numerically simulate two-dimensional wave transformations in a vertical plane.

Introduction

In these years, remarkable progresses have been achieved in techniques for analyzing nearshore wave transformation. However, our knowledge on the fundamental mechanism of wave breaking is rather limited as well as on after-breaking wave transformation. Existing breaking indices are by no means enough for application to a complicated field of composite and/or irregular waves. For future study of this problem, introduction of new methods is of particular importance in both experimental and theoretical approaches.

Since wave transformation such as breaking is quite unstable, exactly the same phenomenon cannot be reproduced even in a well-designed wave flume. It is therefore difficult to obtain an entire picture of interior fluid motion by repeating velocimeter measurements. At least, the accuracy of thus obtained velocity distribution cannot be sufficient for succeeding estimation of pressure, vorticity, and so forth. In this

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context, observation and measurement with visual devices are much more promising.

On the other hand, a Eulerian equation system is normally employed in computational approach to two-dimensional fluid motions. Numerical schemes in the Eulerian framework, however, have a weakness in handling moving boundaries. Note that the fluid motion under a wave is essentially prescribed by surface conditions and that the surface profile rapidly deforms under a breaking condition. Although the Lagrangian equations have some disadvantages in particular in their numerical integration process, they may provide a useful means for wave transformation analysis, allowing much more rigorous treatment of surface boundaries.

Methods of Picture Analysis

Techniques of flow visualization and video image analysis are employed here to investigate the hydraulic mechanisms of solitary wave runup on a vertical wall and its breaking on a uniform slope. Once if a precise velocity field in a wave is obtained, then corresponding distributions of pressure, vorticity, etc., will be easily deduced.

A conventional method of neutral buoyancy float tracing is not quite appropriate for such a purpose. It is not easy to keep the floats with finite sizes in suspension and they may not move in the same manner as water particles. The number of floats is limited for computerized tracing and, furthermore, their location cannot be well controlled. A flow field is more conveniently visualized by densely scattering small solid particles in the fluid. In this case, flow velocity at an arbitrary position is estimated from the variation of a local brightness pattern through either temporal or spatial correlation analysis.

Figure 1 describes the principle of temporal correlation analysis (for example, Miike et al., 1986), in which time-series data of brightness variations at adjacent two points are compared. Time-lag between the variations is considered to be the traveling time of a water particle for the distance between these two points. The flow direction can also be detected since the correlation will be highest between two points along a local path-line. This method is suitable to real time measurement of flow velocity at a fixed point, but is not applicable to strongly unsteady flows, requiring steady shift of brightness variation for a finite time interval.

The spatial correlation analysis (for example, Kimura and Takamori, 1986), in principle, is more similar to the normal float tracing. A recorded image of visualized fluid at each time is converted into a matrix of digital brightness gradation data at all the pixels. A small square body of fluid is specified by a reference frame fixed in a matrix, whereas a test frame with the same size is moved in another matrix for an adjacent time as shown in Figure 2. The spatial correlation between brightness patterns in these two frames is evaluated in terms of the correlation coefficient R defined by

