CHAPTER 9

MODELING OF ENERGY TRANSFER AND UNDERTOW IN THE SURF ZONE

Akio Okayasu[1], Akira Watanabe[2] and Masahiko Isobe[3]

Abstract

A model is presented to describe accurately the energy transfer under breaking waves. In the model, the energy of organized large vortexes as well as those of wave-induced motion and turbulence is taken into account. The model allows to estimate the dissipation rate and distribution of energy, and then the cross-shore two-dimensional distribution of an undertow. The applicability of the model is confirmed by laboratory experiments.

1. Introduction

In order to predict the sediment transport and the material diffusion in the surf zone, it is necessary to estimate the distribution of an undertow. Since the undertow distribution has been evaluated from local properties of waves and turbulence in most of previous models, additional models are needed to evaluate the local properties from incident wave conditions. These undertow models have another disadvantage that they are applicable only to wave breaking in the inner region of the surf zone. Hence, to complete an undertow model which is valid through the whole surf zone, it is necessary to combine it with an accurate description of wave attenuation, energy distributions or generation of turbulence based on the actual breaking mechanism. However, energy of the organized vortexes which were pointed out to be formed around crests of breaking waves was not taken into account in the previous models for wave deformation in the surf zone.

The first objective of the present study is to describe accurately the energy transfer process in the surf zone by taking the energy of the orgauized

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[1] Research Associate, Department of Civil Engineering, Yokohama National University, Hodogaya-ku, Yokohama 240, Japan

[2] Professor, Department of Civil Engineering, University of Tokyo, Bunkyo-ku, Tokyo 113, Japan

[3] Associate Professor, ditto

large vortexes into account. The second objective is to formulate a model to estimate cross-shore vertically two-dimensional distribution of the undertow on arbitrary beach topographies from the energy distributions obtained by the energy transfer model.

2. Energy Transfer Model

2.1 Energy balance in the surf zone

In order to describe the transfer of energy by wave breaking, a model is presented in which the organized large vortexes are taken into account as a transmitter of energy in the energy transfer process from wave motion to turbulence. The total energy in the surf zone is described as

$$E_t = E_w + E_v \tag{1}$$

where E_t is the total energy of wave per unit area, E_w the energy of wave motion and E_v the energy of organized large vortexes.

The energy of the organized vortexes converted from wave energy is transferred to smaller size eddies, and then dissipates. It is assumed that the converted energy from wave motion to the large vortexes never transferred back to the wave motion and the energy which dissipates directly from the wave motion to heat by wave breaking is negligible. By using the energy flux by wave motion $E_w c_g$ and energy dissipation rate by bottom and wall friction D_{b+w}, the transfer rate T_B from the wave energy to the energy of the vortexes per unit length and unit width is expressed as

$$T_B = -\frac{d\,(E_w c_g)}{dx} - D_{b+w} \tag{2}$$

where x is the horizontal coordinate in the shoreward direction and c_g is the group velocity of waves. It is considered that D_{b+w} directly dissipates without conversion through the large vortexes and is not negligible for wave propagation in wave flumes.

The organized large vortexes propagate with the wave crests. Since the phase velocity c is nearly equals to the group velocity c_g in the surf zone, the energy flux by the large vortexes can be approximated by $E_v c_g$, which satisfies the following equation:

$$\frac{d\,(E_v c_g)}{dx} = T_B - D_B \tag{3}$$

where D_B is the dissipation rate per unit area through turbulence by wave breaking. Since the energy once transferred from wave motion to the organized vortexes is kept by the vortexes for a while, difference appears in the spatial distributions between the attenuation of wave energy E_w and the generation of turbulence energy.

2.2 Mass and momentum fluxes by breaking waves

The mass transport by breaking waves consists of those by the wave motion M_w and by the organized vortexes M_v [Okayasu *et al.* (1988)]. By using the linear long wave theory, the mass flux by the wave motion M_w is described as

$$M_w = \rho\frac{c}{h}\overline{(\zeta-\overline{\zeta})^2} = \frac{2c}{gh}E_p \tag{4}$$

where h is the mean water depth, ζ the water surface elevation and E_p the potential energy of waves.

If the length and velocity scales of the organized vortexes can be represented by the wave height H and the wave celerity c, respectively, the mass flux by organized vortexes M_v satisfies

$$M_v \propto \frac{E_v}{c} \tag{5}$$

The total mass flux due to breaking waves can be described in terms of the potential energy and the energy of organized large vortexes as

$$M_t = 1.6\frac{c}{gh}E_p + \frac{2}{c}E_v \tag{6}$$

in which the coefficients were determined so as to fit the measured values.

