CHAPTER 21
SIMILARITY OF VELOCITY PROFILES IN NON-UNIFORM
LONGSHORE CURRENTS

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Abstract
In studying beach changes a law for non-uniform longshore sediment transport is generally needed, which concerns the non-uniform longshore currents. Therefore, experimental study on the similarity of velocity profiles in non-uniform longshore currents was carried out on a plane beach. It was concluded that, the velocity profiles are in similarity alongshore by comparing the nominal velocity at the breaker point. The boundary layer approximation can therefore be applied to the nearshore current equations to derive a single integral equation of non-uniform longshore currents. The theoretical solution of the equation is shown to compare with experimental results. The comparison between the theoretical and experimental non-uniform longshore current velocities are in satisfaction.

1. Introduction
In recent years, the coastal region has become an area of intense human activity for industry, transportation, recreation, as well as coastal protection works. The longshore current has an significant influence on beach changes. The prediction of the longshore current is of great importance for studying beach changes. Many investigators have studied uniform longshore currents, but only a few such as Eagleson\textit{(1965)}, Horikawa and Sasaki\textit{(1968)}, and Gourlay\textit{(1976)} have concentrated on the phenomenon of non-uniform longshore currents. The phenomenon of

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non-uniform longshore currents can be seen clearly in the vicinity of reefs and headlands, as well as coastal structures. The magnitude of the longshore current velocity increases from the upcoast initial current location and increases rapidly in the downcoast direction until a constant profile shape or uniform longshore current is established. Therefore, we may say that the non-uniform longshore currents are responsible for rapid erosion or accretion around coastal structures, where the non-uniform longshore current exist.

The shoreline is simply defined as the boundary between sea and land. Immediately seaward of the shoreline there exists a narrow band of fluid known as the surf zone. The surf zone is characteristic of extremely complex fluid motion induced by the rapid dissipation of the incoming wave energy. Moreover, in the field of hydraulics, particularly in the open channel flow, there exists a thin layer near the bed. Consequently, it may be possible to describe the longshore current motion in the surf zone using the boundary layer concept. Based on this analogy, the concept of boundary layer theory is introduced to the nearshore region. In order to verify this analogy, an experimental study on the velocity profiles in non-uniform longshore currents was performed. If the similarity in velocity profile can be obtained, the boundary layer approximation may lead to a single integral equation of non-uniform longshore currents from the nearshore current equations.

Applying this similarity of velocity profile in non-uniform longshore currents, we have derived the single integral equation of non-uniform longshore currents which is presented by the nominal longshore current velocity at the breaker point (Tsuchiya and Refaat, 1990). The solution of the equation is very briefly shown in the case of plane beaches, and compared with experimental results in good agreement.

2. Experiments
2.1 Experimental setup

The experiments were performed in the fan-shaped wave basin (semicircular part: r=17.5 m and rectangular part: 35 x 10 m) of Ujigawa Hydraulics Laboratory, Disaster Prevention Research Institute, Kyoto University. A smooth concrete beach was constructed with slope 1:10. Two smooth wave guide walls were installed in the normal direction to the wave board. The upstream guide wall was closed to the beach and the downstream one extended to the toe of the fixed bed, see Figure 1. The purpose for having an upstream guide wall closed to the beach is to keep the wave
height uniform along the beach. While, the opening at downstream guide walls helps to minimize the water circulation between the guide walls and to carry it away behind at the still water area.

![Figure 1. Schematic diagram of experimental arrangement of similarity of velocity profiles in non-uniform longshore currents.](image)

2.2 Experimental procedure

The sloping part of the wave basin was marked by a 20 square cm grid. Measurements of current velocity were made at every grid line perpendicular to the shoreline. Two different types of colored tracers were used; 3 cm square shaped paper tracers and 2.5 cm ball tracers. The tracer trajectories within each grid was recorded by using a video camera system. Measurements of wave heights in the constant depth of the basin were made with capacitance type wave gauges. While on the sloping part, measurements of wave heights and mean water level were made over the entire grid area using capacitance type wave gauge and low frequency filter, respectively. The wave gauge and low frequency filter were installed on a vehicle mounted on rails controlled by personal computer. The angles of incoming wave incidence were measured at the constant depth part by measuring the angles of...
inclination of the wave generator to the beach. The measurements of angles of incoming waves were done at the breaker line from the video tape record. Snell's law and linear wave theory were used to estimate and compute the measured angles of wave incidence at the breaker line.

2.3 Experimental results

In order to develop a more realistic model for non-uniform longshore currents, it is necessary to employ regular waves under various conditions. To carry out this investigation, eight experiments were performed under the conditions that the still water depth at the constant depth part is 30.6 cm, the wave period is 1.13 sec, the range of angle of incidence is 15° to 55°, and the wave height is varied from 3.8 cm to 8.5 cm, as shown in Table 1 where \( H \) is wave height, \( \alpha \) is angle of wave incidence, \( h \) is still water depth, \( \gamma \) is breaker index, \( W \) is mean width of the surf zone, \( P \) is Longuet-Higgins parameter and the indices 0 and b refer to values on constant depth part of the wave basin and the breaker line, respectively.

