CHAPTER 50

Estimation of Directional Spectrum Expressed in Standard Form

Masahiko Isobe [1]

Abstract

A standard directional spectrum is determined by a finite number of parameters included in its mathematical expression. In this paper, a theory is developed to estimate the parameters from a given data set on the basis of the maximum likelihood method. The theory is applied to estimating the parameters in the Mitsuyasu-type standard directional spectrum from data obtained by a three-component array. Main results of analysis of field data are 1) the peak wave direction agrees well with the mean direction defined at each wave frequency, 2) the peakedness parameter can be approximated by that estimated from the long-crestedness parameter at each wave frequency, and 3) the peakedness parameter takes its maximum at the frequency slightly lower than the peak frequency.

1. Introduction

A random sea state is described by a directional spectrum. In recent years, field observations of directional spectra have often been carried out using varieties of measuring instruments developed. At the same time, many theories have been developed for the accurate estimation of directional spectrum (Isobe *et al.*, 1984; Kobune and Hashimoto, 1986; Hashimoto and Kobune, 1988). Cross and power spectra, from which the directional spectrum is estimated, are statistically random quantities. However, the statistical variability is not appropriately taken into account in the theories developed so far.

A standard form of the directional spectrum such as the Mitsuyasu-type directional spectrum is determined by a finite number of parameters. This way of describing directional spectra is convenient for practical uses such as establishing a data base of random sea waves or specifying a design wave condition. Hence, it is necessary to develop a theory to estimate the values of these parameters on a reasonable basis.

[1]Associate Professor, Dept. of Civil Eng., Univ. of Tokyo, Bunkyo-ku, Tokyo, 113 Japan

In this paper, a general theory is presented to estimate the parameters included in a standard directional spectrum on the basis of the maximum likelihood method, in which the statistical variability of the Fourier coefficients is taken into consideration. Then, an explicit expression is given for the Mitsuyasu-type directional distribution function. The theory is applied to data obtained from field experiments and the characteristics of directional sea state are discussed.

2. Theory

2.1 Probability density of Fourier coefficients

Suppose M kinds of time series data, $\xi^{(m)}(\boldsymbol{x}_m, t)$ (m=1 to M, $\boldsymbol{x}_m$: coordinates at measuring points, t: time), of various wave properties at various points are given. The time series $\xi^{(m)}$ are expanded into Fourier series as

$$\xi^{(m)}(\boldsymbol{x}_m, t) = \sum_{i=1}^{\infty}(A_{ci}^{(m)} \cos 2\pi f_i t + A_{si}^{(m)} \sin 2\pi f_i t) \tag{1}$$

From the central limit theorem, the coefficients $A_{ci}^{(m)}$ and $A_{si}^{(m)}$ have normal distributions with zero means. Hereinafter we deal only with the ith component, f_i, of the frequency and drop the subscript i. Then the coefficients $A_{ci}^{(m)}$ and $A_{si}^{(m)}$ are expressed in a vector form as

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{A}_c \\ \boldsymbol{A}_s \end{pmatrix} \tag{2}$$

and the cross covariance matrix, $\Xi\Delta f$, of $\boldsymbol{A}$ is expressed in terms of co-spectrum matrix, C, and quadrature-spectrum matrix, Q, as

$$\Xi\Delta f = [< \boldsymbol{A}\boldsymbol{A}^t >] = \begin{bmatrix} C & Q \\ -Q & C \end{bmatrix} \Delta f \tag{3}$$

Where Δf is the frequency range that the coefficient $\boldsymbol{A}$ represents and $< >$ denotes an expected value. The cross-power spectrum matrix, Φ, are defined as $\Phi_{mn} = C_{mn} - iQ_{mn}$ and related with the directional spectrum, $S(f, \theta)$, as

$$\Phi_{mn}(f) = \int_0^{2\pi} H_m(f,\theta)\bar{H}_n(f,\theta) \exp[-i\boldsymbol{k}(\boldsymbol{x}_n - \boldsymbol{x}_m)]S(f,\theta)d\theta \tag{4}$$

(Isobe *et al.*, 1984). Where f is the frequency, θ the wave direction, $\boldsymbol{k}$ is the wave number vector, H_m the transfer function from the water surface elevation to the mth measured quantity, and $\bar{}$ denotes the complex conjugate.

From the cross-covariance matrix expressed by Eq. (3), the joint normal distribution, $p(\boldsymbol{A})$, of the Fourier coefficients $\boldsymbol{A}$ is given as

$$p(\boldsymbol{A}) = \frac{1}{(\sqrt{2\pi\Delta f})^{2M}\sqrt{|\Xi|}} \exp[-\frac{1}{2\Delta f}\boldsymbol{A}^t\Xi^{-1}\boldsymbol{A}] \tag{5}$$

(Rice, 1944). Where $|\Xi|$ and Ξ^{-1} are the determinant and inverse matrix of Ξ.

