CHAPTER 39

WAVE KINEMATICS IN THE SURFACE ZONE

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Abstract

An extensive series of laboratory experiments have been carried out with regular and irregular waves travelling over a horizontal bottom. Velocity measurements of the flow were obtained at numerous vertical positions with major emphasis given to the surface zone. The flow measurements were made possible by use of a custom designed LDV having the special characteristic of being operative very near the free surface. Presented is an overview of the study and some of its majors results.

1 Introduction

The study of the flow within water waves has a long history, yet, it seems, some of the fundamental aspects of the phenomenon are poorly or not satisfactorily understood. Specifically, how reliable is current knowledge on details of wave surface shape, velocity field, induced current, or effects of ambient current over the spectrum of wave heights, periods, and water depths? Even for regular waves, let alone irregular waves, the answers to this question are far from complete. For more complex situations, say, for instance, three-dimensional ocean waves, the present methods for predicting the flow field are, at best, dubious. Clearly the problem is difficult and is one that deserves continued yet careful study—even at the most fundamental level.

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Presently, designers for the offshore environment are often faced with the problem of using flow descriptions, and hence loading information, that might be of uncertain reliability. For fixed structures, one can take steps to alleviate the uncertainties by use of a safety factor and/or a wave theory that has proven to be conservative. Although this approach may solve the problem it can add substantially to overall development costs. For structures affected dynamically by waves, the problem is more acute; an overly conservative approach may render a design impractical due to excessive motion, for instance, when, in fact, if more realistic knowledge of the flow were known, the design might be completely acceptable.

The number of studies concerning measurement of wave kinematics is few and their scope often limited. These facts are a reflection of the difficulties, partially described above, that one encounters in such endeavors. Recent reports on field measurements and comparison to theory have been made, e.g., Lambrakos (1981), Forristall (1986), and one on the WADIC experiment presented at this forum. Among others, laboratory measurements of wave kinematics have been reported by Vis (1980), Bosma and Vugts (1981), Anastasiou *et al.* (1982), Bullock and Short (1985), and Gudmestad *et al.* (1988). Though each of these reports contain interesting and useful information, they still lack unification and presentation of sufficient evidence in a way that, say, designers can utilize the information with confidence. This is a problem, and in fact it is quite unlikely that generalized procedures to tackle the wave kinematics issue will be available soon.

The primary intent of this paper is to present examples of and discuss an extensive series of laboratory experiments that have been carried out with regular and irregular waves travelling over a horizontal bottom. Velocity measurements of the flow were obtained at numerous vertical positions with major emphasis given to the surface zone. The flow measurements were made possible by use of a custom designed LDV having the special characteristic of being operative very near the free surface.

2 Theoretical and Experimental Considerations

Linear Random Wave Theory. As a first approach to the problem of irregular waves, one can assume the flow to be composed with the sum of many individual *linear* waves each with its own frequency, direction, phase and celerity as predicted by the linear dispersion relation. This technique is often referred to as *linear random wave theory* (see Forristall, 1981 for a description). Bosma and Vugts (1981) performed experiments using one-dimensional irregular waves and reported on comparisons of velocity measurements obtained by laser Doppler velocimetry to linear random wave theory. They show that the agreement far

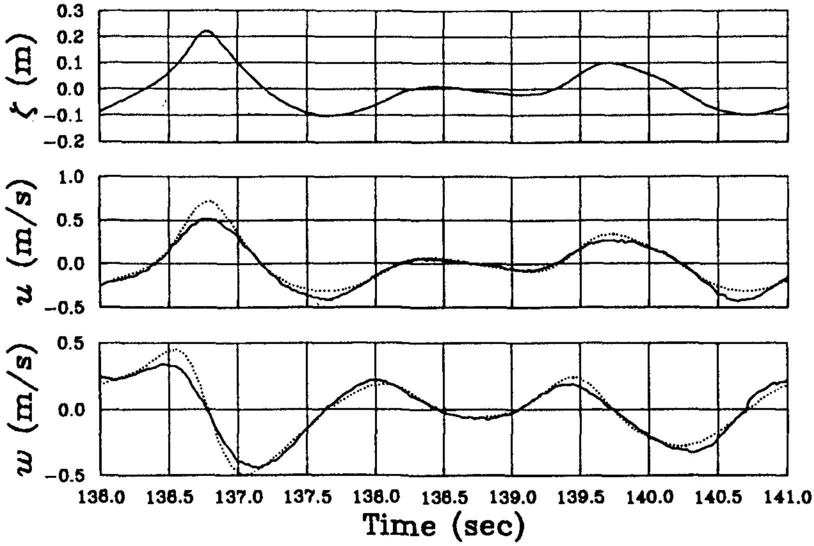

Figure 1 Example of linear random wave theory, — measurement, ··· linear random wave theory (Case 6, $z = -0.10\,\mathrm{m}$).

