CHAPTER 33

2ND ORDER WAVE GENERATION

AND APPLICATION TO SHOALING INVESTIGATIONS

by

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ABSTRACT

Model tests have been carried out in a wave channel with and without consideration of 2nd order wave components in the control signal for the wave paddle. The wave channel was equipped with a 1:30 sloped bottom starting 57.00 m in front of the paddle. The water depth was 1.00 m during all tests.

The Self-Correcting System was proved to be able to suppress unwanted free higher and lower harmonics during the generation process.

On the slope measured amplitudes of the 2nd order waves were lower than predicted by 2nd order theory. Commonly used wave height parameters are not influenced significantly by the type of control signal.

1. Introduction

A large number of shoaling investigations have been carried out in the last two decades. Starting with pure regular wave investigations, later on tests were performed with irregular seastate and finally with directional spectra in the increasing number of 3D wave basins in the world. On the other hand with the increasing amount of computer hardware power available, software tools were developed to calculate shoaling phenomena in 2D or 3D models. But still there is a lack of information concerning the behaviour of non-linear waves generated in a wave channel either in a horizontal section and especially over a sloping bottom.

This paper deals with the generation of 1st and pertinent 2nd order waves in a wave channel. A technique is presented leading to proper 2nd order wave components without unwanted free waves in the horizontal section of the channel. Measured values of 2nd order wave amplitudes and commonly used wave height parameters are plotted for the shoaling area.

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2. 2nd Order Wave Theory for Wave Groups and Spectra

The theoretical solution of 2nd order wave phenomena is referred to SAND (1982,"Long Wave Problems ...") and SAND/MANSARD (1986,"Reprod. of Higher Harmonics..."). They present transfer function formulas which allow the computation of the amount of 2nd order wave components in irregular wave trains. Fig. 1 shows a time series of a simple wave group, composed of two frequencies f1 and f2. The extension of 1st order theory to 2nd order shows that additional wave components appear at frequencies f2-f1 (bounded long wave) and f1+f2 (higher harmonic). Furthermore the higher harmonics f1+f1 and f2+f2, known from the regular wave theory, are 2nd order effects related to each basic frequency component. The superposition of 1st and 2nd order waves shows the typical flatter throughs and sharper crests compared to the linear wave train. Lower and higher harmonics are bound to the group of the basic linear waves, i.e. they do not propagate as linear waves with similar frequencies but with the group velocity.

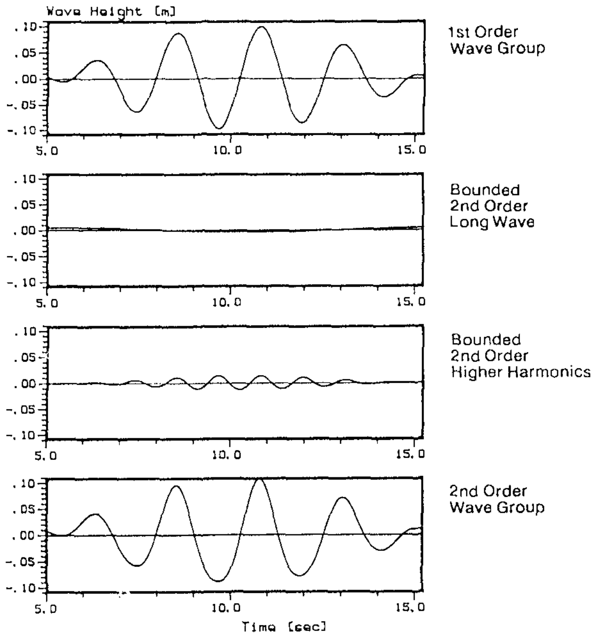
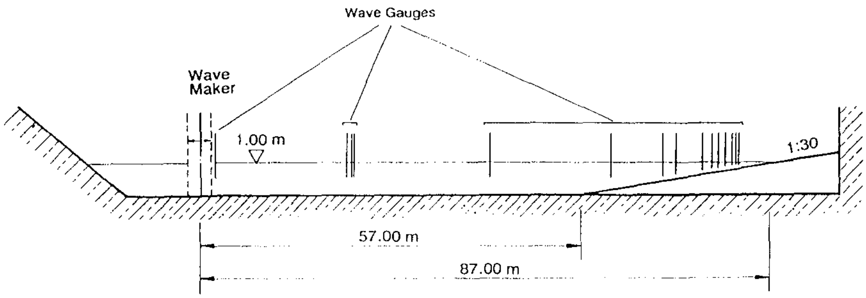


