CHAPTER 27

EXTREME WAVES AND WAVE COUNTS IN A HURRICANE

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Abstract

Estimates of wave conditions within a storm event must recognize the variability of wave conditions during the storm and not just the peak conditions. A methodology is presented for the estimation of the probability distribution of the maximum wave in a storm event and the wave count in the same storm event. The predictions are compared with field observations of wave conditions in tropical cyclones from Australia's North West Shelf.

1. INTRODUCTION

The spatial and temporal complexity of wave conditions within a hurricane must be succinctly summarized for the benefit of designers of offshore structures. Conventional limit design is commonly based on a characterization of the wave climate in terms of the maximum wave height and the associated period, whereas fatigue analysis utilizes a wave count distribution.

The classical analysis of wave record statistics involves an explicit assumption that the typical 20 minute wave record is one realization of a stationary stochastic process. Analysis leads to a prediction of the probability density function for the maximum wave in the record, whose mean value is highly sensitive to both the intensity of the sea state and the duration adopted for comparative purposes (e.g. 20 minutes or 3 hours or whatever).

In a storm event however, the sea state is not a stationary stochastic process. Reasonable estimates of maximum wave conditions and wave counts must recognize the slowly-varying evolution of the sea state. Attention must also be given to the restrictions imposed by gravitational instability, especially in

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regard to estimates of maximum wave conditions which likely approach limiting conditions.

A methodology is presented for the estimation of the probability distribution of the maximum wave in a storm event and the wave count in the same storm event. The predictions are compared with field observations of wave conditions in tropical cyclones from Australia's North West Shelf.

2. WAVE OBSERVATIONS DURING TROPICAL CYCLONES

Tropical cyclone (or hurricane or typhoon) conditions are extreme and mercifully infrequent. The intensity is often maintained over a relatively large time and spatial scale and large wave conditions are experienced even at sites relatively distant from the storm track. The available data base of wave observations during hurricane conditions is not extensive. This is a consequence of the rarity of the meteorological event and the erratic nature of storm paths, together with the instrument survival threat posed by such extreme conditions when they do materialize.

In many cases, the measured record includes only a handful of standard 20 minute records during the life of the storm, which generally extends over several days. Where the record is relatively comprehensive, it is often a distant or minor storm. Among the most comprehensive records available are those collected by Woodside Offshore Petroleum on Australia's North West Shelf. The better records are not especially extreme when measured against the brief segment recorded during the infamous Hurricane Camille. They nonetheless loom large in the local wave climate on the North West Shelf and are sufficiently large to characterize expected response patterns.

Wave data from Tropical Cyclone Victor is representative of North West Shelf conditions. TC Victor in early March 1986 reached a minimum central pressure of 930 mb and the storm track was 200 km from the North Rankin data site at closest approach. The standard measuring program of 20-minute records every 6 hours was increased to continuous recording for over two days during the closest approach of the storm. Despite some small data loss, the overall record is comprehensive and provides an excellent illustration of the wave conditions at the North Rankin site during the storm. The water depth at the data site is 125 m, essentially deep water for the measured wave conditions.

The mean rms wave height $H_{\rm rms}$ evolution of the separate 20-minute records is shown in Figure 1. The evolution in the wave height follows the hydrograph pattern of river elevations or flows during flood (i.e. storm) conditions. There is a rapidly rising limb towards a peak, followed by a rather more slowly falling limb to ambient conditions.

There is no such clear trend in the mean zero-up-crossing period τ_z evolution of the separate 20-minute records, which is shown in Figure 2. Also included on Figure 2 are the one standard deviation error bands, which are relatively wide. The change in period during the storm is apparently quite small, which conflicts with the expectation of wind-generated waves in more uniform wind conditions. This somewhat unexpected feature of wave conditions in

hurricanes had previously been noted (Sobey and Young 1986) in the context of peak period. It is also predicted by the Sobey and Young discrete spectral model of wind wave generation in hurricanes.

3. PREDICTIVE CAPABILITY OF RAYLEIGH THEORY FOR STATIONARY RECORDS

Before attempting to accommodate time-varying wave conditions in an analysis of extreme waves and wave counts, it is useful to review the predictive capability of existing approaches to the prediction of extreme waves and wave counts in stationary sea states.

The familiar approach follows Longuet-Higgins (1952) in representing the probability distribution of wave heights within a stationary sea state by the Rayleigh distribution, with probability density function (PDF) and cumulative distribution function (CDF) respectively as

$$f(R) = 2Re^{-R^2}$$

$$F(R) = 1 - e^{-R^2}$$
(3.1)

where $R = H/2(2m_0)^{\frac{1}{2}}$ is the normalized wave height. Comparisons of field data provide reasonable visual support for this distribution for near-average wave conditions, with the data being sparse and scattered for the more extreme waves.