below the mean water level is reasonable, but tends to deviate increasingly with increasing elevation. This tendency of deviation can be explained, at least partially, by the significance of the higher wavenumber components *riding atop* the lower wavenumber components and hence may be displaced many of their own wavelengths from the mean water line. For large wavenumber, k, the amplitude of the velocity contribution is proportional to e^{kz}, where z is the coordinate measured vertically upward from the mean water line. The higher wavenumber contributions can be become excessively large when $z > k^{-1}$ and similarly, significantly diminished when $z < -k^{-1}$. The net effect is that contributions from the higher wavenumber components will be over-represented in crests and under-represented in troughs. A good example of this can be seen in Figure 1. Shown are measurements of ζ, u, and w, the surface elevation, and the horizontal and vertical velocities respectively. The dotted line is the velocity obtained using linear random wave theory. The velocity shown in Figure 1 was measured at 0.1 m below the mean water line using the facility described in the next section; the wave conditions correspond to Case 6 of Table 1. We see significant departure of measurement from theory in the crests and troughs, on the other hand when ζ is near zero the comparison is better. This latter feature should be expected if one believes in the linear superposition of waves since all wave components are, at least momentarily, at the elevation that linear theory presumes they are, namely $z = 0$, the mean water line.

Stretching Approximations. The notion of predicting kinematics beneath irregular waves from a spectral decomposition of the surface elevation, such as that done with linear random wave theory, is very attractive for its simplicity and speed of computation. Suggestions to cope with the problems inherent with linear wave theory have been made by Chakrabarti (1971) and Wheeler (1979), though both approaches fail to satisfy the Laplace equation Forristall (1981) mentions that use of Wheeler's method results in a lower error in the kinematics boundary condition when compared to linear random wave theory. With Wheeler's approach the variable z in the linear solutions for flow velocity is substituted as follows:

$$z \leftarrow \frac{z - \zeta}{1 + \zeta/d} \quad , \tag{1}$$

where d is the undisturbed water depth. The new expression for z is never greater than zero and for each wavenumber will effectively *stretch* or *compress* the velocity profile from the mean water line to the instantaneous free surface. For the case of irregular waves the higher wavenumber components, which tend to be of lesser amplitude than the low wavenumber components they ride atop, will be more reasonably represented. Taking the same example as shown in Figure 1 and using Wheeler's approach, we see in Figure 2 that a better

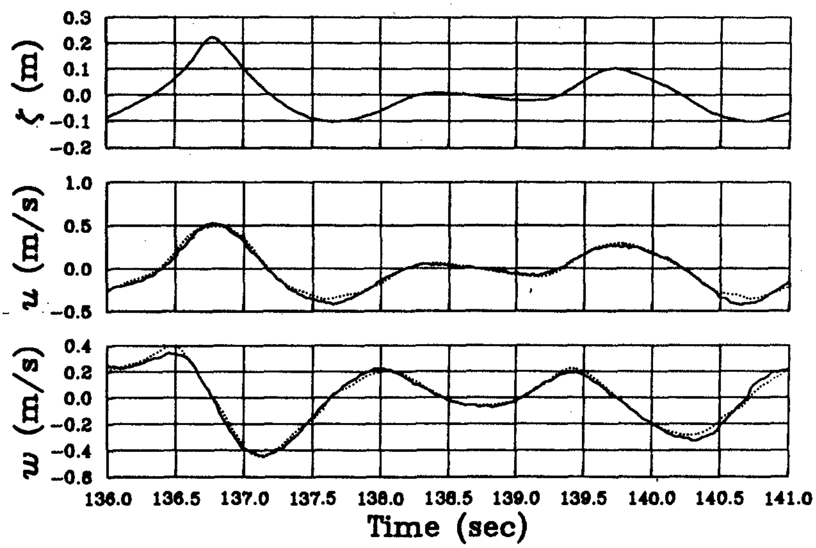

Figure 2 Example of a stretching approximation, — measurement, ··· Wheeler's method (Case 6, $z = -0.10\,$m).

approximation to the measured velocity over linear random wave theory is obtained. Still there are differences and this is but one comparison. The velocity in this case was computed using similar steps as for linear random wave theory; the procedure is complicated somewhat by the fact that ζ now appears in the equations for flow velocity such that it is not possible to form a simple transfer function relating surface elevation information to the flow velocity beneath.