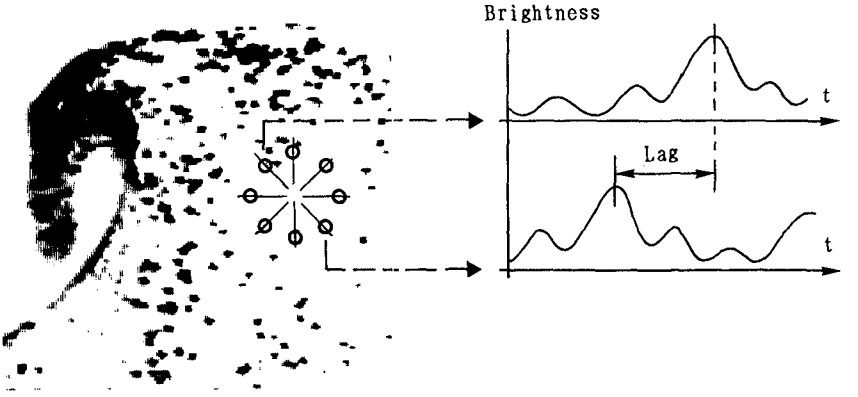


Figure 1. Temporal Correlation Analysis

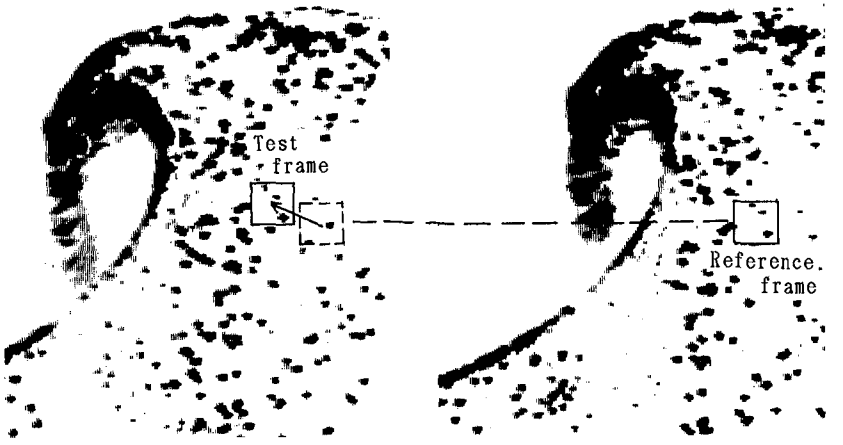


Figure 2. Spatial Correlation Analysis

$$R = \sum(m_i-\overline{m})(n_i-\overline{n})\Big/\sqrt{\sum(m_i-\overline{m})^2(n_i-\overline{n})^2} \qquad (1)$$

where m_i and n_i represent the brightness gradations at the i-th pixels in the reference and test frames, and $\sum$ and $\overline{\ }$ respectively denote summation and average over the frames. Higher correlation will be obtained as the test frame gets closer to the new location of the fluid body in question. Spatial displacement of the fluid body during a prescribed time interval is thus estimated. This method is adopted in the following, since it is obviously applicable even to unsteady flows.

When brightness variation is smooth both in time and in space, introduction of gradient method is recommended to notably reduce the computational load for the above-explained fitting test. Provided that the apparent brightness of a water particle is kept unchanged as it moves, the total derivative of the brightness B should vanish: that is,

$$dB/dt = B_t + UB_x + VB_y \qquad (2)$$

where t is the time, x and y are the horizontal and vertical coordinates, U and V are the velocity components in the x- and y-directions, and suffices denotes partial derivatives. The above relationship gives the flow direction from temporal and spatial gradients of brightness.

It should be also noted that the angle of fluid particle rotation can be directly detected by rotating the test frame and seeking for better fit, where the brightness values have to be interpolated for the pixels in the rotated frame. The local vorticity can be thus estimated possibly more accurately than from velocity gradients. Pre-processing of image data using filters may be helpful in these advanced analyses (Huang, 1981).

Experimental Setup and Data Processing

For each case of experiment, either a vertical wall or a uniform slope were installed in a wave flume 17m long, 40cm wide and 60cm deep. A solitary wave was generated by a single forward motion of a piston-type wave board driven by compressed air. In order to lessen the effect of parallax in video recording, the flume width was reduced to 10cm by a partition wall with a gradual contraction in front of the wave generator.

The time-scale of the present experiments required the use of high-speed monochromatic video system, by which 1000 scenes can be recorded per second. A scene was resolved into 256x256 pixels when converted to a

matrix of digital brightness gradations. Flow was visualized by scattering yellow polyethylene beads with diameter of roughly 0.5mm in the water. The beads are originally heavier than water, but their specific gravity is adjustable by heating because of gas containment. The density of bead distribution was determined on trial and error basis so as to produce the best video images. The material of background wall and the directions of lighting had also to be carefully decided. Reflected lights from water surface and basin walls badly disgrace the reliability of image data.

Figure 3 shows the outline of data acquisition and processing system, where the arrows indicate the flow of data. A set of digitized image data is enormous in its volume, while a high-speed video tape can be rewound only several times. Obtained analogue data, therefore, were first copied on a normal tape, and only necessary parts of the data for each analysis were stored on a hard disk after A-D conversion. For efficient processing of data by a micro-computer, it is desirable to write programs in a machine language.