The wave count, the number of waves n in a stationary record of N waves whose height equals or exceeds a given height, is related to the exceedence probability as



FIGURE 1 Rms wave height evolution in TC Victor during March 1986.



FIGURE 2 Zero-up-crossing period evolution in TC Victor during March 1986.

$$n(R) = N[1 - F(R)]$$
(3.2)

The present writers are not aware of any published comparisons of this wave count distribution with field data. Its reliance however on the Rayleigh distribution leads to the expectation that there would be reasonable visual support for near-average waves with a wider scatter for more extreme waves.

Assuming that consecutive waves are independent, the CDF for the maximum wave in a stationary record of N waves is (Longuet-Higgins 1952)

$$F_{\max}(R) = F^{N}(R) \tag{3.3a}$$

The PDF for the maximum wave in a stationary record of N waves is then

$$f_{\max}(R) = \frac{d}{dR} F_{\max}(R) = Nf(R)F^{N-1}(R)$$
(3.3b)

where f(R) and F(R) are the PDF and CDF for individual wave heights in the record. This distribution for the maximum wave in a stationary record of N waves is strongly dependent on N, as shown in Figure 3 for N values from 10 to 1000, a value of 100 being typical of a 20-minute record. The distribution remains relatively wide for all wave counts in the expected range. The Rayleigh distribution has been included in Figure 3 for perspective.

Evaluation of this prediction with field data is complicated by the sparse data set that is available, each 20-minute data set for example yielding only a single observation for wave counts of order 100. Estimates of the distribution are not available and observations of the maximum wave can only be compared with the mean of the distribution, estimated as (Longuet-Higgins 1952)



FIGURE 3 Rayleigh-theory predictions of maximum wave in N waves.

$$\bar{R}_{\max} = \int_{0}^{\infty} r f_{\max}(r) \, dr \sim \ln^{\frac{1}{2}} N + \frac{\gamma}{2} \ln^{-\frac{1}{2}} N \tag{3.4}$$

where γ is Euler's constant.

The predictive capacity of Equations 3.3 and 3.4 was evaluated from the TC Victor data set introduced as Figures 1 and 2. Each of the separate 20minute records was considered as representative of a stationary sea state. The theoretical estimate of the expected value of $H_{\rm max}$ for each segment was computed from Equation 3.4 and the Rayleigh distribution, using the measured $H_{\rm rms}$ and N. This was compared with the measured $H_{\rm max}$ in Figure 4. Although the distribution of $H_{\rm max}$ is relatively wide, the TC Victor data base nonetheless identifies a systematic overprediction of order 10% by the "Rayleigh" theory.

As this systematic overprediction would likely feed through to predictions of extreme waves in a complete storm, alternatives to the Rayleigh distribution were reviewed. Field observations of amplitude distributions in intense seas have been represented as the empirical Weibull distribution

$$F(\rho;\alpha,\beta) = 1 - \exp(-\frac{8}{6}\rho^{\alpha})$$
(3.5)

where $\rho = H/H_{\rm rms}$ and $H_{\rm rms}$, α and β are the parameters defining the distribution. α and β parameter values of 2 and 8 respectively retrieve the Rayleigh distribution. Forristall (1978) estimates 2.13 and 8.42 from Gulf of Mexico data and Krogstad (1985) typically estimates 2.38 and 12.9 from North



FIGURE 4

Comparison of Rayleigh-theory predictions of maximum wave in N waves and measurements for stationary 20-minute record segments from TC Victor.

Atlantic data. Interestingly, these empirical distributions bracket the Rayleigh distribution in the high amplitude tail, the Forristall distribution predicting a lower probability for the higher waves in the sea state and the Krogstad distribution a higher probability. These distributions are based on theoretical arguments or data that are biased towards average conditions in an albeit intense sea state. Confidence levels in the tail are not especially high, although it is the region of potentially major concern in consideration of maximum wave conditions in a storm. Estimates of the PDF for the maximum wave in N waves follow directly from Equation 3.3b and are compared with the "Rayleigh" estimate in Figure 5. As expected, they bracket the Rayleigh estimate. An estimate based on the Krogstad distribution would increase the overprediction to approximately 20%. An estimate based on the Forristall distribution would marginally decrease the overprediction but not significantly.