3 Experimental Equipment and Procedures

The experiments for this investigation were carried out using the wave tank illustrated in Figure 3, which is located at the Norwegian Hydrotechnical Lab-

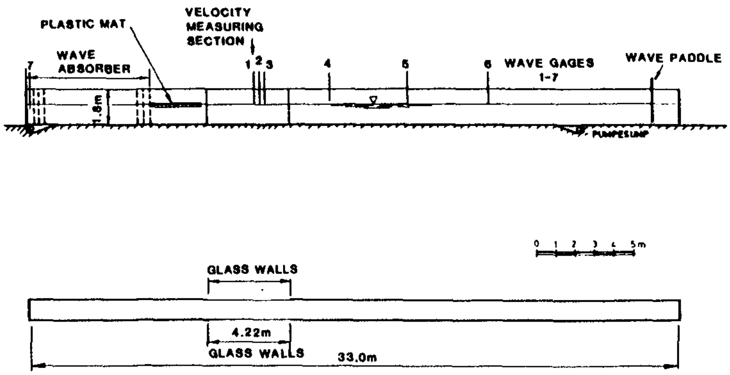

Figure 3 Wave Tank.

oratory in Trondheim. The tank is 33 m long, 1.02 wide and 1.8 m deep. It is constructed of concrete with a glass section 4.22 m wide, located approximately 10 m from the end of the tank, that allows viewing over the full depth. The wave generator is hydraulically driven and can be discretely varied from a pure hinge mode to a pure piston mode. At the end of the tank, opposite the wave generator, is located a passive wave absorber developed by the National Research Council of Canada. This device consists of a series of vertical perforated steel plates and has a reflection coefficient of approximately 5% over a broad frequency range that encompassed the range of frequencies with significant wave energy typical in this study. (For more details on this device see Jamieson and Mansard, 1987). Just ahead of the absorber, in the direction of the wave generator, is a positively buoyant mat 2.5 m in length that floats on the water surface. Usage of the mat significantly reduces high frequency reflections from the absorber and helps to reduce cross waves.

Flow velocity was measured on the centerline at a single longitudinal position along the tank, coinciding with gage 1, but at several different elevations (see next section) by a two-component laser Doppler velocimeter (LDV). The two velocity components measured were in a plane parallel to the side walls of the tank with a measurement volume cross-section of approximately 100 μm in diameter. Frequency shifting was utilized enabling the resolution of directional ambiguities in the flow. The LDV was specifically designed for this study and has the special feature of using only a *single* laser beam in the flow. This feature allows measurements very near arbitrarily oriented surfaces, including the free surface. This device is similar to and based on one described by Skjelbreia (1987).

The surface elevation was measured with standard resistive-type gages having streamlined supports to minimize disturbances. One wave gage was position at the LDV station, it was displaced from the wave tank's centerline 0.34 m to avoid disturbance to the flow in the vicinity of the LDV measurement point. The wave generator control signal was constructed from a JONSWAP target wave spectrum using $\gamma = 3.0$. The spectrum was divided into 1000 frequency components and each component assigned a random phase. Great care was given to maintaining reproducible wave conditions in the tank. This was necessary to construct a *snap-shot* of the flow throughout the depth since the LDV is capable of only a single point measurement. Careful control of the water depth was found to be very important for reproducibility and it was maintained to within ± 1 mm by an overflow connected by tubing to the tank and by having a water inflow of 0.1 l/min into the tank to offset small leaks in the overall system.

4 Test Program

The complete test program consisted of nine wave conditions, six irregular wave cases, two regular wave cases, and one case that the control signal to the wave generator was constructed from two sinusoids, these are listed in Table 1. In total there were 269 runs recorded.

Listed in Table 1, the peak period, T_p, is the value used to create the control signal for driving the wave generator and the significant wave height, H_s, was determined from the measured energy spectrum at the LDV station. For the two period case, the values for H_s/gT_p^2 and d/gT_p^2 are based on the average of the two periods and for the regular wave cases, T_p and H_s are simply T and H, the period and waveheight.

Velocity measurements of the flow were obtained at as many as fifteen separate elevations for each of the nine cases considered. The majority of the elevations were clustered in the surface zone with a few covering the flow on down to the bottom.