As for the radiation stress, the total radiation stress S_t can also be divided into two parts which are the excess momentum flux by wave motion S_w and that by the organized large vortexes S_v. The small amplitude wave theory is adopted for convenience to calculate S_w in this study. The radiation stress by the organized large vortexes S_v is evaluated as

$$S_v = \frac{5}{3}E_v \tag{7}$$

3. Estimation of Energy Distribution

In the present model, the one dimensional time-dependent mild slope equation is adopted to calculate the distribution of the energy of wave motion E_w. The energy of organized large vortexes E_v supplied from E_w at a certain point is assumed to be transferred to the turbulence energy within some distance determined in terms of the local water depth. The energy dissipation rate D_B is obtained by integrating the transferred energy.

3.1 Estimation of energy of wave motion

The time-dependent mild slope equation was originally derived by using the small amplitude wave theory. However, waves in the surf zone can no more be regarded as small amplitude waves. In that sense, the time-dependent mild slope equation may not be appropriate for waves in the surf zone, but it is also a fact that there is no wave theory which can express the waves in the surf zone adequately. It has been reported that the wave energy calculated by the time-dependent mild slope equation fits well with the measured wave energy [see e.g. Watanabe and Dibajnia (1988)]. Hence, in this study, the time-dependent mild slope equation is adopted as the governing equations to estimate the wave energy variation in the surf zone.

The following time-dependent mild slope equation with the dissipation term is used after Watanabe and Dibajnia (1988):

$$\frac{\partial Q}{\partial t} + c^2 \frac{\partial \zeta}{\partial x} + f_A Q = 0 \tag{8}$$

$$\frac{\partial \zeta}{\partial t} + \frac{1}{n}\frac{\partial (nQ)}{\partial x} = 0 \tag{9}$$

in which t is the time, Q the flow rate, $n = c_g/c$. The attenuation factor f_A by wave breaking is the sum of f_T and f_{b+w}, where f_T is the energy transfer factor from wave motion to large vortexes and f_{b+w} the energy dissipation factor due to energy loss by bottom and wall friction. f_T is expressed as

$$f_T = \alpha_T \tan\beta \sqrt{\frac{g}{h}\left(\frac{\gamma - \gamma_r}{\gamma_s - \gamma_r}\right)} \tag{10}$$

which was given by Watanabe and Dibajnia. In Eq. (10), α_T is a parameter which linearly increases from 0 to 2.5 around the breaking point, then takes a constant value 2.5 in the inner region. The bottom slope $\tan\beta$ is the average value of the bottom slope near the breaking point, g the acceleration of gravity, γ the ratio of water particle velocity to wave celerity. The symbols γ_s and γ_r are γ on constant slope and for wave recovery zone, respectively. The energy dissipation factor f_{b+w} is obtained from the laminar solution by Iwagaki and Tsuchiya (1966) as

$$f_{b+w} = \frac{1}{n}\sqrt{2\nu\sigma}\left(\frac{1}{B} + \frac{k}{\sinh 2kh}\right) \tag{11}$$

where ν is the kinematic viscosity, σ angular frequency of wave, k the wave number, B the width of wave flume. In the surf zone, the bottom and wall boundary layers may not be laminar flow, but the dissipation by wave breaking is so large that the damping due to the bottom and wall friction is negligible.

The breaking point x_b is determined as the point where the wave height takes its maximum value. Isobe (1987) approximated it by the following formula:

$$\begin{aligned}\gamma_b = {} & 0.53 - 0.3\exp(-3\sqrt{h_b/L_0}) \\ & + 5(\tan\beta)^{3/2}\exp\{-45(\sqrt{h_b/L_0} - 0.1)^2\}\end{aligned} \tag{12}$$

where L_0 is the deep-water wavelength and subscript b denotes the quantity at the breaking point.

The breaking points slightly differ depending on its definition. Since the energy transfer from the wave motion to the organized large vortexes and also the energy dissipation should occur from the point where the wave crests begin to break, the coefficient α_T in Eq. (10) is set to be 0 at the crest breaking point x'_b and to be the maximum value 2.5 at the transition point x_t. Considering the results by Seyama and Kimura (1988), the crest breaking point is given as

$$x'_b = x_b - 2h_b \tag{13}$$

in this model.