Fig. 1: Example of a Wave Group and Pertinent 2nd Order Components

2nd order waves of an irregular wave train are calculated by superposition of lower and higher harmonics of all possible combinations of frequencies in the 1st order spectrum.

The amplitudes of lower and higher harmonics are calculated from transfer functions given in the above mentioned papers.

When a 1st order wave train is generated in a hydraulic model, a transformation process occurs near the wave paddle generating the group bounded long wave (f2-f1) and also the bounded higher harmonics f1+f1, f1+f2, and f2+f2. During this process also free long waves and free higher harmonics are created which are about 180° out of phase at the paddle compared to the bounded wave components and move independent from the basic linear wave combinations. At a certain frequency (i.e. f2+f1) two waves are existent in the channel: the bounded one which belongs to the 1st order waves and should not be suppressed and the unwanted free one which should be suppressed because it is only due to the technique of wave generation in the model and may cause problems in model test interpretation.



Gauge No.	Distance to Paddle	Water-depth	Gauge No.	Distance to Paddle	Water-depth
[-]	[m]	[m]	[-]	[m]	[m]
1	1.00	1.000	9	78.20	0.360
2	23.01	1.000	10	79.90	0.205
3	23.41	1.000	11	80.92	0.170
4	23.67	1.000	12	81.88	0.135
5	45.22	1.000	13	82.54	0.115
6	63.62	0.750	14	83.03	0.100
7	71.85	0.478	15	83.63	0.080
8	73.94	0.410			

Fig. 2: Experimental Setup

3. Model Test Layout

Fig. 2 shows the experimental setup during the tests in the wave channel at FRANZIUS-INSTITUT. The channel has a horizontal bottom of 57.00 m length and a 1:30 sloped plywood beach at the end. The width of the channel is 2.20 m and the water depth is 1.00 m. The wave paddle was driven in "pusher mode" during all tests. Some wave gauges are installed in the area with the horizontal bottom. The greater part is concentrated on the slope, mainly in the region with low water depth. Test seria contained regular waves, bichromatic waves (groups composed of two frequencies) and spectra with different incident wave steepness and peak enhancement factors.

The sequence length of the wave groups was 20.48 sec and the sequence length of the spectra was 102.4 sec. All signal creation and measurement analysis work was done on a HP1000/A600+ computer equipped with DA/AD converter. The sampling rate of signal input and output was 80 msec in the case of wave groups and 100 msec in the case of spectra. The measurement of the gauge signals starts when the wave paddle starts to move and lasts 4 minutes if wave groups are generated and 6 minutes for wave spectra respectively.

4. Generation of Wave Groups

4.1. Analysis over Test Duration

In order to value the existence of bounded and free waves an FFT-analysis of the measured time seria was performed for several time windows. Fig. 3 shows results from tests with 1st order wave generation. The variation of the measured amplitudes of the frequency component f2-f1 with increasing test duration is plotted in different water depths (f1=0.25 Hz, f2=0.30 Hz). In comparison the theoretical amplitudes according to the formula of SAND applied to the measured components f1 and f2 is shown. The control signal in this test contains no 2nd order amplitude, especially no (f2-f1)-component. It clearly can be stated, that the amplitude of this component is different in space and time dimension. The top frame contains the component at gauge 1 (water depth h=1.00m) near the paddle. The amplitude is about 2 mm, increasing to 15 mm after 70 sec test duration due to reflection. At gauge 4 (h=1.00m) the amplitude increases from 4 to 13 mm and at gauge 9 (h=0.27m), where still no breaking occurs, the amplitude reaches 25 mm and decreases to 18 mm after 100 sec. From the results in the time windows without reflection it can be seen that in the channel a system of two waves, both with the frequency f2-f1, a bounded and a free one, exists. The superposition leads to different resultant amplitudes in space because of different propagation velocities in the channel.