Table 1 Test Program.

Case	Type	T_p (s)	H_s (m)	d (m)	H_s/gT_p^2	d/gT_p^2
1	Irregular	1.2	0.11	1.3	0.0078	0.092
2	Irregular	1.4	0.16	1.3	0.0083	0.068
3	Irregular	1.65	0.17	1.3	0.0064	0.049
4	Irregular	1.65	0.17	0.6	0.0064	0.022
5	Irregular	1.8	0.21	1.3	0.0066	0.041
6	Irregular	2.4	0.25	1.3	0.0044	0.023
7	Two Per.	2.1&2.4	0.18	1.3	0.0036	0.026
8	Regular	1.5	0.26	1.3	0.0118	0.059
9	Regular	1.5	0.23	0.6	0.0104	0.027

5 Presentation and Discussion of Results

The results to be presented have been selected from Case 6 listed in Table 1. There have not been any adjustments to the velocity measurements to account for a current that may be present or to remove the effects of reflected or resonant waves. Instead, all measurements presented are the raw, unfiltered information that was obtained by the wave gages and the LDV. A note on the flow velocity time traces to be presented: measurements near the surface have varying degrees of intermittent behavior depending on what level they were obtained. This occurs when the water surface drops below the probe volume of the LDV and the measurement ceases. Until the water surface moves back up to the level of the LDV the signal holds at the last measured value, these ***drop-out*** periods will be noted as the flat portions in the time traces. Occasionally when the water surface just crosses the LDV level, a spike in the measurement occurs, these should not be interpreted as valid measurements.

Irregular Waves. We will present a few comparisons to contrast the differences between linear random wave theory and Wheeler's method. In Figure 4 are shown measurements taken from Case 6 of the horizontal velocity at elevations throughout the depth together with predictions using linear random wave theory. The upper measurements illustrate the well know fact of the unsuitability of linear random wave theory in the surface region (the ordinates for z = 0.20, 0.10, and 0.05 m have multiplying factors of $10^{21}, 10^9$, and 10^3 respectively). Beginning at $z = 0.00$ (mean water level) we note the near 250% over-prediction of velocity beneath the crest located at 136.7 s. The surface is relatively steep about this crest and hence has energy at the higher wavenumbers as compared to other portions of the signal where there is not the same