According to the small amplitude wave theory, the value of the potential energy E_p is equal to that of the kinetic energy E_k. However, E_k is larger than E_p for non-linear waves in general. Dibajnia *et al.* (1988) obtained the result that the maximum ratio of E_k to E_p calculated by the finite amplitude wave theory is up to 1.2 as far as the calculation converged. Since the breaking waves in the surf zone can be considered as non-linear, the ratio $R_p \equiv E_p/E_k$ is made to change linearly from 1 at x'_b to its minimum value at the wave plunging point x_p in this model. E_p is reduced to about 90% of $E_w/2$ in the inner region of the surf zone.

The values γ_s and γ_r in Eq. (10) have been determined so that the potential energy agreed well with the measured value. They should be reduced in proportion to the decrease of E_p, because the variation of R_p accelerates the decrease of E_p apparently. Hence, in this study, they are determined as

$$\gamma_s = 0.35\,(0.57 + 5.3\tan\beta) \tag{14}$$

$$\gamma_r = 0.45\left(\frac{a}{h}\right)_b \tag{15}$$

where a is the wave amplitude.

The time-dependent mild slope equation can deal with wave reflection, however, a rapid change of the energy transfer factor f_A generates considerable numerical reflection in the offshore side of the breaking point. Since the energy dissipation by wave breaking occurs with little wave reflection, it is necessary to keep it small for the accurate description of the wave field. It is possible to decrease the energy of the reflected waves by changing the wave number in the onshore region according to the value of f_T. The modified wave number k' is given as

$$k' = \frac{a_k}{2\,(a_k^2 + b_k^2)}\,k \tag{16}$$

$$a_k = \frac{\sqrt{1 + \sqrt{1 + \frac{f_T^2}{\sigma^2}}}}{2\sqrt{2}}, \quad b_k = \frac{f_T}{2\sqrt{2}\sigma\sqrt{1 + \sqrt{1 + \frac{f_T^2}{\sigma^2}}}} \tag{17}$$

If $f_T = 0$, k' is equal to k which is calculated by the small amplitude wave theory. In the surf zone, k' is smaller than k in general. The smaller wave number results larger wave celerity. Horikawa and Isobe (1980) found that the wave celerity in the surf zone can be predicted fairly well by the solitary wave theory. It means that the modification agree with the reality. However, k/k' is kept to be less than 1.2 to prevent the wave celerity from increasing too much due to the extremely large value of f_T close to the shoreline.

The offshore and shoreline boundary conditions, the method of numerical computation and the adopted mesh scheme are the same as those employed by Watanabe and Dibajnia (1989). The solution is assumed to be converged when the absolute errors between the values obtained from two successive cycles of the calculation at every point are less than a tolerance error throughout the field. In the present study, the required absolute error is equal to 1% of the incident wave height for both the wave amplitudes and the setup calculation.

3.2 Estimation of energy dissipation rate

In order to evaluate the dissipation rate D_B, the vortex energy supplied from wave energy by wave breaking is assumed to be equally transferred to the turbulence energy in a distance l_d given in terms of the local water depth h. The distance l_d is determined as

$$l_d = \begin{cases} 4\left(\dfrac{x_t - x}{x_t - x_b'} + 1\right) h & (x \le x_t) \\ 4h & (x > x_t) \end{cases} \tag{18}$$

where x_t is the wave transition point which is the boundary between the outer and inner regions in the surf zone. By using l_d, D_B is calculated as

$$D_B(x) = \int_{-\infty}^{x} t_d(x, x')\, dx' \tag{19}$$

where

$$t_d(x, x') = \begin{cases} 0 & (x \le x') \\ \dfrac{T_B(x')}{l_d(x')} & (x' \le x < x' + l_d(x')) \\ 0 & (x' + l_d(x') \le x) \end{cases} \tag{20}$$

3.3 Determination of transition and plunging points

The determination of the transition and plunging points is necessary for evaluating the energy distribution in the present model. For that sake, the transition and plunging points were measured for various incident waves on different bottom slopes.