Results of Experiments

Photo 1 shows video images describing runup and return flow motions of a solitary wave on the vertical wall. This can also be regarded as the formation process of a standing wave. It was observed that the lower layer water starts to fall slightly before the maximum runup height is reached. A very slender runup profile is thus formed. After that, the crest water gets into nearly

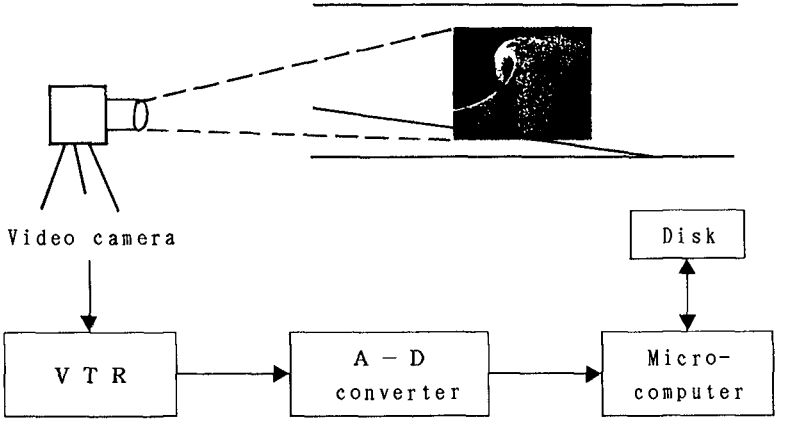
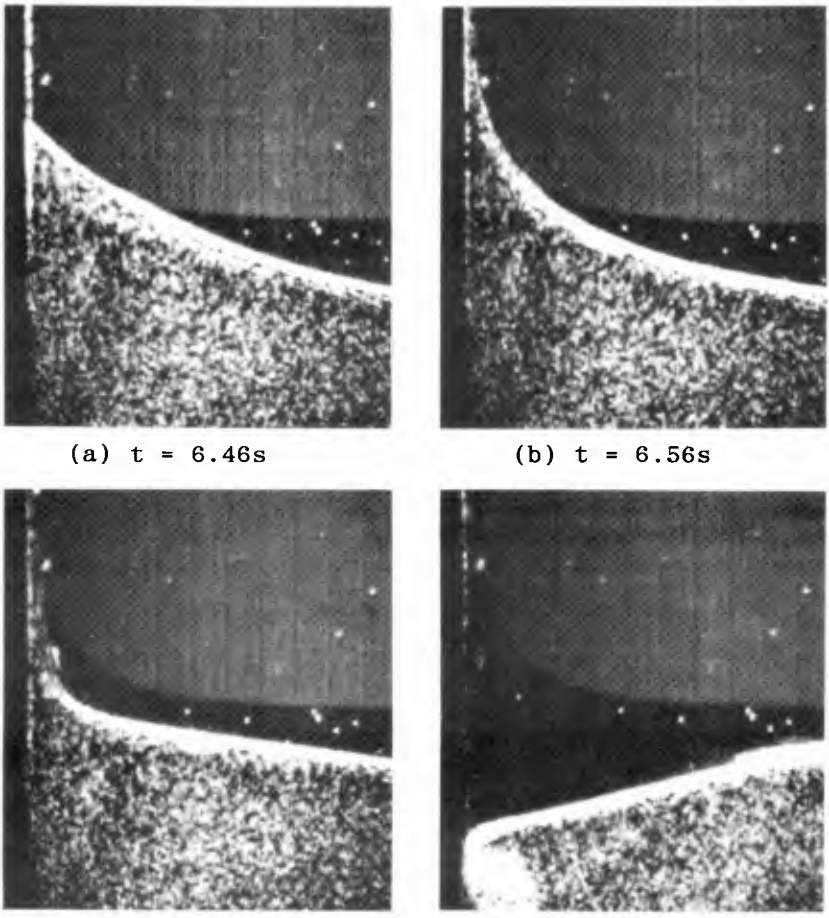


Figure 3. Data Acquisition and Processing System

free fall motion and plunges into lower water layer, yielding a vortex motion at the foot of the wall. Slow replay of video record in fact well exhibit such details of rapid wave deformation. A result of image analysis is given in Figure 4, which shows an instantaneous flow field in the rising stage. Obvious misevaluations of velocity appear at several points in the figure. In the fitting tests, the frame size of 9x9 pixels were adopted, and displacements of water particles were evaluated in pixel unit. More precise velocities will be obtained by estimating sub-pixel fractions of displacements through two-dimensional interpolation.