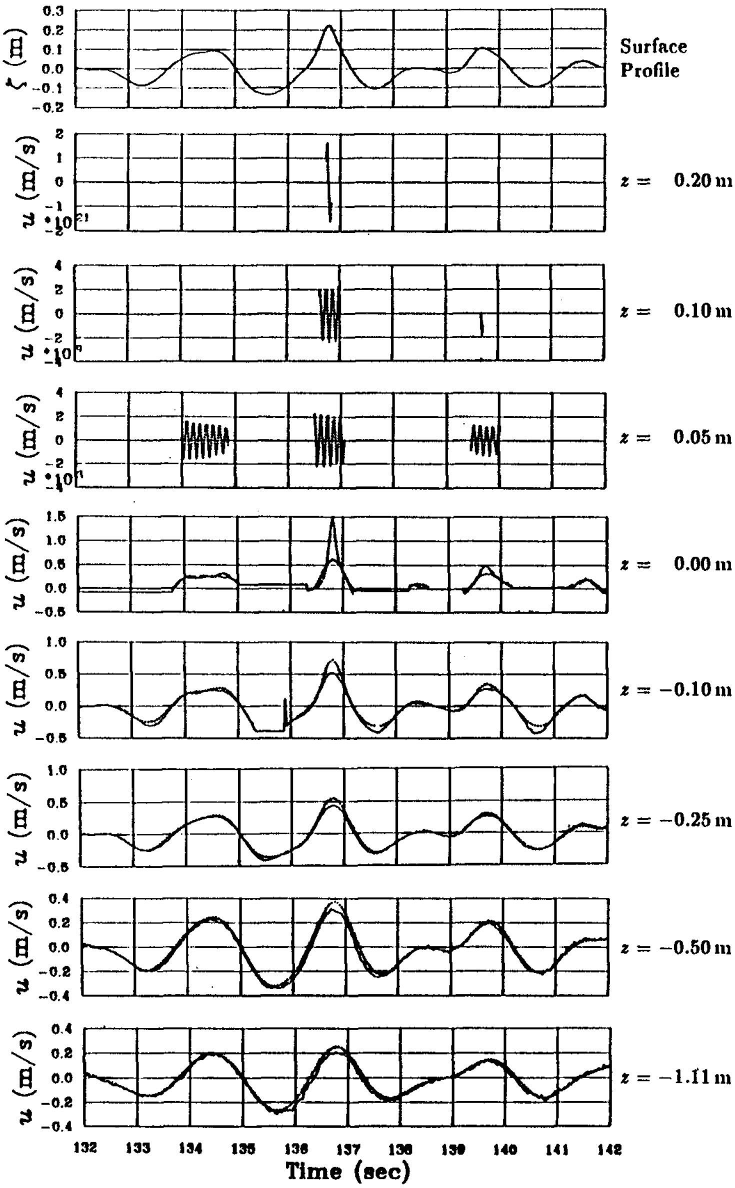

Figure 4 Measurements (—) of surface elevation and horizontal velocity compared with linear random wave theory (···), Case 6.

degree of error. The measurement level at $z = -0.10$ m is beneath nearly all the troughs in this segment, note that the velocity is consistently under-predicted in magnitude. Moving further down the comparison improves, still the velocity at $z = -1.11$ m under the crest at 136.7 s is over-predicted by 25%.

In Figure 5 we move on with the same case and time segment as the previous figure but now the horizontal velocity is compared with Wheeler's method. There is general improvement in the comparison. In the surface region reasonable values are now predicted and the actual measurement have become visible. In particular, the velocity measurements beneath the crest at 136.7 s are all within 8% of the predicted values with the exception of the two measurements nearest the bottom. The trough values at $z = -0.10$ m are under-predicted as was the case with linear random wave theory. There are a number of interesting features we can observe in this figure, for instance note the rapid increase in horizontal velocity as the surface is approached in the crest located at 136.7 s, pointing out the importance of wave steepness and the need for measurements near the surface. A good example of the frequency dependence of the velocity can be seen by comparing the wave with its crest at 134.5 s to the one with its crest 136.7 s. For example the measurement made at $z = 1.11$ m shows the horizontal velocity in each wave to be similar while the surface profile indicates that one wave has nearly twice the waveheight as the other. One can see this type of feature elsewhere in the signal as well.

To complete our comparison, shown in Figure 6 are measurements of vertical velocity and predictions according to Wheeler's method corresponding to the same time segment as in the two previous figures. In general, the comparison is good. The maximum vertical velocity is consistently over-predicted just ahead of the crest at 136.7 s in the surface region, $z = 0.05$ to -0.10 m. At $z = -1.11$ m the magnitude of the flow is low and near the resolving level of the LDV, the jaggedness in the signal is not necessarily turbulence. In contrast to the horizontal velocity, the vertical velocity is better predicted at this level.