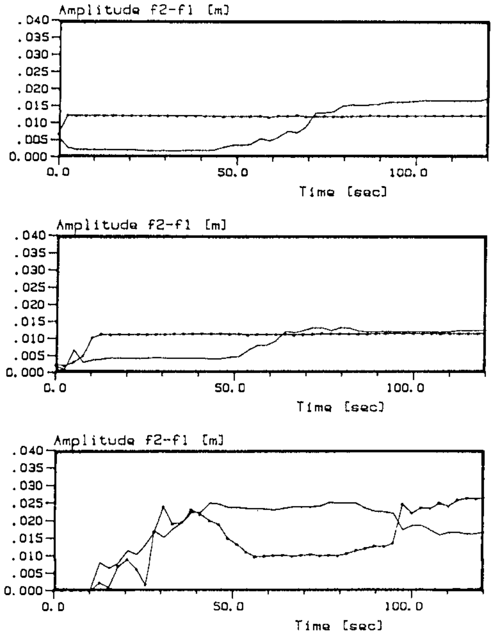


Fig. 3: Amplitude of 2nd Order Frequency Component f2-f1 versus Test Duration without Compensation (f1 = 0.24 Hz, f2 = 0.29 Hz, dotted line: Theory)

A proper generation method should create the bounded lower and higher components without parasitic free waves. Measured amplitudes would then be constant in space and, as long as there is no influence from reflection, in time. Ignoring the theoretical solution of 2nd order theory for computation of the control signal we have used the "Self-Correcting System" (DAEMRICH,EGGERT, 1981) for the generation of the control signal with the higher and lower harmonic components.

The target wave train was computed first with lower and higher harmonics according to the theory. Then a control signal for the wave paddle was calculated using linear hydromechanic transfer functions as a first input. 3 or 4 test runs with the Self-Correcting System were made to find the best fitting of desired and target wave train at the reference gauge. For an easy and definite detection of deviations from the target wave train the reference wave gauge has to be chosen close to the wave paddle. In our test it was chosen to be the gauge 1 m in front of the wavemaker.

Fig. 4 shows the component f2-f1 now with compensation of 2nd order waves in the above mentioned way. Top and middle curve are gauges in the horizontal section of the channel. The third graph is again gauge 9 on the slope. The wave group reaches this point after 30 sec, at that time the amplitude of the component f2-f1 starts to increase.

Amplitudes fit the theoretical values very well in the horizontal section up to the time step 40 sec. The amplitudes of the basic linear components f1 and f2 are stable during the whole test run. The increasing amplitude after 40 sec indicates a reflected wave at the frequency component f2-f1. From our analysis it cannot

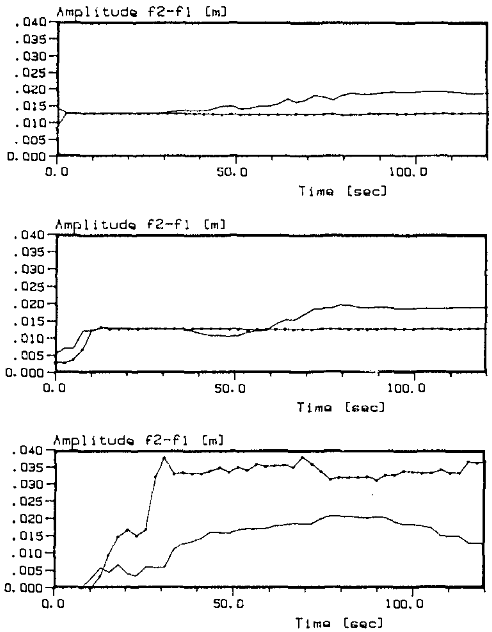


Fig. 4: Amplitude of 2nd Order Frequency Component f2-f1 versus Test Duration with Compensation (f1 = 0.24 Hz, f2 = 0.29 Hz, dotted line: Theory)