Figure 1 shows the relation between the averaged value of l_t/h_{0b} and the bottom slope $\tan\beta$, where l_t is the distance from the breaking point to the transition point and h_{0b} the still water depth at the breaking point. The marks

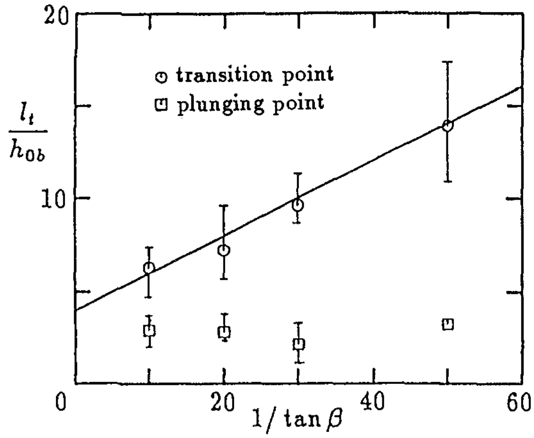

Fig. 1 Relationships between bottom slope and transition and plunging distance.

in the figure express the averages and the vertical lines express the fluctuations. The solid line expresses

$$l_t = \left(\frac{1}{5\tan\beta} + 4\right) h_{0b} \tag{21}$$

by which the transition point can roughly be estimated.

As for the plunging points, the averages are nearly constant and do not depend on the bottom slope. The distance l_p from the breaking point to the plunging point is expressed roughly as

$$l_p = 2.5 h_{0b} \tag{22}$$

4. Undertow Model

4.1 Vertical variation of mean shear stress and eddy viscosity

The Reynolds stress and the eddy viscosity coefficient are quantitatively evaluated from the energy dissipation rate by means of dimensional analysis in the model. By using dimensional analysis, Battjes (1975) obtained the representative velocity of turbulence q as

$$q \approx \left(\frac{D_B}{\rho}\right)^{1/3} \tag{23}$$

The vertically averaged mean shear stress τ_m in the horizontal plane and the vertically averaged mean eddy viscosity ν_m are expressed as

$$\tau_m = C_\tau \rho^{1/3} D_B^{2/3} \tag{24}$$

$$\nu_m = C_\nu \rho^{-1/3} h D_B^{1/3} \tag{25}$$

respectively. C_τ and C_ν are constant and taken to be 0.02 and 0.03, respectively, in this model.

The mean shear stress τ and the eddy viscosity ν_e are assumed to be linear functions of the vertical elevation z' from the bottom [Okayasu *et al.* (1988)] and are expressed as

$$\tau = a_\tau z' + b_\tau = \frac{0.04}{3} \rho^{1/3} D_B^{2/3} \left(\frac{5}{d_t} z' - 1 \right) \tag{26}$$

$$\nu_e = a_\nu z' = 0.06 \rho^{-1/3} D_B^{1/3} \frac{h}{d_t} z' \tag{27}$$

where d_t is water depth at wave troughs.

4.2 Vertical variation of undertow

Though the molecular viscosity ν is far smaller than the eddy viscosity ν_e in the surf zone in general, it cannot be neglected near the bottom or in the offshore region. The total viscosity is therefore defined as follows:

$$\nu_t = \nu_e + \nu \tag{28}$$

By using the eddy viscosity model, the relation between the mean shear stress τ acting on the horizontal plane and the steady current U in x-direction is expressed as

$$\tau = \rho \nu_t \frac{\partial U}{\partial z} \tag{29}$$

Substituting Eqs. (26), (27) and (28) into Eq. (29), the steady current U can be expressed as

$$U = \frac{a_\tau}{a_\nu} z' + \frac{a_\nu b_\tau - a_\tau \nu}{a_\nu^2} \log(a_\nu z' + \nu) + C_1 \tag{30}$$

where C_1 is an integral constant which is obtained in terms of the mass transport by waves M_t as

$$C_1 = -\frac{1}{2} \frac{a_\tau}{a_\nu} d_t - \frac{a_\nu b_\tau - a_\tau \nu}{a_\nu^3 d_t} (a_\nu d_t + \nu) \log(a_\nu d_t + \nu)$$
$$- \nu \log \nu - a_\nu d_t - \frac{1}{h} M_t \tag{31}$$

In order to obtain the same vertical distribution as what was proposed by Longuet-Higgins (1953) when $D_B = 0$, and to give a continuous solution in and out the surf zone, Eq. (30) is modified as

$$U = \frac{a'_\tau}{a_\nu}\left(z' - \frac{d_t}{2}\right) + \frac{a_\nu b'_\tau - a'_\tau \nu}{a_\nu^2}\left(1 + \log\frac{a_\nu z' + \nu}{a_\nu d_t + \nu} - \frac{\nu}{a_\nu d_t}\log\frac{a_\nu d_t + \nu}{\nu}\right) - \frac{1}{h}M_t \tag{32}$$

$$\left.\begin{aligned} a'_\tau &= a_\tau + \frac{\nu a^2 \sigma k}{2h^2 \sinh^2 kh}\left(3kh\sinh 2kh + \frac{3\sinh 2kh}{2kh} + \frac{9}{2}\right) \\ b'_\tau &= b_\tau - \frac{\nu a^2 \sigma k}{4h\sinh^2 kh}\left(2kh\sinh 2kh + \frac{6\sinh 2kh}{2kh} + 9\right) \end{aligned}\right\} \tag{33}$$

