CHAPTER 222

A NUMERICAL WAVE PREDICTION MODEL FOR PERSONAL COMPUTERS[1]

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ABSTRACT

A two-dimensional wave prediction model suitable for use on personal computers is described. The model requires the two-dimensional time-dependent wind field as input. Output consists of wave height, wave period, and wave direction estimates at all grid points on a computational grid representing an enclosed or semi-closed basin. Model predictions compare favorably with observations from a wave research tower in Lake Erie. A formula is provided to estimate how long a model simulation would take on a personal computer given the surface area of the computational domain, the grid size, and the computer clock speed.

1. Introduction

Numerical wave prediction models have been categorized in various ways according to their development as one-dimensional or two-dimensional, parametric or spectral, discrete or hybrid, and first, second, or third generation. The SWAMP Croup, 1985, provides a summary and intercomparison of the various model types. The simple empirical formulas for wave height and period estimation such as the SMB formulas (Bretschneider, 1970) or JONSWAP relations (Hasselmann et al., 1973) are basically one-dimensional in that they depend only on local wind speed and upwind fetch distance. Two-dimensional models take account of advective affects on the wave field by predicting the spatial distribution of wave energy over an entire two-dimensional domain. Until recently, the two-dimensional prediction models have only been implemented on multi-user mainframe or minicomputers while microcomputer (PC) users were limited to using empirical, one-dimensional wave prediction formulas for wave estimation. With improvements in the capabilities of the personal computers, it is now

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possible to make practical use of some two-dimensional models with relatively inexpensive computing machines.

This paper describes the formulation and implementation of a particular two-dimensional wind wave prediction model (Donelan, 1977, Schwab et al., 1984a and Liu et al., 1984) for use on personal computers. The model is a parametric type that numerically solves a local momentum balance equation on a computational grid covering a closed or semi-closed basin. Only actively generated wind waves are predicted, swell is not included. The model is driven by the two-dimensional, time-dependent wind field. Output includes estimates of significant wave height, peak energy period, and wave propagation direction at each grid point. The fact that the model is a parametric model and only needs to retain a few parameters describing the wave spectrum at each grid point instead of a complete spectral description makes it practical for use on personal computers.

2. Techniques

The basic model equations relate the time rate of change of wave momentum and the divergence of the wave momentum flux to wind forcing as follows:

$$\frac{\partial M_x}{\partial t} + \frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} = \frac{\tau_w^x}{\rho_w}$$

$$\frac{\partial M_y}{\partial t} + \frac{\partial T_{yx}}{\partial x} + \frac{\partial T_{yy}}{\partial y} = \frac{\tau_w^y}{\rho_w} \qquad (1)$$

The x and y momentum components are defined as

$$M_x = g \int_0^\infty \int_0^{2\pi} \frac{F(f,\theta)}{c(f)} \cos\theta \, d\theta \, df$$

$$M_y = g \int_0^\infty \int_0^{2\pi} \frac{F(f,\theta)}{c(f)} \sin\theta \, d\theta \, df. \qquad (2)$$

$F(f,\theta)$ is the wave energy spectrum as a function of frequency, f, and direction, θ. $c(f)$ is the phase speed. T_{xx}, T_{xy}, T_{yx}, and T_{yy} are the components of the momentum flux tensor defined as:

$$T_{xx} = \frac{g}{2} \int_0^\infty \int_0^{2\pi} F(f,\theta) \cos^2\theta \, d\theta \, df$$

$$T_{xy} = T_{yx} = \frac{g}{2} \int_0^\infty \int_0^{2\pi} F(f,\theta) \cos\theta \sin\theta \, d\theta \, df \qquad (3)$$

$$T_{yy} = \frac{g}{2} \int_0^\infty \int_0^{2\pi} F(f,\theta) \sin^2\theta \, d\theta \, df$$