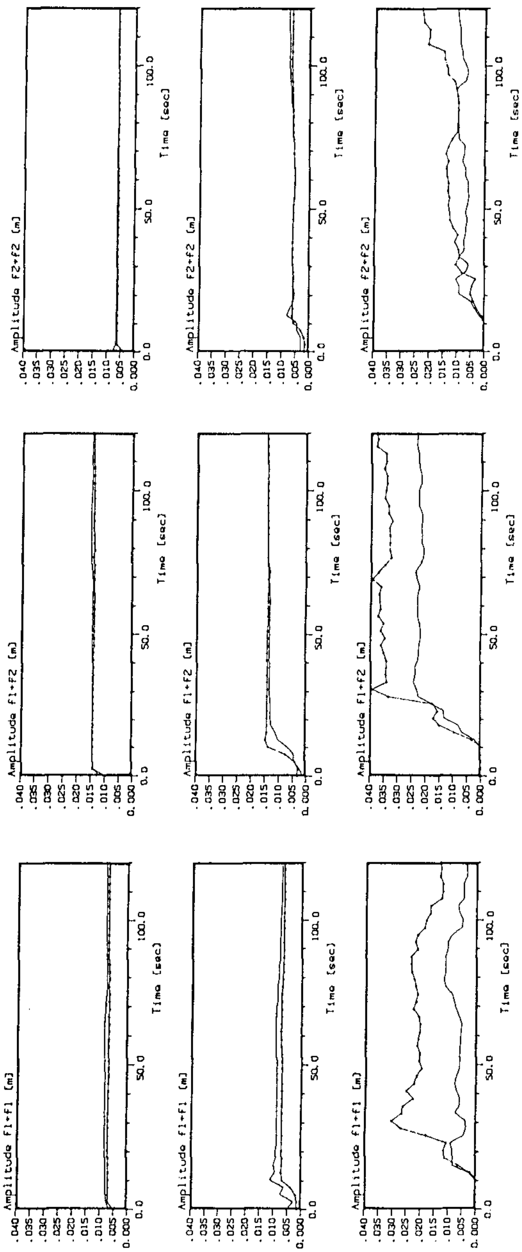


Fig. 5: Amplitude of 2nd Order Frequency Components f1+f1, f1+f2, and f2+f2 versus Test Duration with Compensation (f1 = 0.24 Hz, f2 = 0.29 Hz, dotted line: Theory)

be clearly stated whether it is a reflected bounded long wave or a reflected free wave, created during the propagation of the basic linear waves over the sloping bottom at the end of the channel.

At gauge 9 in a water depth of 0.27 m (bottom curve, Fig. 4) the measured amplitude was always lower then the theoretical one, but both curves are nearly stable over time when the group reaches this point.

The time variation of the amplitudes of the higher harmonics are similar to the variation of the lower component in Fig. 4. In Fig. 5 the higher harmonics are plotted over test duration for the case with 2nd order compensation in the control signal. The measurements fit very well with calculations and are stable over time. In the higher harmonic components almost no free parts are detectable. On the slope the measured amplitudes are always lower than predicted by theory.

4.2. Analysis over Channel Length

In Fig.6 the measured amplitude of the lower harmonic (f2-f1) is plotted over the channel length with (Fig. 6, top) and without (Fig. 6, bottom) compensation of 2nd order waves. The FFT-analysis starts 40 sec after start of the wave paddle movement. Test conditions are the same as in the previous plots. If no free wave component exists in the channel, the curves labeled "theory" and "model test" should be parallel to x-axes and have similar course in the horizontal section of the channel. Without compensation of 2nd order waves the amplitudes differ in space, indicating the existence of both bounded and free components. With compensation nearly no free component is detectable.