Now a complex variable, ζ, is defined as

$$\zeta = \boldsymbol{A}_c - i\boldsymbol{A}_s \tag{6}$$

then the right hand side of Eq. (5) is expressed in terms of ζ as

$$p(\boldsymbol{A}) = \frac{1}{(2\pi\Delta f)^M|\Phi|}\exp[-\frac{1}{2\Delta f}\zeta^t\Phi^{-1}\zeta] \tag{7}$$

in which the dimension of the matrix reduces to M. In deriving the above equation,

$$\Phi\Delta f = [<\bar{\zeta}\zeta^t>/2] = (C - iQ)\Delta f \tag{8}$$

and the relationship, $|\Xi| = |\Phi|^2$, is used.

2.2 Definition of likelihood function

If the directional spectrum is given, Φ can be calculated by Eq. (4) and the joint distribution function of $\boldsymbol{A}$ is determined by Eq. (7). Since $\boldsymbol{A}$ is obtained by the Fourier analysis of given time series data, Eq. (7) gives the probability density with which the given $\boldsymbol{A}$ occurs.

A set, $\boldsymbol{A}^{[j]}$ (j=1 to J), of the Fourier coefficients $\boldsymbol{A}$ is obtained by dividing the time series into segments or from a small but finite range of frequency. The joint probability with which the set $\boldsymbol{A}^{[j]}$ (j=1 to J) occurs at the same time is obtained by multiplying each probability density. Its Jth root, L, is expressed as

$$\begin{aligned} L(\boldsymbol{A}^{[j]};\Phi) &= \{p(\boldsymbol{A}^{[1]}) \times p(\boldsymbol{A}^{[2]}) \times \cdots \times p(\boldsymbol{A}^{[J]})\}^{1/J} \\ &= \frac{1}{(2\pi\Delta f)^M|\Phi|}\exp[-\sum_{m=1}^{M}\sum_{n=1}^{M}\Phi_{mn}^{-1}\hat{\Phi}_{nm}] \end{aligned} \tag{9}$$

where

$$\hat{\Phi}_{nm} = \frac{1}{2J\Delta f}\sum_{j=1}^{J}\hat{\zeta}_n^{(j)}\zeta_m^{(j)} \tag{10}$$

From the above definition, $\hat{\Phi}_{nm}$ is regarded as the quantity which can be obtained by operating a rectangular filter to the periodogram $\hat{\zeta}_n^{(j)}\zeta_m^{(j)}/2\Delta f$ and this means that $\hat{\Phi}_{nm}$ is the cross or power spectrum obtained by the Fourier analysis.

The quantity L expressed by Eq. (9) means the possibility or likelihood that the set $\boldsymbol{A}^{[j]}$ occur at the same time and called the likelihood function. From field measurements, $\boldsymbol{A}^{[j]}$ (j=1 to J) is given but Φ is unknown. In the maximum likelihood method, Φ is determined so that the possibility becomes maximum.

As seen from Eq. (4), the cross-power spectrum matrix Φ is a function of the parameters included in the standard directional spectrum. However, we

first assume that every component in the cross-power spectrum matrix changes independently and find the maximum value of the likelihood function in a global sense. To obtain the value of Φ which maximizes L, we take the derivative of Eq. (9) with respect to every component, Φ_{kl} (k and l=1 to M), of the matrix Φ and make every derivative vanish.

Let the cofactor of the matrix Φ be denoted by ϕ, the following relationships are obtained from mathematical theorems concerning a matrix.

$$|\Phi| = \sum_{k'=1}^{M} \phi_{lk'} \Phi_{k'l} \tag{11}$$

$$\Phi_{mn}^{-1} = \frac{\phi_{mn}}{|\Phi|} \tag{12}$$

And hence

$$\frac{\partial |\Phi|}{\partial \Phi_{kl}} = \phi_{lk} = |\Phi| \Phi_{lk}^{-1} \tag{13}$$

$$\frac{\partial \Phi_{mn}^{-1}}{\partial \Phi_{kl}} = -\Phi_{ln}^{-1} \Phi_{km}^{-1} \tag{14}$$

Now the derivative of Eq. (9) is expressed as

$$\frac{\partial L}{\partial \Phi_{kl}} = L \times \{-\Phi_{lk}^{-1} + \sum_{m=1}^{M} \sum_{n=1}^{M} \Phi_{ln}^{-1} \hat{\Phi}_{nm} \Phi_{mk}^{-1}\} \tag{15}$$

To make the derivative vanish, we obtain

$$\Phi_{nm} = \hat{\Phi}_{nm} \tag{16}$$

and then the maximum value of L is obtained as

$$L_{\max} = \frac{e^{-M}}{(2\pi \Delta f)^M |\Phi|} \tag{17}$$

Equation (16) means that the cross-power spectra Φ_{nm} should agree with the quantity which is obtained by operating a rectangular filter to the periodogram.