(a) t = 6.46s (b) t = 6.56s

(c) t = 6.64s (d) t = 6.84s

Photo 1. Solitary Wave Runup on a Vertical Wall

Photo 2 shows a visualized breaking motion of a solitary wave on the 1/20 slope. A shadow region appears under the overturning wave crest, since light source was located on the upper flange of wave flume. Figure 5 is an example of obtained velocity field. The velocity vectors were plotted only for the points where larger correlation coefficients than 0.7 was obtained in the fitting tests. This criterion of minimum reliability could not be attained for the shadow region, indicating the fatal importance of lighting technique for thorough estimation of velocities over the whole region.

Lagrangian Equation System

For theoretical treatment of fluid motion in a vertical plane, it is here assumed that the fluid is inviscid and incompressible. A further assumption is that only the gravity is exerted as an external force.

The continuity equation in the Lagrangian framework is given as

$$X_a Y_b - X_b Y_a = J \tag{3}$$

where a and b are the Lagrangian parameters to identify a water particle, X and Y are the horizontal and vertically upward coordinates to indicate the location of the water particle at an arbitrary time, J is at most a function of a and b, and suffices again denotes partial derivatives. The above equation is differentiated with

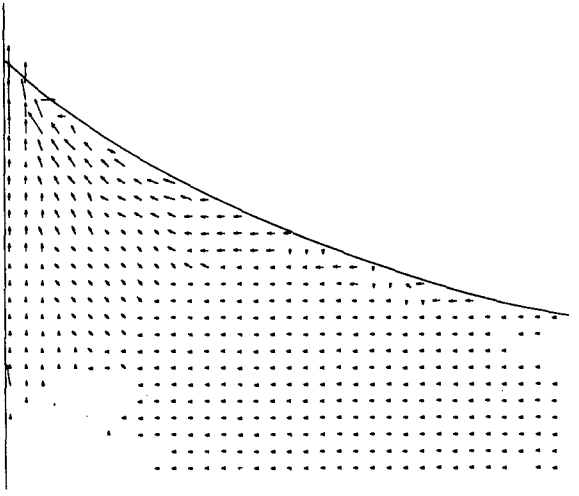


Figure 4. Observed Flow Velocities in a Solitary Wave Running Up on a Vertical Wall

(a) t = 5.140s

(b) t = 5.165s

Photo 2. Solitary Wave Breaking on a Uniform Slope

respect to time t to yield

$$U_a Y_b - U_b Y_a - V_a X_b + V_b X_a = 0 \tag{4}$$

in which the velocity components U and V are simply related to X and Y as

$$U = X_t , \quad V = Y_t \tag{5}$$

On the other hand, the momentum equations are written as

$$X_{tt} X_a + (Y_{tt} + g) Y_a = -P_a / \rho \tag{6}$$

$$X_{tt} X_b + (Y_{tt} + g) Y_b = -P_b / \rho \tag{7}$$

where P is the pressure, g is the gravitational acceleration, and ρ is the fluid density. Elimination of force terms in the above equations through cross-differentiation and single integration of the obtained equation with respect to time lead to the following law of vorticity conservation.

$$U_a X_b - U_b X_a + V_a Y_b - V_b Y_a = C \tag{8}$$

where C is an arbitrary function of a and b.