In Fig. 7 the 2nd order higher harmonic wave amplitudes are plotted from tests with and without compensation. The tendency is the same as for the lower harmonic: Without compensation of 2nd order waves, the amplitude differs over the channel length, a free wave exists. With compensation almost no free wave is detectable in the horizontal section.

Fig 8 and 9 are similar plots like Fig. 6 and 7 but for a shallow water wave group.

The results reported were obtained from tests with groups that only contain two frequencies. The tests with spectra were performed in a similar way with the Self-Correcting System. However, the principle of generating free and bounded harmonics can be studied easier in

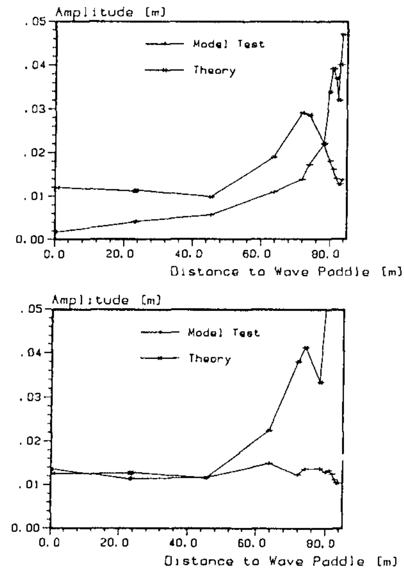


Fig. 6:
Amplitude of 2nd Order
Component f2 - f1
without (top) and with
(bottom) compensation
(f1 = 0.24 Hz, f2 = 0.29 Hz)

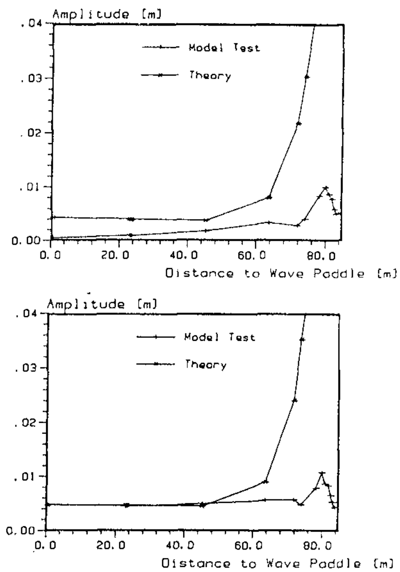


Fig. 8:
Amplitude of 2nd Order
Component f2 - f1
without (top) and with
(bottom) compensation
(f1 = 0.20 Hz, f2 = 0.24 Hz)