2.3 Estimation of parameters in standard directional spectrum

Once we choose a standard directional spectrum, the components of the cross-power spectrum matrix are not independent of each other. The cross-power spectrum matrix, Φ_{nm}, is now a function of the parameters, λ_i (i=1 to I), in the standard directional spectrum through Eq. (4) and is written as $\Phi_{nm}(\lambda_i)$. Hence, to determine the value of λ_i by the maximum likelihood method, we take the derivative of Eq. (9) with respect to λ_i (i=1 to I) and make every derivative vanish.

The derivative of the likelihood function with respect to λ_i can be obtained by the chain rule as

$$\frac{\partial L}{\partial \lambda_i} = \sum_{k=1}^{M}\sum_{l=1}^{M} \frac{\partial L}{\partial \Phi_{kl}} \frac{\partial \Phi_{kl}}{\partial \lambda_i} \tag{18}$$

On substituting Eq. (15) into the above equation and setting the derivative equal to zero, the following equation results:

$$\sum_{k=1}^{M}\sum_{l=1}^{M}\{-\Phi_{lk}^{-1} + \sum_{m=1}^{M}\sum_{n=1}^{M} \Phi_{ln}^{-1}\hat{\Phi}_{nm}\Phi_{mk}^{-1}\}\frac{\partial \Phi_{kl}}{\partial \lambda_i} = 0 \tag{19}$$

From the above equation with i=1 to I, we can determine the values of λ_i (i=1 to I) and then the directional spectrum.

Once λ_i are determined, the value, $\hat{L}_{\max}$, of the likelihood function corresponding to λ_i is calculated. Since the global maximum $L_{\max}$ is given by Eq. (17), the adaptability of the standard directional spectrum can be defined as $\hat{L}_{\max}/L_{\max}$. This is an advantage of the present method.

The concept of the maximum likelihood used in the present study is similar to that used in Hashimoto and Kobune (1988). However, in the present study, the joint normal distribution of the Fourier coefficients is taken into consideration, whereas in Hashimoto and Kobune (1988) the cross-power spectra are assumed to be independent of each other. For example, when two time series data are obtained at very close locations, it is more rational to take into consideration the correlation between the Fourier coefficients as in the present study.

In the numerical calculation of Eq. (19), the Newton-Raphson procedure can be employed. Let the left hand side of Eq. (19) be denoted as

$$f_i(\lambda_{i'}) = \sum_{k=1}^{M}\sum_{l=1}^{M}\{-\Phi_{lk}^{-1} + \sum_{m=1}^{M}\sum_{n=1}^{M} \Phi_{ln}^{-1}\hat{\Phi}_{nm}\Phi_{mk}^{-1}\}\frac{\partial \Phi_{kl}}{\partial \lambda_i} \tag{20}$$

then the second derivatives are expressed as follows:

$$\begin{aligned}\frac{\partial f_i}{\partial \lambda_{i'}} = {} & \sum_{k=1}^{M}\sum_{l=1}^{M}\{-\Phi_{lk}^{-1} + \sum_{m=1}^{M}\sum_{n=1}^{M} \Phi_{ln}^{-1}\hat{\Phi}_{nm}\Phi_{mk}^{-1}\}\frac{\partial^2 \Phi_{kl}}{\partial \lambda_{i'}\partial \lambda_i} \\ & + \sum_{k'=1}^{M}\sum_{l'=1}^{M}\sum_{k=1}^{M}\sum_{l=1}^{M}[-\Phi_{l'k}^{-1}\Phi_{lk'}^{-1} + \{\Phi_{lk'}^{-1}\sum_{m=1}^{M}\sum_{n=1}^{M}\Phi_{l'n}^{-1}\hat{\Phi}_{nm}\Phi_{mk}^{-1} \\ & + \Phi_{l'k}^{-1}\sum_{m=1}^{M}\sum_{n=1}^{M}\Phi_{ln}^{-1}\hat{\Phi}_{nm}\Phi_{mk'}^{-1}\}]\frac{\partial \Phi_{k'l'}}{\partial \lambda_{i'}}\frac{\partial \Phi_{kl}}{\partial \lambda_i}\end{aligned} \tag{21}$$

By the linear approximation of $f_i(\lambda_{i'})$ the following iteration formula can be obtained:

$$\lambda_i^{(j+1)} = \lambda_i^{(j)} - \sum_{i'=1}^{I}[\frac{\partial f_i}{\partial \lambda_{i'}}]^{-1} f_{i'}|_{\lambda_i = \lambda_i^{(j)}} \tag{22}$$

where $\lambda_i^{(i)}$ and $\lambda_i^{(i+1)}$ are the values of λ_i at ith and $(i+1)$th iteration steps. The above formula is repeatedly used until a converged solution is obtained.