If we assume that deep water linear theory applies and that wave energy is distributed about the mean wave direction as cosine squared, i.e.,

$$F(f,\theta) = \frac{2}{\pi} E(f)\cos^2(\theta - \theta_o) \quad \text{for } |\theta - \theta_o| \le \frac{\pi}{2} \qquad (4)$$

then the momentum fluxes can be expressed as (Schwab et al., 1984a)

$$T_{xx} = \frac{g\sigma^2}{4} \left[\cos^2\theta_o + \frac{1}{2}\right]$$

$$T_{xy} = T_{yx} = \frac{g\sigma^2}{4} \left[\cos\theta_o \sin\theta_o\right] \qquad (5)$$

$$T_{yy} = \frac{g\sigma^2}{4} \left[\sin^2\theta_o + \frac{1}{2}\right]$$

where σ^2 is the variance

$$\sigma^2 = \int_0^\infty \int_0^{2\pi} F(f,\theta) \, d\theta \, df. \qquad (6)$$

In the numerical model, only the two components of the wave momentum vector (M_x, M_y), the wave phase speed at the peak of the spectrum (c_p), and the total energy in the spectrum (σ^2) are calculated at each grid point. To calculate the wave momentum vector components, the time derivatives in (1) are represented by central differences. An upwind difference scheme is used for the momentum flux advection terms. The time step is determined dynamically as

$$\Delta t = \Delta s/\sqrt{2}c_m \tag{7}$$

where c_m is either the maximum wind speed during that time step or the maximum wave phase velocity calculated at the previous time step, whichever is greater and Δs is the grid interval. The wave momentum fluxes in (4) are calculated using the approximate relation

$$\sigma^2 = \frac{c_p|M|}{g} \quad . \tag{8}$$

Although this relation between variance (or energy), peak energy frequency ($f_p=g/2\pi c_p$), and integrated wave momentum is basically empirical, it applies quite well over a wide range of conditions.

The numerical model is based on a wave momentum conservation equation rather than wave energy conservation so that the following formula can be used for the wind input source function:

$$\frac{\tau_w}{\rho_w} = 0.028\ c_d|\vec{u} - 0.83\ \vec{c}_p|(\vec{u} - 0.83\ \vec{c}_p) \tag{9}$$

where u is the 10 m wind vector and c_d is the form drag coefficient defined as

$$c_d=[0.4/\ln(50/\sigma)]^2 \tag{10}$$

with σ in meters. This formula for the drag coefficient is based on a logarithmic atmospheric boundary layer profile with a surface roughness height of $\sigma/5$. Note that when the wind and waves are travelling in the same direction,the momentum input function goes to zero when $u = 0.83\ c_p$ or $c_p = 1.2\ u$ (full development). This form of wind input is useful in describing the relative amounts of atmospheric momentum being expended on wave generation and the amount going into the water column to generate currents (see Donelan, 1977).

In the numerical scheme, after the new wave momentum vector is calculated from (1), peak energy frequency is found from the empirical relation

$$\frac{g^2\sigma^2}{u^4} = 6.23\mathrm{x}10^{-6} \left(\frac{f_p u}{g}\right)^{-10/3} . \qquad (11)$$

By using (8) this relation can also be written as

$$f_p = 0.0179 \left(\frac{u^2}{|M/g|^3}\right)^{1/7} . \qquad (12)$$

It should also be pointed out, however, that other values of the coefficient and exponent in the empirical relation (11) between nondimensional energy and nondimensional peak energy frequency could be used and, in fact, are currently being evaluated in the model.

3. Results

The numerical model has been implemented on an IBM-XT type personal computer with an 8087 numeric coprocessor running at a clock rate of 4.77 MHz. With this configuration, a thirty day hindcast for a 10 km grid covering Lake Erie (253 grid squares) takes about one hour. Because the time step (7) in the numerical model is proportional to the grid interval, the total computation time is proportional to the cube of the grid interval, i.e., doubling the resolution of the computational grid (halving the grid interval) increases computation time by a factor of eight. Model tests on several different types of mainframe and microcomputers using various lake grids led to the development of a rough formula for the expected computation time. As shown in (7), the total computation time in seconds for one day of simulation, T, is going to depend on the maximum phase speed of the waves predicted during a run, but for typical wind speeds of 5-10 m s^{-1}, T can be estimated for a lake of surface area A with a computational grid of grid interval Δs and a computer running at a CPU clock rate of R Mhz as

$$T = \frac{1.7 \times 10^4 A}{(\Delta s)^3} \left(\frac{4.77}{R}\right) \frac{\text{m-sec}}{\text{day}} . \qquad (13)$$

For Lake Erie with $A=2.54\mathrm{x}10^{10}$ m^2, Table 1 shows some typical expected run times on a 4.77 Mhz machine with various grid intervals.