<table>
<thead>
<tr>
<th>Ex. No.</th>
<th>( H_0 ) (cm)</th>
<th>( \alpha_0 ) deg.</th>
<th>( H_b ) (cm)</th>
<th>( h_b ) (cm)</th>
<th>( \gamma ) ((H_b/h_b))</th>
<th>( \alpha_b ) deg.</th>
<th>( W_b ) (cm)</th>
<th>( P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>3.80</td>
<td>17.0</td>
<td>5.68</td>
<td>6.05</td>
<td>0.94</td>
<td>7.32</td>
<td>80</td>
<td>0.20</td>
</tr>
<tr>
<td>R2</td>
<td>5.10</td>
<td>17.0</td>
<td>6.73</td>
<td>7.87</td>
<td>0.86</td>
<td>8.36</td>
<td>100</td>
<td>0.09</td>
</tr>
<tr>
<td>R3</td>
<td>8.50</td>
<td>17.0</td>
<td>11.61</td>
<td>11.72</td>
<td>0.99</td>
<td>10.22</td>
<td>150</td>
<td>0.10</td>
</tr>
<tr>
<td>R4</td>
<td>4.50</td>
<td>35.0</td>
<td>7.98</td>
<td>8.80</td>
<td>0.91</td>
<td>17.56</td>
<td>90</td>
<td>0.05</td>
</tr>
<tr>
<td>R5</td>
<td>5.20</td>
<td>45.0</td>
<td>7.21</td>
<td>8.03</td>
<td>0.90</td>
<td>20.81</td>
<td>100</td>
<td>0.10</td>
</tr>
<tr>
<td>R6</td>
<td>7.75</td>
<td>45.0</td>
<td>9.63</td>
<td>11.80</td>
<td>0.82</td>
<td>25.51</td>
<td>130</td>
<td>0.10</td>
</tr>
<tr>
<td>R7</td>
<td>5.50</td>
<td>55.0</td>
<td>7.74</td>
<td>9.48</td>
<td>0.82</td>
<td>26.56</td>
<td>100</td>
<td>0.10</td>
</tr>
<tr>
<td>R8</td>
<td>6.65</td>
<td>55.0</td>
<td>8.60</td>
<td>10.03</td>
<td>0.86</td>
<td>27.38</td>
<td>120</td>
<td>0.08</td>
</tr>
</tbody>
</table>

The experimental results are classified mainly into two categories, they are:

1. Similarity of velocity profiles

Figures 2 and 3 show the examples of the measured longshore velocities, for two incident wave angles 17° and 45°, compared with the theoretical curves of Longuet-Higgins (1972). The rate of agreement between measured and computed longshore current profiles is influenced by the choice of Longuet-Higgins parameter
LONGSHORE CURRENTS PROFILES

Examples of measured longshore velocity profiles compared with theoretical curves of Longuet-Higgins model in the case of incident wave angle $\alpha_0 = \pi / 6$.

(a) In the case of Run 1

(b) In the case of Run 3
Examples of measured longshore velocity profiles compared with theoretical curves of Longuet-Higgins model in the case of incident wave angle $\alpha_0 = 45^\circ$.

Figure 3.
The curves were fitted to the data using the method of least square. It is noted that for all experiments Longuet-Higgins parameter $P$ is less than 0.2. From these figures it is obvious that:

a) The similarity of velocity profiles is quite satisfactory. The shape of the velocity distribution starts to grow from the initial boundary until reaching a constant shape, which represents uniform velocity profile. Therefore, the integration coefficients, which appeared in the equation of non-uniform longshore currents to be described in Eq. (1), are not functions of longshore direction nor time.

b) For a small angle of incidence, the agreement between the measured velocities and Longuet-Higgins curve is good. While in the case of large angle of incidence, the agreement is less.

The development of non-uniform longshore current velocity at the breaker line is shown in Figure 4, where the velocity is normalized by the one at breaker point and the longshore distance is normalized by the factor $B$ to be described in Eq. (4). The longshore current velocity increases rapidly from zero, at the initial condition, to a constant value which corresponds to the uniform longshore current.

[Graph: Uniform longshore current velocity]

Figure 4. Comparison of measured and computed non-uniform longshore current velocity.

(2) Uniformity of wave set-up

Figure 5(a) and 5(b) illustrate the distribution of measured wave height for Run 2 and the mean water level for Run 3 over the entire sloping bed area. In these figures the horizontal axis represents the cross-shore direction, while the longshore direction is represented by the third axis. The uniformity of wave height and wave
Figure 5. Distribution of measured wave height and mean water level over the entire sloping bed area.