2.4 Expression for Mitsuyasu-type directional spectrum

The Mitsuyasu-type directional distribution function is expressed as

$$S(f,\theta) = P(f)\frac{2^{2s+1}\Gamma^2(s+1)}{\pi\Gamma(2s+1)}[\cos\frac{\theta-\theta_o}{2}]^{2s} \tag{23}$$

where $P(f)$ is the frequency spectrum, and Γ represents the gamma function. To estimate the directional spectrum for a certain frequency f, the unknown parameters are P, θ_o and s which are denoted by λ_1, λ_2 and λ_3, respectively.

In the following, we derive the equations for data obtained by a three-component array in which the water surface elevation, η, and two components, u and v, of the horizontal water particle velocity are measured. By denoting $\xi^{(1)} = \eta$, $\xi^{(2)} = u$ and $\xi^{(3)} = v$, the cross-power spectra for Eq. (23) are

$$\Phi_{11} = P(1+\epsilon) \tag{24}$$

$$\Phi_{12} = PH_u m_1 \cos\theta_o \tag{25}$$

$$\Phi_{13} = PH_u m_1 \sin\theta_o \tag{26}$$

$$\Phi_{22} = PH_u^2(\frac{1}{2} + m_2\cos 2\theta_o)(1+\epsilon) \tag{27}$$

$$\Phi_{33} = PH_u^2(\frac{1}{2} - m_2\cos 2\theta_o)(1+\epsilon) \tag{28}$$

$$\Phi_{23} = PH_u^2 m_2 \sin 2\theta_o \tag{29}$$

where

$$m_1 = \frac{s}{(s+1)} \tag{30}$$

$$m_2 = \frac{s(s-1)}{2(s+1)(s+2)} \tag{31}$$

and H_u is the transfer function form the water surface elevation to the magnitude of the horizontal water particle velocity. Though the quantity H_u can be evaluated by the small amplitude wave theory, the following formula obtained from the relationship among the power spectra is used to eliminate the error included in the data.

$$H_u = \sqrt{\frac{P_u(f)+P_v(f)}{P_\eta(f)}} \tag{32}$$

Equations (24), (27) and (28) are expressions for power spectra and are multiplied by $(1+\epsilon)$ on assuming that a noise component with a relative power of ϵ is included in the data. Hence, ϵ becomes the fourth unknown parameter, λ_4, to

be determined by the maximum likelihood method. If another assumption that the noise component is due to a component of the directional spectrum with a uniform directional distribution is adopted, ϵP, $\epsilon P H_u^2/2$ and $\epsilon P H_u^2/2$ should be added to the power spectra Φ_{11}, Φ_{22} and Φ_{33}, respectively. Which assumption should be adopted can be discussed in terms of the adaptability $\hat{L}_{\max}/L_{\max}$.

It is easy to differentiate Eqs. (24) to (29) with respect to λ_i ($\lambda_1 = P, \lambda_2 = \theta_o, \lambda_3 = s, \lambda_4 = \epsilon$) up to the second order and by substituting the derivatives into Eqs. (20) and (21) the right hand side of Eq. (22) can be determined and then modified values $\lambda_i^{(i+1)}$ are calculated. The initial values of λ_i (i=1 to I) are taken to be the power spectrum of η, the mean direction, the value of s obtained from the long-crestedness parameter, and zero, respectively.

Numerical instability sometimes occurred in the algorithm of the original Newton-Raphson method. Hence additional procedures are employed. When modified λ_i gives a smaller value of the likelihood function, the magnitude of the modification by Eq. (22) is made half. When a convergent solution is not obtained within 10 iterations, the procedure is continued by neglecting the second derivatives in Eq. (21). This modification made the numerical calculation convergent for all the cases tried.