Table 1. Typical PC run times for various grid sizes on Lake Erie

Δs (km)	T (sec/day)
15	128
10	432
5	3456

In September and October of 1981, a solar-powered wave research tower was deployed in the eastern part of the central basin of Lake Erie (see Schwab et al., 1984b for the details). The tower was 6 km offshore in 14 m of water. Instruments on the tower measured wave height, wave period, and wave direction at hourly intervals. The numerical model was run for this entire period using the wind measured at the tower for the forcing function (9). The results from the model are compared to observed wave parameters in Fig. 1.

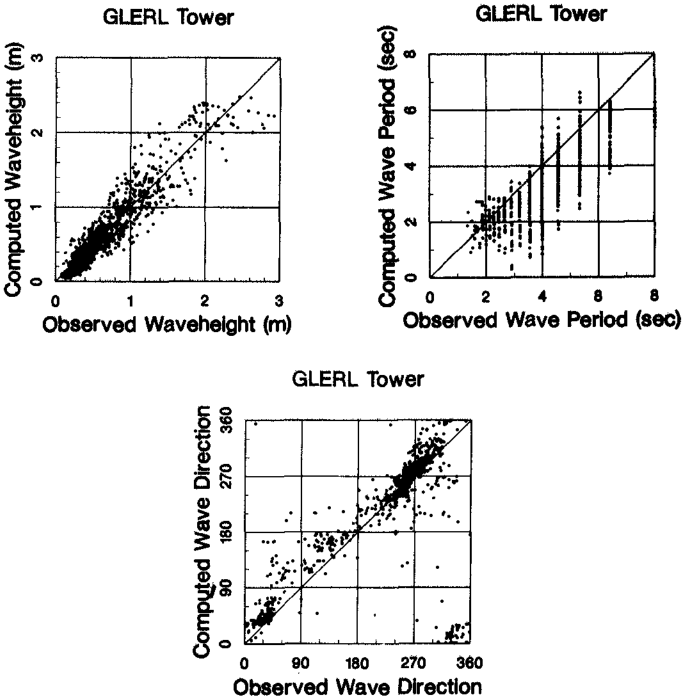

Figure 1. Comparison of observed and computed wave height, period, and direction in Lake Erie for September and October, 1981.

The agreement for wave height is quite good, with a root mean square error of 0.20 m and a correlation coefficient of 0.93 between computed and observed values. The computed wave periods, however, tend to underestimate the observed wave period. This may be due to using (12) to estimate wave period. Other formulas of the general form of (14) are currently being tested. Wave direction is modeled quite well, except for waves travelling in the 90-180^o range. These are small waves generated from winds coming directly offshore (6 km fetch) so that it is very difficult to measure dominant wave direction accurately.

The model has also been extensively tested against other field data from the Great Lakes (typical fetch distances 100-300 km) and has been applied to Puget Sound, Hudson's Bay and the Beaufort Sea. Calibration and verification runs from deep and intermediate water depths show typical accuracies of 10-20% for wave height and 20-30% for wave period. The model is currently being used operationally by the National Weather Service for routine wave forecasting for the Great Lakes.

4. Conclusions and Recommendations

Although this type of model applies only to active wave generation, it can provide practical estimates of design waves for lakes, bays, estuaries, and semi-enclosed coastal areas. The main limitations of the model are that the wave spectrum is assumed to be single-peaked (no swell or multiple wind seas) and that shallow water effects are ignored. It is possible to extend the parametric formulation of the model to finite depth water by incorporating depth-dependent phase speed and depth-dependent spectral shape functions in the model equations along with a bottom dissipation term. Experiments with a shallow water model and comparisons of the results with field measurements are currently underway.

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