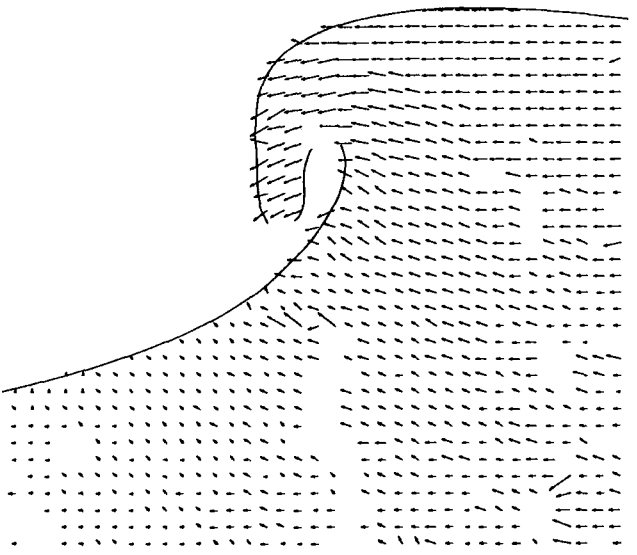


Figure 5. Observed Flow Velocities in a Breaking Solitary Wave

For simplicity, a and b are defined as the positional coordinates of a fluid particle at a reference time t=0 when the fluid is at rest. lt is most convenient to define the coordinate system in such a way that either a or b takes a constant value along every boundary. For a solid boundary, the expression of boundary configuration in terms of X and Y by itself gives a kinematic condition. Along a free surface, P=0 is imposed as a dynamic condition, which can be easily rewritten in terms of the other unknown variables with reference to the momentum equations (6) and (7).

Numerical Simulation Using Lagrangian Equations

Numerical analysis with experimental verification is a useful means for investigating the detailed dynamics of wave transformation. Behavior of surface waves is essentially governed by the boundary conditions at the free surface. Although the Lagrangian coordinate system is possessed of a definite advantage in describing a moving boundary (Chan, 1975), it has been rarely employed in wave computations, perhaps because of 1) second order derivatives with respect to time involved in the equations of motion, 2) nonlinearity of both the equations of motion and continuity, and 3) weakness against large deformation of computational grids. These difficulties, however, became less significant nowadays as computational technology rapidly progressed. For this sort of simulation, two alternative formulations are available.

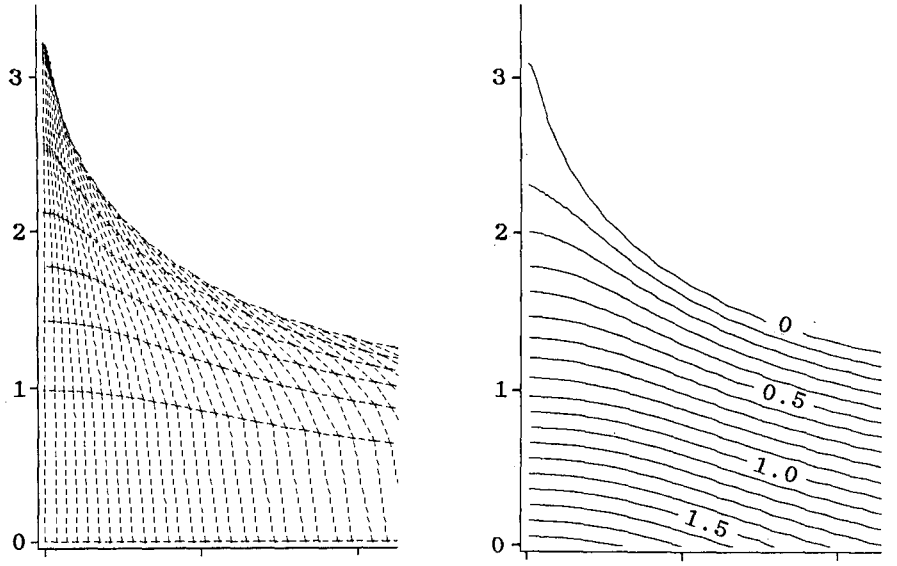
One is a velocity solution method, where the continuity equation (4) and the irrotationality constraint (8) are discretized and solved simultaneously with given boundary conditions for velocity components at all the grid points. The computation proceeds to the next time step through integration of Eq. (5). lt is obvious that this method is applicable only to irrotational fluid motion.

The other is a pressure solution method, in which the original set of equations is directly treated, that is, discretized pressures in the momentum equations (6) and (7) are determined so that the resultant deformation of every fluid particle satisfies the continuity equation (3). This formulation for the pressure yields in principle a Poisson-type equation, which is numerically solved through easier operation of matrices with a smaller dimension. Furthermore, the pressure formulation is more flexible in taking into account secondary factors such as fluid viscosity and surface tension. A shortcoming of this formulation lies in weaker constraint against the generation of non-physical vorticity,

which often results in computational instability of checkerboard-split type.