3. Application

3.1 Field experiment

The data used in application of the present theory were obtained from three series of field experiments. Sets of an ultra-sonic wave gage and an electromagnetic current meter were installed at the sea bottom. The first and second field experiments were performed from 1 to 2 of October, 1987, and from 29 to 30 of August, 1988, respectively, at Oarai beach, Ibaraki prefecture, Japan, facing the Pacific Ocean. One segment of data record contains 2046 data with the sampling frequency of 0.5s, and 95 and 75 segments were recorded in Oarai 1987 and 1988 experiments, respectively. The bottom topography and the arrangement of the instruments are shown in Fig. 1 for the Oarai 1988 experiment, as an example. The third experiment was performed from 11 to 18 of January, 1989, at Ogata beach, Niigata prefecture, Japan. This experiment was performed as a part of a comprehensive, inter-institutional field experiment, SWAN 89 (Sediment, Wave and Nearshore Circulation Observation, 1989), managed by Professor Tsuchiya, Kyoto University. Data segments with a 17 min. duration were recorded every one hour and total number of segments is 134. The average wave statistics during the observation are shown in Table 1.

3.2 Result of data analysis

Assumptions of noise components are examined first. Figure 2 shows the

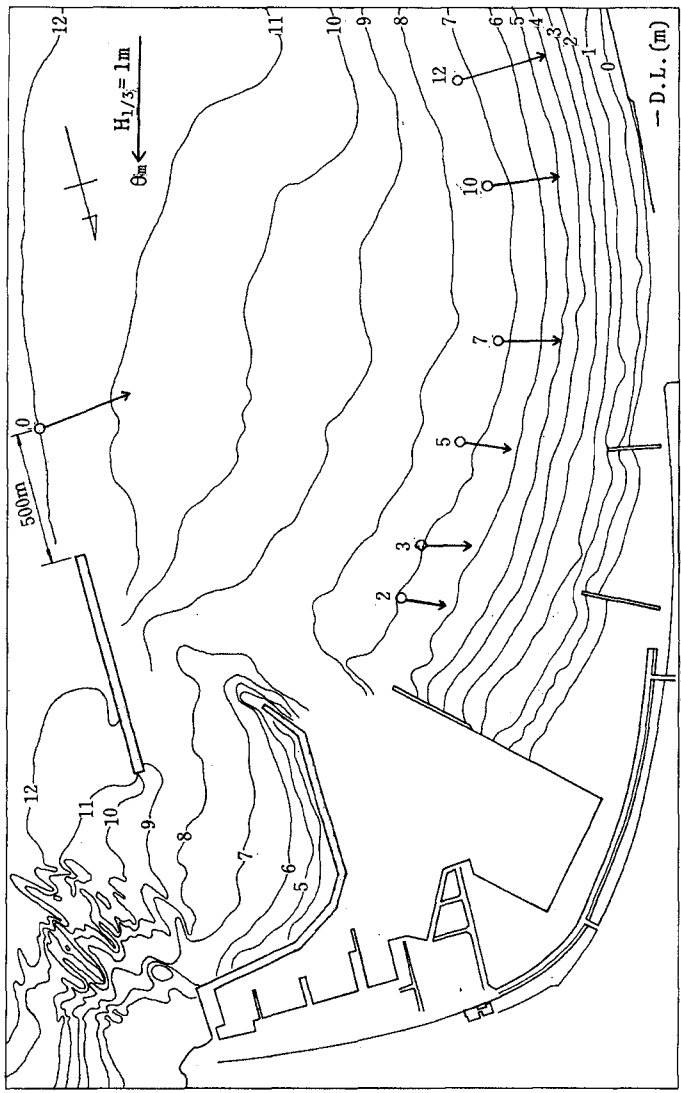

Fig. 1 Arrangement of measuring points (Oarai, 1987.10.01-02).

Table 1: Wave statistics during field experiments.

Observation	Measuring point	h (m)	$H_{1/3}$ (m)	$T_{1/3}$ (s)	$\bar{\theta}_m$	$\bar{\gamma}$
OA87	0	12.5	0.63	6.3	174	0.61
OA88	1	13.2	1.09	7.4	165	0.43
	2	12.1	-	-	-	-
	3	10.9	1.04	7.5	175	0.40
	4	7.3	1.02	7.6	-179	0.33
OG89	1	25.0	1.67	6.8	-77	0.44
	2	15.3	1.31	6.5	-51	0.42
	3	11.3	1.19	6.2	-50	0.40

OA: Oarai, OG: Ogata, h: water depth,

$H_{1/3}$: significant wave height, $T_{1/3}$: significant wave period,

$\bar{\theta}_m$: mean direction measured anti-clockwise from east,

$\bar{\gamma}$: long-crestedness parameter.

result of data analysis for three assumptions of noise component. From the top to bottom figures, average values during the observation are shown for the frequency spectrum, $P(f)$, the peak direction, θ_o, the peakedness parameter, s, and the adaptability, $\hat{L}_{max}/L_{max}$, are plotted. In each figure, the solid line corresponds to the assumption of the noise component proportional to the frequency spectrum, and the dotted line to the assumption of the noise component with uniform directional distribution. The broken line represents the result when the noise component is neglected. Without noise component, the adaptability $\hat{L}_{max}/L_{max}$ decreases significantly. The adaptabilities are almost the same for the two assumptions of noise component, but the difference of the estimated s between the two assumptions is large. Since the assumption of noise component proportional to the frequency spectrum gives a slightly higher adaptability and gradual increase of s with decreasing water depth, this assumption is adopted in the following analysis.