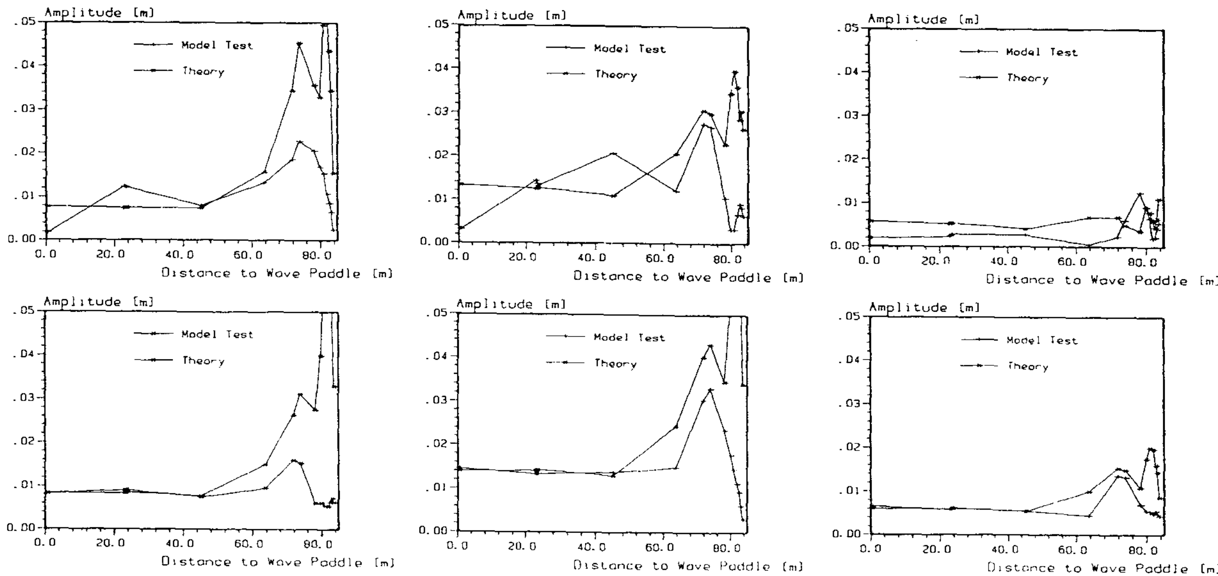


Fig. 7: Amplitude of 2nd Order Component
f1 + f1, f1 + f2, f2 + f2
without (top row) and with (bottom row)
compensation (f1 = 0.24 Hz, f2 = 0.29 Hz)

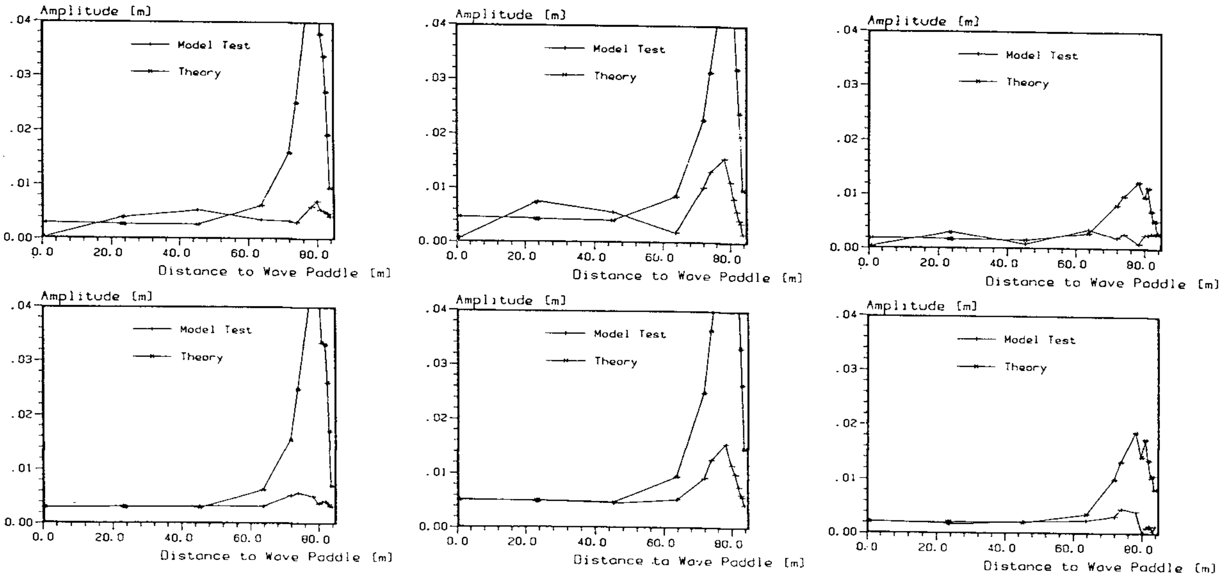


Fig. 9: Amplitude of 2nd Order Component
f1 + f1, f1 + f2, f2 + f2
without (top row) and with (bottom row)
compensation (f1 = 0.20 Hz, f2 = 0.24 Hz)

simple wave groups because every higher and lower harmonic has only one pair of basic linear waves as source. In a spectrum a lot of combinations of frequency components will contribute to a lower and higher harmonic frequency, each with a bounded and free part. A seperation to each source couple f1 and f2 and verification and anlysis in the hydraulic model seems to be a sisyphean task.