The values of the second terms in Eq. (33) are far smaller than those of the first terms in the inner region of the surf zone.

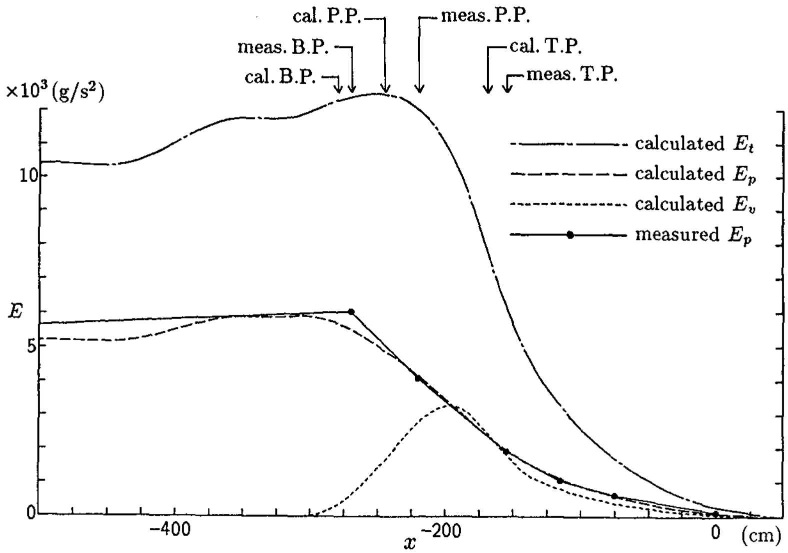

Fig. 2 Calculated and measured energy variations (1/20 slope).

5. Results

Figure 2 shows variations of the calculated values of the potential energy of wave motion E_p, the energy of the organized large vortexes E_v and the total energy of wave E_t for wave breaking on 1/20 constant slope. Measured potential energy is also shown in the figure. The period and height of the incident waves are 2.00 s and 8.50 cm, respectively. The calculated and measured value of E_p agree well. E_v takes almost the same value as E_p at the transition point (indicated by T.P.). E_p does not attenuate so much up to the plunging point (P.P.). The calculated values of the energy transfer rate T_B and the energy dissipation rate D_B are shown in Fig. 3. The hatched area corresponds to the rate of change of the vortex energy. The small value of D_B near the plunging point is consistent with the reality.

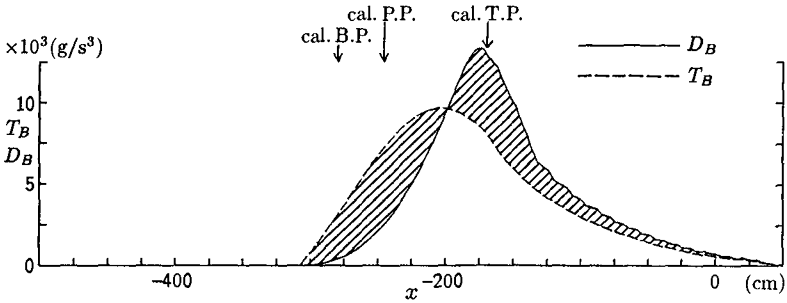

Fig. 3 Rate of energy transfer and dissipation.

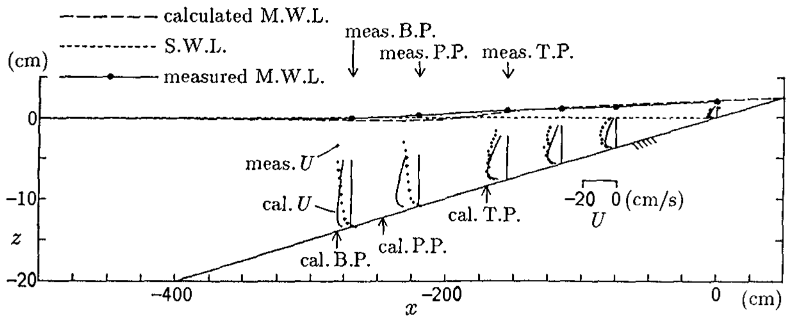

Fig. 4 Calculated and measured undertow distributions (1/20 slope).