Also included on Figure 5 is an estimate based on a tabulated f(R) established by Sobey (1990). This estimate is based on a mean Jonswap spectrum and was established by simulation of one hundred separate realizations of wave sequences, each containing in excess of one thousand waves. This methodology gives reasonable recognition to the more extreme waves in the stationary sea state. The tabulated f(R) used together with Equation 3.3 still appears to predict high, though it does come rather closer to the TC Victor than any of the other predictors.

4. A SLOWLY-VARYING SEA STATE

Wave conditions in a stationary sea state may be characterized by the probability distribution of wave amplitudes, represented by either the probability density function f(a) or the cumulative distribution function F(a), where a = H/2 is the wave amplitude and H is the wave height. In a slowly-varying seas state, this distribution can be expected to vary slowly with time, the cumulative distribution function, for example, becoming F(a;t).

Within a short interval of time Δt , the wave count is $\Delta t/\tau_z(t)$ where $\tau_z(t)$ is the mean zero-crossing period in the short interval. Under the assumption (Longuet-Higgins 1952) that consecutive wave in the interval are independent, the cumulative distribution function for the maximum wave amplitude in the interval is

$$F_{max}(a;t) = [F(a;t)]^{\Delta t/\tau_{x}(t)}$$
(4.1)

A storm event from time A to time B may be characterized as J consecutive intervals of duration Δt_i centered at time t_i such that

$$B - A = \sum_{j=1}^{J} \Delta t_j = \int_{A}^{B} dt$$
(4.2)



FIGURE 5 Alternative predictions of PDF for maximum wave in 100 waves.

If wave conditions between consecutive intervals remain independent, then the cumulative distribution function for the maximum wave in the storm event is (Borgman 1970)

$$F_{\max}(a) = [F(a;t_1)]^{\Delta t_1/\tau_z(t_1)} [F(a;t_2)]^{\Delta t_2/\tau_z(t_2)} \dots [F(a,t_j)]^{\Delta t_j/\tau_z(t_j)}$$

$$= \exp \int_{A}^{B} \frac{1}{\tau_z(t)} \ln F(a;t) dt$$
(4.3)

The wave count distribution with amplitude within a storm event will be termed n(a) to distinguish it from the total mean wave count N within the storm event. The number of waves in the short interval Δt that exceed a is

$$\Delta n(a) = [1 - F(a; t)] \frac{\Delta t}{\tau_{\tau}(t)}$$
(4.4)

For a storm of duration B - A, again characterized as J consecutive intervals of duration Δt_j centered at time t_j , the number of waves in the complete storm that exceed a is

$$n(a) = \Delta n_1(a) + \Delta n_2(a) + ... \Delta n_j(a) = \int_{A}^{B} \frac{1}{\tau_z(t)} [1 - F(a;t)] dt$$
(4.5)

where it has again been assumed that wave conditions between consecutive intervals remain independent.

The time history of the local zero-crossing period and the local cumulative distribution function remains to be specified.

5. PARAMETERIZATION OF SEA STATE IN A STORM EVENT

Statistical summaries (such as significant wave height and zero-crossing period) of measured wave conditions at a fixed site during a storm event typically display the familiar "hydrograph" shape from surface water hydrology. This is especially true of significant wave height and the equivalent rms amplitude. There is a base wave amplitude (analogous to the base flow in surface water hydrology) that is sustained at a reasonably constant level well before and after the storm event. The storm hydrograph itself has the familiar asymmetric rising and falling limbs. In some cases, the time scale of the falling limb is significantly longer that the rising limb but in other cases it is more nearly equal. Storm duration above the base level may range from several hours to several days. The zero-crossing period might be expected to follow a similar trend, although the variation would be much less dramatic. Figures 1 and 2 provide examples of measured hydrographs during TC Victor.

A suitable schematic form for these hydrographs would be

$$q(t) = q_b + \frac{1}{2}(q_p - q_b) \left[1 + \cos(2\pi \frac{t - t_p}{4\tau_*}) \right]$$
(5.1)

where q represent either the local rms wave amplitude $a_{\rm rms}(t)$ or the local zerocrossing period $\tau_{\rm z}(t)$. The subscripts b and p identify the base and peak hydrograph levels respectively, $t_{\rm p}$ is the time of the hydrograph peak and τ_* is the half life of the hydrograph, as sketched in Figure 6. Separate half lives of the rising and falling limbs (τ_1 and τ_2) are defined such that the storm event extend from time $t_{\rm p} - 2\tau_1$ to time $t_{\rm p} + 2\tau_2$. The finite duration of the storm event is a significant feature in defining the expected maximum wave conditions. A truncated Gaussian form would be equally appropriate.