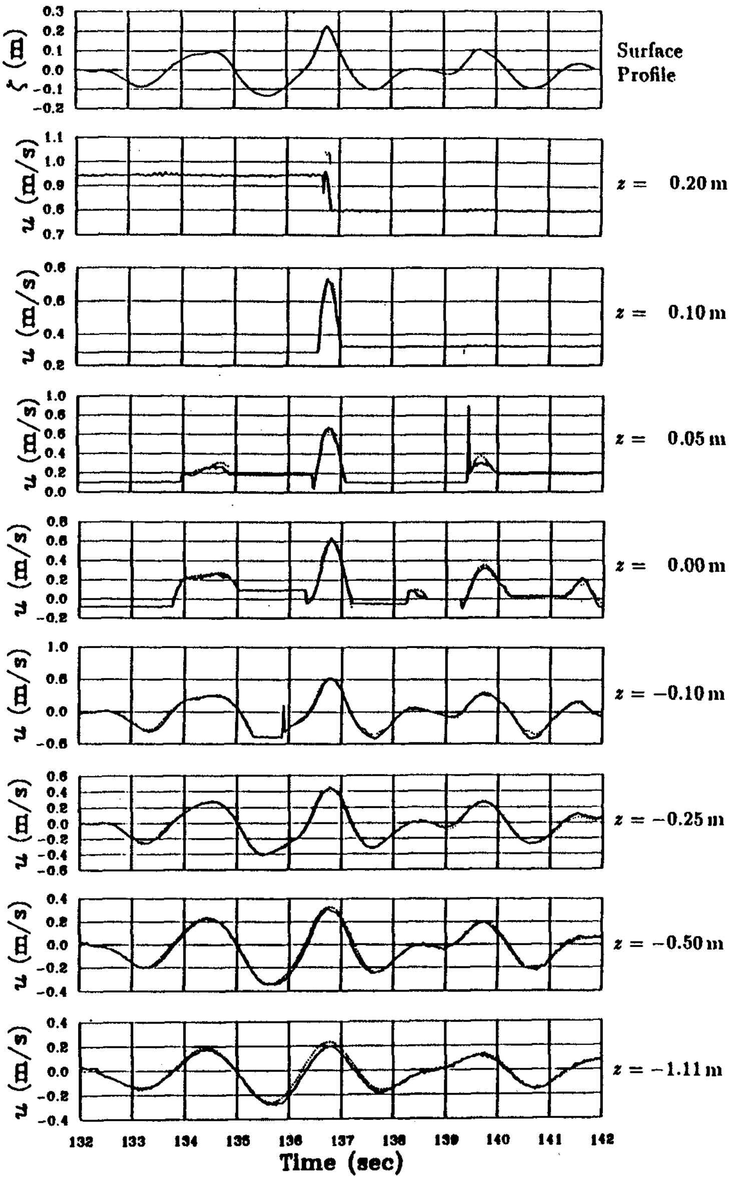

Figure 5 Measurements (—) of surface elevation and horizontal velocity at various levels compared with Wheeler's method (···), Case 6.

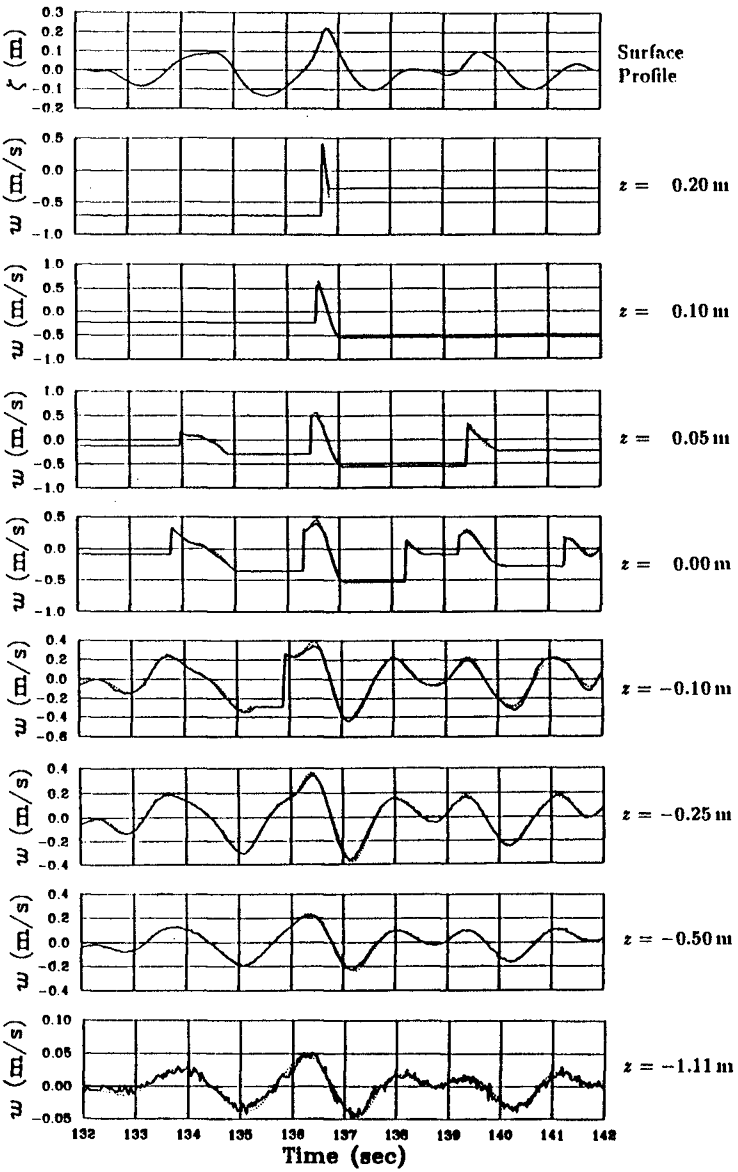

Figure 6 Measurements (—) of surface elevation and vertical velocity at various levels compared with Wheeler's method (···), Case 6.

6 Conclusions

Presented has been an overview of, what we consider, a systematic and well planned series of experiments to investigate the wave kinematics problem. Preliminary analysis has shown Wheeler's method to compare well with the presented measurements, including a regular wave case during the early stages of a run.

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