In the top of each figure through Figs. 3 to 6, P is the frequency spectrum, ϵP the noise component, and P_η the frequency spectrum of the water surface elevation. In the second figure, θ_o denotes the estimated peak direction. The symbols θ_m and θ_p represent the mean and principal directions, respectively, which are calculated from the power and cross spectra (Horikawa, *ed.*, 1985). In the third figure, s is the estimated peakedness parameter, and s_γ represents s

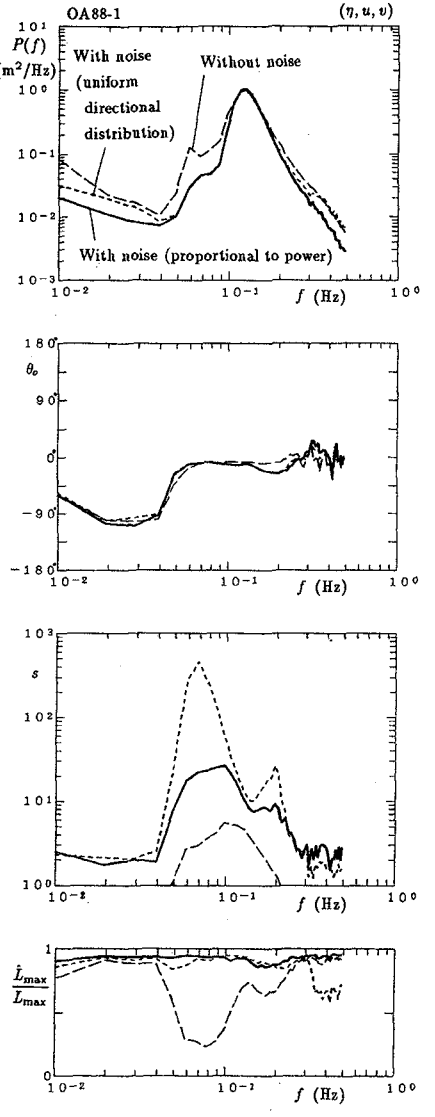

Fig. 2 Effect of formulation of noise component on directional spectrum estimation.

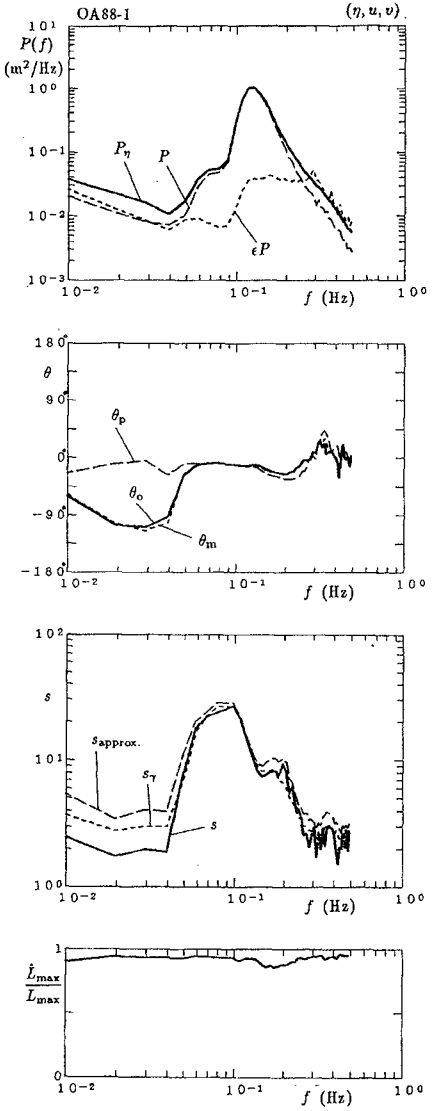

Fig. 3 Result of directional spectrum estimation (1).