4.3. Analysis in the Shoaling Area

The difference of 2nd order wave amplitudes between measurement and theory is obvious in the area of the slope and detectable in all cases (Fig. 6 - 9).

In Fig. 8 and 9 the incident linear waves are shallow water waves. There is good agreement in the horizontal bottom section but no agreement of theory and measurement in the area with low water depth. Although the measured amplitudes increase, they never reach the theoretical ones.

The analysis of 2nd order waves on the slope is dominated by the fact that measured amplitudes are lower than the predicted ones. Theoretical values are obtained by using only the two basic linear components as a source, although spectral deformation on the slope occurs (s.Ch. 5). Incorporating the complete spectrum to calculate the higher harmonics on the slope as well as considering wave setup only leads to insignificant reduction of theoretical values.

This leads to the question whether 2nd order waves can be generated stable in time and space dimension for significant lower water depth.

5. Spectral Deformation

It was the aim of the test to prove whether in the shoaling area a difference occurs in measured wave parameters with and without compensation of 2nd order waves in the control signal.

In a first step the ratio of H1/3 over Hmo is plotted against relative water depth (Fig. 10). The two graphs in one frame are tests without and with compensation. Starting with the common value of 0.95 for deep water the ratio increases up to 1.2 in the first case and 1.10 in the second. A general tendency in the behaviour of 2nd order and 1st order spectra is not detectable.

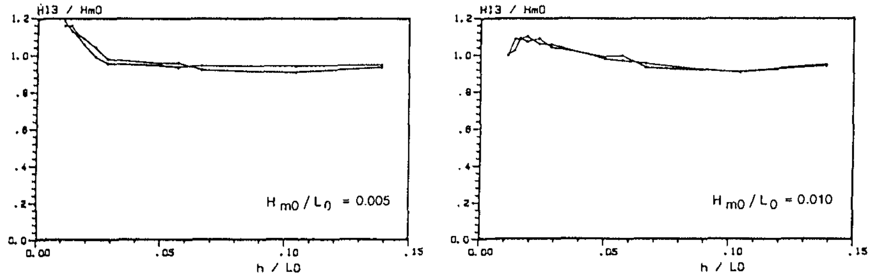


Fig. 10: Ratio of Waveheight Parameters

Fig. 11 shows a plot of H1/3 over H1/30 against the ratio of water depth over incident wave height. Again no difference can be detected whether compensation of 2nd order was used or not. The curve without dots shows the result of GODA's theory for the irregular wave deformation process. Regarding the results of all tests it can be stated that theory and measurements are in good agreement in shallow water but for values of h/Ho between 1.6 and 4.0 the theory overestimates the measurement.

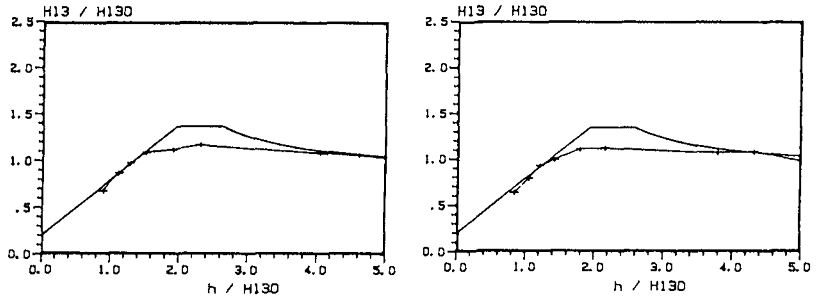


Fig. 11: Variation of Significant Waveheight H1/3

6. Conclusion

The Self-Correcting System was found to produce irregular waves with strongly reduced unwanted free 2nd order waves when theoretical 2nd order time series served as target wave trains.

It has been shown, that in the case of wave groups the amplitudes of the 2nd order waves in the shoaling area differ dependent on the consideration of 2nd order waves in the control signal. In both cases they are lower than predicted by theory.

In a wave spectrum, wave height parameters seem to be not significantly changed whether a compensation of 2nd order waves was used or not.

7. Acknowledgement

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