The adopted form for the local cumulative distribution function has been based on the review of predictive capabilities for extreme waves in stationary conditions, summarized in Figure 4 and 5. It is apparent from these figures that all of the viable alternatives resulted in overprediction of the maximum wave in a typical 20-minute record. The tabulated distribution of Sobey (1990) gave the best agreement with measured data and has been used in the present analysis. It must be anticipated that the overprediction apparent in the stationary sea state analysis will be at least equally apparent in the slowly-varying sea state analysis.

Finally, in recognition of the upper bound imposed by gravitational instability, an empirical modification to the probability distribution was introduced to restrict the maximum wave conditions to the limit wave. The adopted local cumulative distribution function is





where $F'(\rho)$ is the tabulated CDF for a stationary sea state and $\rho_{\rm m} = H_{\rm m}/(2a_{\rm rms})$, $H_{\rm m}$ being the limit wave height appropriate for the local wave period, water depth and current. This expedient has the additional benefit of limiting the significance of the tail to the local population distribution.

The limit wave height has been defined by interpolation from the Williams (1985) tables. This limit wave cutoff should have only marginal influence on the wave count and will likely influence the maximum wave distribution only near the hydrograph peak of the more intense storms.

6. WAVE CONDITIONS IN A HURRICANE

In application to tropical cyclones or hurricanes on Australia's North West Shelf, it was noted in Figure 2 that measured time histories of zero-crossing periods did not change significantly during the storm. Accordingly, the zerocrossing period was assumed to be constant for the duration of the storm. With the rms wave amplitude following the cosine hydrograph, the standard deviate of the local probability distribution becomes

$$\rho(t) = \frac{A}{B + \frac{1}{2}(1 - B)(1 - \cos s)}$$
(6.1)

where $A = a/a_p$, $B = a_b/a_p$ and $s = 2\pi (t - t_p)/4\tau_*$. The cumulative distribution function for the maximum wave in the storm then becomes

$$F_{\max}(A) = \exp\left[\frac{N}{\pi}\Phi(A, B, \alpha, \beta, A_m)\right]$$
(6.2)

where N is the mean wave count in the complete storm, $A_{\rm m} = (H_{\rm m}/2)/a_{\rm p}$ and Φ is the dimensionless definite integral

$$\Phi = \int_{0}^{\pi} \ln F(s; A, B, \alpha, \beta, A_m) \, ds \tag{6.3}$$

In like manner, the wave count becomes

$$n(A) = \frac{N}{\pi} \Psi(A, B, \alpha, \beta, A_m)$$
(6.4)

where Ψ is the dimensionless definite integral

$$\Psi = \int_{0}^{\pi} \left[1 - F(s; A, B, \alpha, \beta, A_m)\right] ds$$
(6.5)

Both integrals were evaluated numerically.

7. COMPARISONS WITH FIELD DATA

Tropical cyclone Victor in February-March 1986 was a reasonably intense storm, having a minimum central pressure of 930 mb. The peak and base rms wave heights were 3.16 m and 0.70 m respectively and the mean wave count in the hydrograph was 62000. The predicted probability density function for the maximum wave in the storm is shown in Figure 7. The distribution is moderately The maximum wave measured during the storm was 8.04 m, which wide. corresponds to a $H_{\rm max}/H_{\rm p}$ of 2.53. There is order of magnitude agreement between this prediction and the single measurement during TC Victor. It would appears however that the maximum wave in the storm is over-predicted by the theory, as was anticipated with the adoption of the tabulated CDF. The adoption of the Forristall, Rayleigh and Krogstad distributions for the CDF further widens the gap between theory and measurement. A similar trend resulted from the only other storm in the North Rankin data base (TC Ilona) with near-continuous records during the storm.

Figure 7 compares the measured and predicted wave count distributions. Agreement is good for the more frequent waves of average height but the trend at the more extreme wave heights is again over-predicted by the theory.

8. CONCLUSIONS

A methodology has been established for the prediction of the PDF for the maximum wave in a storm event. The maximum wave in the storm appears to



FIGURE 7 Predicted PDF for maximum wave in TC Victor.



FIGURE 8 Predicted and measured wave count in TC Victor.

be marginally over-predicted for hurricane waves on Australia's North West Shelf. Almost exactly the same systematic trend is identified in the prediction of the maximum wave in the (stationary) 20-minute record segments making up the complete data base.

A parallel methodology for the wave count in the same storm event provided good agreement with field data for average wave heights but again over-predicts at the more extreme waves.

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