Figure 4 shows a comparison of the calculated and the measured distribution of an undertow together with the bottom profile. The variation of the mean water level is also given in the figure. The undertow profiles are well evaluated except near the plunging point. The discrepancy near the plunging point may be caused by the insufficient accuracy of the evaluation of the energy dissipation rate or the energy and mass flux by the organized large vortexes in the outer region. Discrepancy is also found in the variation of setup around the plunging point, but the accuracy is good near the shoreline.

Figure 5 shows a result on a step-type beach. Period and height of the incident waves are 1.20 s and 9.24 cm in this case. The agreement between the calculated and measured values is good, although the calculated energy oscillates in the wave recovery zone. Comparisons of the distributions of undertow and variations of the mean water level are shown in Fig. 6. They agree well in the whole surf zone except around the plunging point, although the calculations were carried out only from the incident wave conditions. It can be concluded that the present model can compute the steady current distributions on various beach topographies with a good accuracy.

5. Concluding Remarks

In order to describe accurately the mechanism of the energy transfer during wave breaking, a model was presented in which the organized large vortexes were taken into account as a transmitter of energy in the energy transfer process from wave motion to turbulence. The mass and momentum fluxes by the organized large vortexes were also discussed.

By using the models of the energy distribution and the mass transport, a model was presented for the two-dimensional distribution of the undertow. The Reynolds stress and the eddy viscosity coefficient were quantitatively evaluated from the energy dissipation rate on the basis of the dimensional analysis. The variation of the mean water level in the surf zone was also predicted with a good accuracy by considering the momentum flux by the organized vortexes. The model can evaluate the distribution of the undertow on an arbitrary beach topography from the incident wave condition.

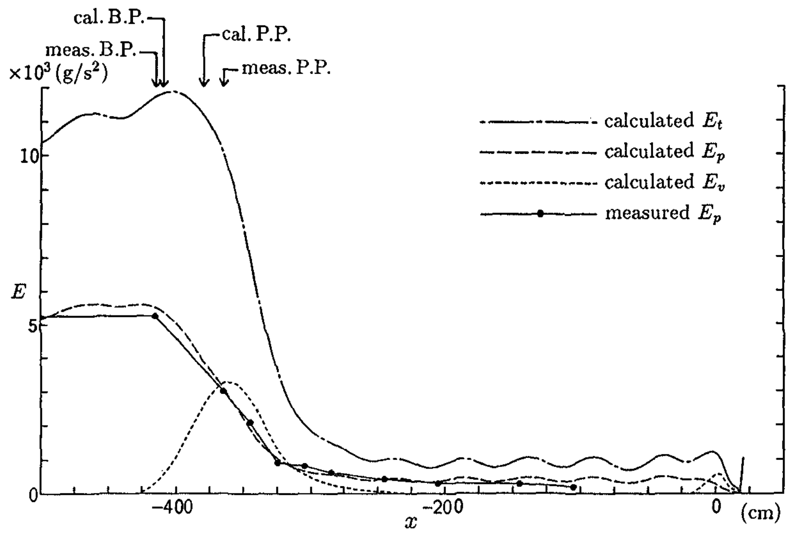

Fig. 5 Calculated and measured energy variations (step type).

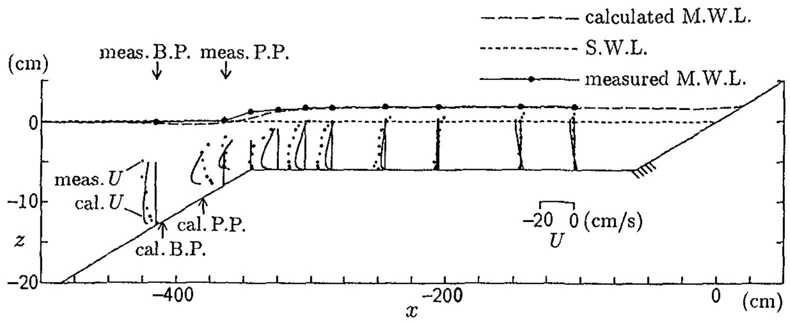

Fig. 6 Calculated and measured undertow distributions (step type).

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