Set-up alongshore are clearly seen in these figures. Figure 6 shows the cross-shore measurements of the wave field for Run 2. The lower part of the figure shows the measured wave height computed with the calculated one by linear wave theory. In the experiments, the breaking wave is a plunging type, this means that the wave height continues to increase up to the breaker point. The effect of nonlinearity is so small that the measured values fit the calculated ones well. The upper part of the figure shows the changes in the mean water level presented by waves. Bowen, Inman and Simmons's (1968) formula for wave set-down was used to fit the measured data. It is clear from this figure that wave set-up increases linearly toward the shoreline. Therefore we can conclude from Figures 5 and 6, that wave set-up is uniform alongshore and increases linearly toward the shoreline.
Figure 6. Cross-shore measurements of wave height and mean water level in the case of Run 2.

Figure 7 illustrates the variation of the measured wave set-up along the shoreline. The measured values used in this figure are 20 cm from the initial shoreline because the limitation of measurements at this region. But, fortunately, the wave set-up increases linearly, thus we can use this value instead of the maximum wave set-up value. In this figure Run 2 shows the maximum variation of wave set-up which is less than 7%, other cases are much smaller. Thus the wave set-up
will be uniform alongshore. Furthermore, it is well known that wave set-up is a function of breaker index, $H_b/h_b$. Figure 8 demonstrates the uniformity of breaker index, $H_b/h_b$, along the shoreline. In this figure, the value of the breaker index, $H_b/h_b$, is laid between 0.8 and 1.0.

![Figure 8. Longshore variation of measured breaker index, $H_b/h_b$.](image)

3. The Theoretical Solution of Non-Uniform Longshore Current

Based on the similarity of velocity profiles in the non-uniform longshore currents, as previously discussed, a single equation of non-uniform longshore currents can be derived. Tsuchiya and Refaat (1990) have recently applied the so-called boundary layer approximation to the nearshore current equations to obtain the equation of non-uniform longshore currents. This equation can be written as:

$$\frac{\partial}{\partial x} \left( U_0^2 h_b^2 \right) + \alpha_3 \frac{\gamma C_f c_b}{\pi} U_0 h_b = f(x)$$  \hspace{1cm} (1)

where

$$f(x) = \frac{\gamma^2}{16} g h_b^2 \left\{ \sin 2\alpha_b \cdot 2 \cos^2 \alpha_b \frac{\partial y_b}{\partial x} \cdot (5 \beta_1 - 6 \beta_2) + 6 \beta_3 \sin^2 \alpha_b \right\}$$  \hspace{1cm} (2)

where $U_0$ is the non-uniform longshore current velocity at the breaker line, $\gamma = H_b/h_b$ is the breaker index where $H_b$ and $h_b$ are the breaker height and depth respectively, $y_b$ is the position of the breaker line from the datum line, $\alpha_b$ is the breaking wave
angle, $C_f$ is the bed shear stress coefficient, $c_b = \sqrt{gh_b}$ is the breaking wave celerity, $g$ is the acceleration of gravity, and $\alpha_i$, $\beta_i$ ($i = 1, 2, 3$) are the integration coefficients. These integration coefficients depend only on the velocity profiles of longshore currents. Regarding the similarity of the velocity profiles of longshore currents, these integration coefficients are not functions of the longshore direction nor time.

This equation includes many parameters, but in the case of plane beach it is only one unknown $U_0$, because the right hand side becomes constant. With the boundary condition that at $x = 0$, $U_0 = \overline{U_0}$ the solution of this equation is derived as:

$$\frac{(1 - \overline{U_0} / U_b) \exp (1 - U_0 / U_b)}{(1 - U_0 / U_b) \exp (1 - \overline{U_0} / U_b)} = \exp (B x)$$

where

$$U_b = \frac{5 \pi m}{16 C_f} g h_b \left( \frac{\sin \alpha_b}{c_b} \right) \cos \alpha_b$$

$$B = \left( \frac{8 C_f}{5 \pi m} \right)^2 \frac{2}{\alpha_b h_b \sin 2\alpha_b}$$

In Figure 4, a solution to Eq. (3) for boundary condition at $x=0$, $U_0 = 0$, is shown and compared with the experimental values. Each point in the figure is identified by the plotting symbol corresponding to each run in the experimental measurements. It is shown that most of the points lie around the theoretical curve expressed by Eq. (3). The theoretical curve given by Eagleson (1965) is also shown in the figure for comparison. Based on the similarity of velocity profiles of longshore currents, the theoretical solution given by Eq. (3) describes not only the non-uniform longshore velocity at the breaker line but also at any given relative local depth ratio.

4. Conclusions

In this paper, an experimental study on the velocity profiles in non-uniform longshore currents was performed in order to investigate their similarity. From the comparison of the experimental measurements of longshore currents with the theoretical curves of Longuet-Higgins, it is observed that the degree of agreement between measured and computed longshore current profiles is influenced by the choice of Longuet-Higgins parameter $P$, and while the shape of the velocity
distribution starts to grow in a similar shape from the initial boundary until reaching a constant shape which represents uniform velocity profile, the similarity of velocity profiles is very satisfactory. The theoretical solution of non-uniform longshore currents on a plane beach was compared with the experimental result. The solution describes not only the non-uniform longshore velocity at the breaker line but also at any given relative local depth ratio.

References