It proved from trial computations of the solitary wave transformations that two methods mentioned above yield no significant difference in their results. Computed distributions of velocity and pressure for the case of solitary wave runup are shown in Figure 6, where both the coordinates and pressure are nondimensionalized with the still water depth as length-scale. Water particles are either at rest or slowly falling except near the wave crest as observed in the corresponding experiment. Extremely mild pressure gradient in the upper layer gives a theoretical background to the observed free-fall motion of crest water. The computation fails immediately after this as fatal distortion of computational grids emerges in the crest region.

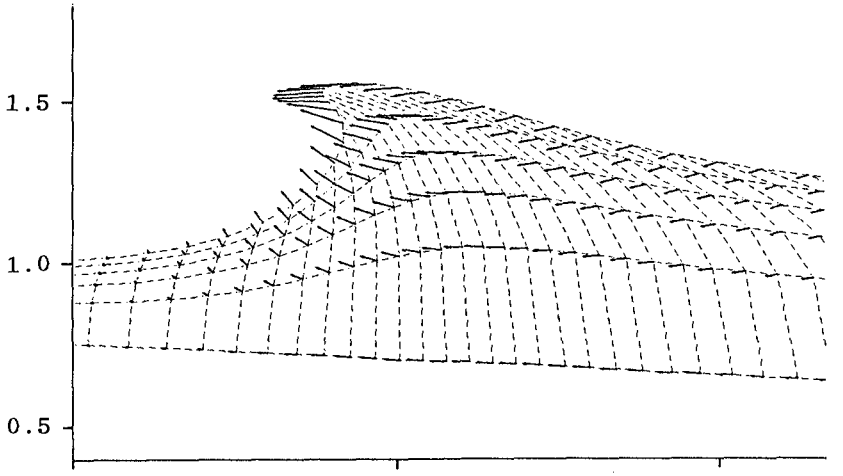
Computed velocity and pressure in the breaking wave are shown in Figure 7. Mild pressure gradient in the crest region implies less dynamical interaction between water particles. Further decrease in the local pressure will lead to a free motion of the crest water and possibly to air entrainment. On the contrary, verti-



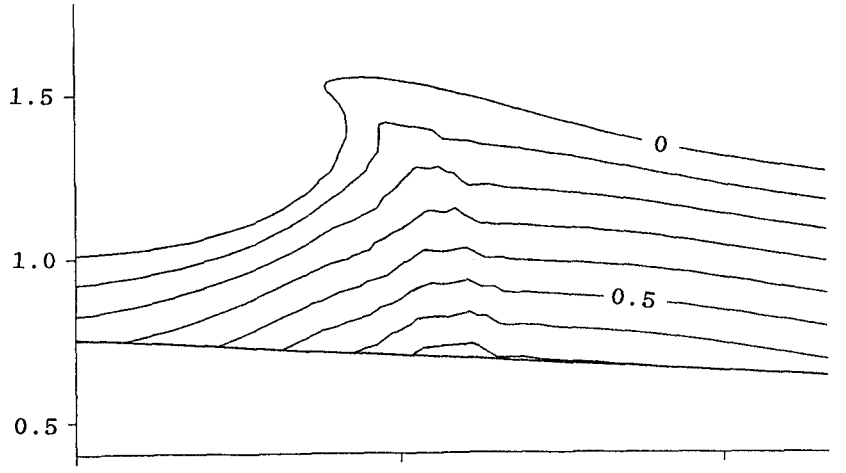
(a) Velocity vectors (b) Isobars

Figure 6. Computed Distributions of Velocity and Pressure in a Solitary Wave Running Up on a Vertical Wall

and possibly to air entrainment. On the contrary, vertical pressure gradient at the foot of the breaker front is rather steep, providing a good circumstance for sediment suspension. In general, numerical simulation excluding the effect of surface tension allows unrealistic formation of a sharp plunging nose.



(a) Velocity vectors



(b) Isobars

Figure 7. Computed Distributions of Velocity and Pressure in a Breaking Solitary Wave

Concluding Remarks

For better understanding of wave breaking phenomena, combined experimental and theoretical investigations are required particularly on their dynamical aspects with more attention to the process of local pressure reduction. The spatial correlation analysis of video images provides a useful means in such studies as well as the numerical analysis in the Lagrangian framework. The accuracy of velocity estimation by means of image analysis is expected to largely improve in the near future as video systems with higher resolution become available.

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