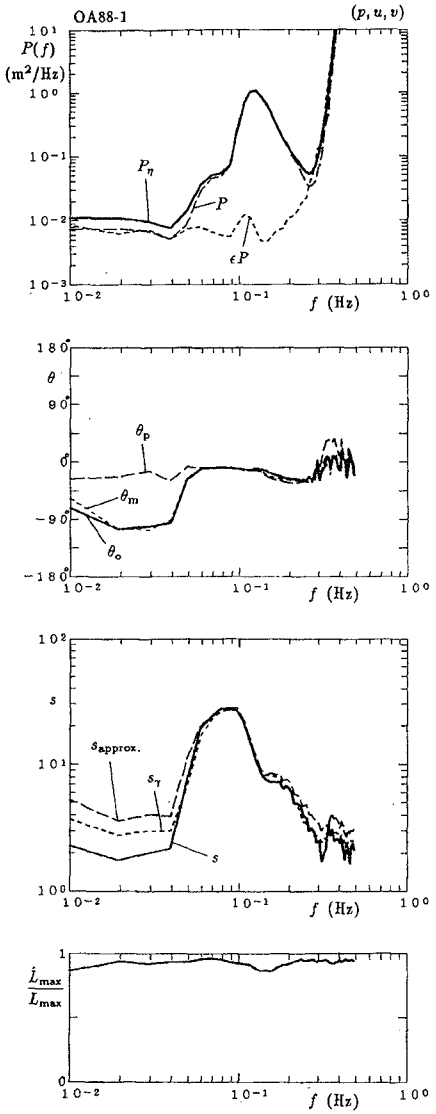

Fig. 4 Result of directional spectrum estimation (2).

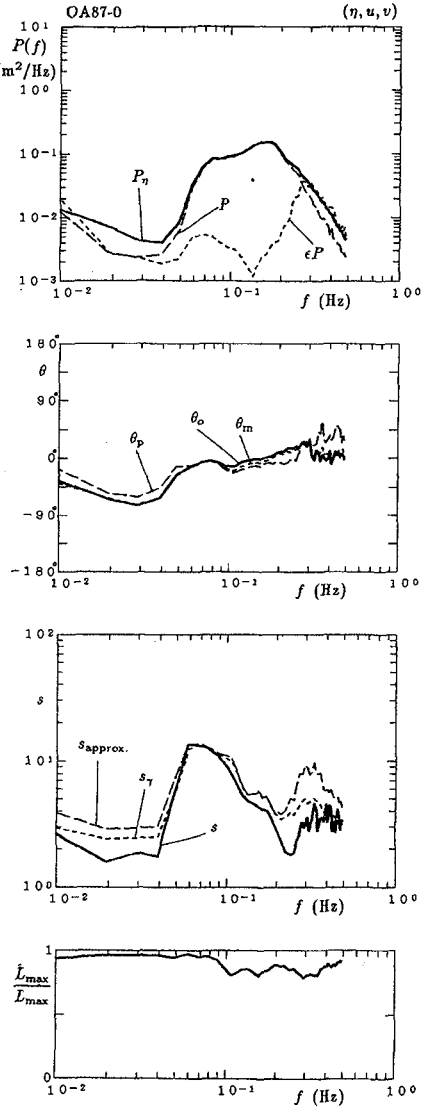

Fig. 5 Result of directional spectrum estimation (1).

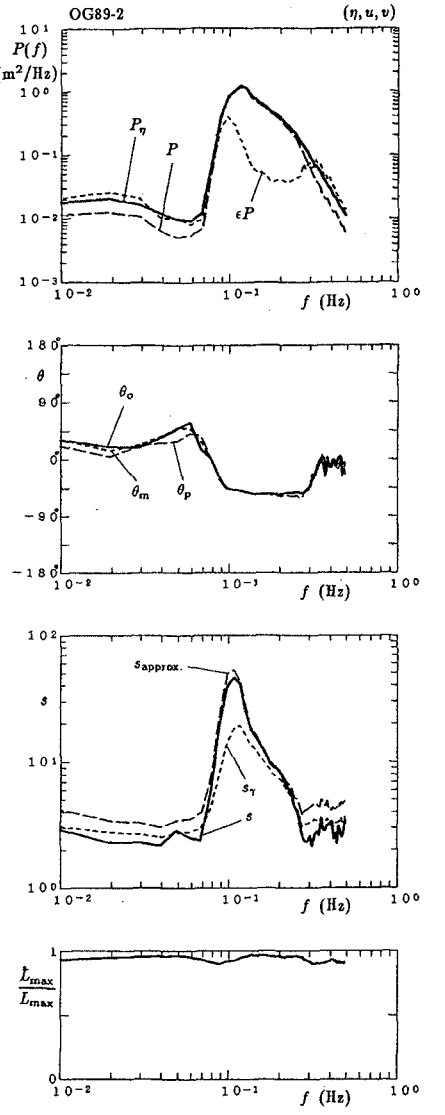

Fig. 6 Result of directional spectrum estimation (2).

calculated by the following equations:

$$\gamma^2 = \frac{\hat{\Phi}_{20} + \hat{\Phi}_{02} - \sqrt{(\hat{\Phi}_{20} - \hat{\Phi}_{02})^2 + 4\hat{\Phi}_{11}^2}}{\hat{\Phi}_{20} + \hat{\Phi}_{02} + \sqrt{(\hat{\Phi}_{20} - \hat{\Phi}_{02})^2 + 4\hat{\Phi}_{11}^2}} \tag{33}$$

$$\gamma^2 = \frac{2s+1}{s^2+s+1} \tag{34}$$

Equation (33) is the definition of the frequency-dependent long-crestedness parameter and Eq. (34) gives the relationship between the long-crestedness parameter and the peakedness parameter. The quantity $s_{\text{approx.}}$ is an approximation of s which is obtained by substituting γ_α for γ in Eq. (34). The quantity γ_α is calculated by the following equations:

$$(1 - \frac{\gamma_\alpha^2}{\gamma^2}) = \alpha \frac{\sin^2 2\theta_{\text{m}}}{4 - 2\sin^2 2\theta_{\text{m}}} (1 - \frac{\gamma'^2}{\gamma^2}) \tag{35}$$

$$\alpha = 0.859 - 0.174\gamma^2 - 0.104 \sin^2 2\theta_{\text{m}} \tag{36}$$

where γ is calculated by Eq. (33) and γ' denotes the value of γ which is calculated from Eqs. (34) and (37).

$$\frac{s}{s+1} = \sqrt{\frac{\hat{\Phi}_{10}^2 + \hat{\Phi}_{01}^2}{\hat{\Phi}_{00}(\hat{\Phi}_{20} + \hat{\Phi}_{02})}} \tag{37}$$

For Fig. 4, the pressure variation is used instead of the water surface elevation. The small amplitude wave theory is used to determine the transfer function from the water surface elevation to the pressure variation. The results are almost the same as those shown in Fig. 3 in which the water surface elevation is used.

Figures 5 and 6 shows the results for Oarai 1987 and Ogata 1989 experiments, respectively. From these figures, it can be concluded that the estimated value P of the frequency spectrum, the peak direction θ_{o}, and the peakedness parameter s agree well with the power spectrum P_η of the water surface elevation, the mean direction θ_{m}, and the approximated value $s_{\text{approx.}}$, respectively. Since the latter parameters are very easy to calculate, the computational time can greatly be reduced.

In the frequency spectrum, the high frequency range seems to follow the -4 power law. The maximum s occurs at the frequency slightly lower than the peak frequency and the dependence of s on the frequency is different from the formula proposed by Mitsuyasu *et al.* (1975). Accumulation of much more data is necessary for a further and more reliable discussion.

4. Conclusion

A general procedure is given on the basis of the maximum likelihood method for estimating the parameters included in a standard directional spectrum. As

an example, formulas are given for estimating the parameters in the Mitsuyasu-type directional spectrum from the data of the water surface elevation and two components of the horizontal water particle velocity. The present method is applied to the data obtained by three series of field experiments. It is found that the parameters in the Mitsuyasu-type directional spectrum can be approximated by the parameters which can be calculated with much less computational time. Accumulation of much more data is necessary to discuss the characteristics of directional sea states such as the functional dependence of the peakedness parameter s on the frequency. The appropriate assumption on the noise component remains to be studied in the future work.

Acknowledgement

This study is financially supported by the Grant-in-Aid for Scientific Research, Ministry of Education, Science and Culture.

Reference

Hashimoto, N., and K. Kobune (1988): Estimation of directional spectrum through a Bayesian approach, Coastal Eng. in Japan, Vol. 31, pp. 183-198.

Horikawa K., (*ed.*) (1988): Nearshore Dynamics and Coastal Processes, Univ. Tokyo Press, 522p., Part V, Chap. 3, pp.407-422.

Kobune, K., and N. Hashimoto (1986): Estimation of directional spectra from the Maximum Entropy Principle, Proc. 5th Int. Offshore Mech. and Arctic Eng. Symp., Tokyo, pp. 80-85.

Isobe, M., K. Kondo, and K. Horikawa (1984): Extension of MLM for estimating directional wave spectrum, Proc. Symp. on Description and Modelling of Directional Seas, Copenhagen, pp.A-6-1 - A-6-5.

Mitsuyasu, H. *et al.* (1975): Observation of the directional spectrum of ocean waves using a cloverleaf buoy, J. Phys. Oceanogr., Vol. 5, pp. 750-760.

Rice, S. O. (1944): Mathematical analysis of random noise, Bell Syst. Tech. J., Vol. 23, pp. 282-332.