CHAPTER 210

COASTAL GENESIS

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ABSTRACT
In order to investigate the effect of enchanced sea level rising (greenhouse effect), of sand mining in the North sea and the large-scale effects of beach nourishment, the Rijkswaterstaat (Governmental Board for Ways, Waterways and Harbours) started the project "Coastal Genesis". This paper deals about research, carried out at the Delft University of Technology in the scope of this project.
The behaviour of the Holland coast at three timescales, i.e. 15 years, 1000 years and 6000 years is considered. Use is made of line models. New techniques are developed for finding the coastal constants of these models (the use of Kalman filtering) and for simulating the effect of sea level rise on coastal accretion and erosion. An extensive set of data on coastal topography, the JARKUS dataset is analyzed.
It is concluded that the effect of cross-shore transport becomes more and more dominant compared to the longshore transport when the time scale increases. A physical conception is displayed, which explains more or less the accretion of the Dutch coast between 4000 Before Christ and the Roman times, and the erosion later on.

I. INTRODUCTION

The coastal management of Rijkswaterstaat requires a strategy of overall coastal defence, starting from the general idea that the Netherlands should be kept in shape, however not against unlimited expense. According to a statement of the Dutch queen "the coast is safe" since the stormsurge barrier in the Eastern Scheldt came in operation. What should be done to keep it safe and how should aspects of increasing importance of nature preservation and conservation of areas needed for water purification and recreation be encountered? How to cope with enchanced sea level rise?
Good predictions are necessary. Understanding of how the present was generated from the past might be of help, and for this goal Dutch geologists, geographers and civil engineers of Rijkswaterstaat, Delft Hydraulics Laboratory and a number of universities are collaborating in the project "Coastal Genesis". For a further description may be referred to Vellinga and Zitman (1988).
This paper deals with research at the Delft University of Technology carried out in the scope of this project. The behaviour of the

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uninterrupted central part of the Dutch coast in 6000 years is considered. Although "the" truth never will be found, it will be seen, that the effect of onshore-offshore transport dominates more and more the effect of longshore transport when larger time scales are considered. In this paper the general conception is captured by the use of line models. Constants occurring in these models are found from hindcasting coastal behaviour. Another paper (Bakker et al.,1988) also gives the relations between these constants and the wave characteristics. Ch.II gives a description of the Dutch coast in the last 15 years and ch.III describes a newly developed way of finding the coastal constants. In ch.IV a one-line model concerning the behaviour in the last 1000 years is given, leading to constants, comparable with the ones for the last 15 years. In ch.V the qualitative effects of sea level rise are modelled (only considering cross-shore transport) and finally ch.VI contains some conclusive remarks.

II. 15 YEARS

Coastal behaviour during the last two decennia can be investigated rather accurately due to the Jarkus dataset, which contains yearly measurements of coastal profiles along the entire dutch coast, up to a depth of 5 a 6 m below MSL. The distance between the ranges is 200 m. A more detailed description is given by Bakker and de Vroeg (1987,1988[a,b]).
This dataset can be used to study the bottom topography and changes of the coastal profile. Also, the measurements provide a useful source for verification and calibration of coastal models.

First the bottom topography is considered. Fig 9[a] shows the situation. Fig 1[a] shows a three-dimensional picture of the coastal area between Den Helder and IJmuiden (see also fig 9[a]), as measured in the year 1984. The scales are distorted: the distance between Den Helder and IJmuiden is 55 km, while the width of the considered coastal zone is 800 m. The landward boundary is chosen near the dune foot. In the north, at Den Helder, the influence of the Texel inlet is visible. At Petten a large seawall prevents the coast from moving landward. Due to a structural regression of this part of the coast the wall is now situated seaward compared to the rest of this region. At IJmuiden there is a physical boundary, formed by large harbour moles.
The picture clearly shows the coast is not uniformly shaped. Not only differences in the slope of the profile are visible, but also shape and number of breaker bars vary along the coast. It appears that a gentle beach slope results in a great number of bars. The same can be concluded from fig 1[b], which shows the coastal area south of IJmuiden over a distance of 59 km. (At the harbour of Scheveningen no measurements were available).
As a result of the breaker bars the coast is constantly transforming, which can be derived from fig 2[a,b,c]. These figures show "3-D-time diagrams" in which the changing of one coastal profile during 15 years is displayed by plotting the profile in successive years next to each other, thus resulting in three-dimensional figures, however, with the time instead of the distance along the coast as third dimension. These diagrams enable the investigation of the migration of breaker bars. It appears that the bars are moving in a seaward direction, however, with strongly varying velocities. In the north, between Den Helder and Callantsoog, the bars are very stable (fig 2[a]). Between Callantsoog and IJmuiden a seaward velocity of 15 a 20 m/year is found (fig 2[b]). South of IJmuiden the bars are moving very fast: 50 m/year (fig 2[c]). No correlation is found between the structural movement of the coast and the migration of the bars.

The changes of the profile due to the breaker bars can be expressed as a fluctuating cross-shore transport within the surfzone. In fig 3 this transport is compared to other sources of transverse transport. Without any doubt some interaction between the tidal zone and the inshore exists, however according to present estimates it is of no great importance where the behaviour of the beach on a time scale of decennia is concerned. On this time scale gradients in the wave induced longshore transport give a possible explanation of the development of the curved Dutch coastline (although not the only possibility). Fig 4 shows an estimate of the magnitude of this transport, based upon Bakker (1971) and Bakker and de Ras (1971). Some simple models based on this longshore transport exist, and will be discussed in this paper.

Though many of the observed characteristics are not yet fully understood, knowledge derived from this "phenomenological approach" can be made of use for statistical purposes. The 3-D-time diagrams in fig 2 show it is difficult to determine any structural movement of the profile. This structural movement is small compared to the fast movement of large amounts of sand due to the breaker bars. This is a problem when trying to use the Jarkus dataset for the calibration of models. One can treat the motions of the bars as a periodical feature or as a random noise, each treatment giving its specific problems (Bakker and de Vroeg (1987,1988[b]). Kalman filtering appears to be a tool for finding the vital characteristics.

III. THE LINEAR KALMAN FILTER

The Kalman filter combines the advantages of the deterministic and the statistical approach, and has proven to be very suitable for recursively updating parameter estimates or time-varying model coefficients, based on the latest data information. It is an algorithm which uses measurements, a physical model of the system, and knowledge of the inaccuracy (noise) of both the measurements and the model, in order to achieve an optimal estimate of system behaviour (Kalman,1969). The method is based on two linear equations:

$$\text{System Equation} \quad : \underline{x}(t) = F(t)\,\underline{x}(t\text{-}1) + \underline{w}(t) \qquad (1)$$

$$\text{Measurement Equation} \quad : \underline{z}(t) = H(t)\,\underline{x}(t) + \underline{v}(t) \qquad (2)$$

where $\underline{x}(t)$ is the system state at time t.
$\underline{z}(t)$ is a measured state, not necessarilly the same as the system state, however, it has to be linearly related to the system state (in the linear filter).
$F(t)$ is a transformation matrix, transforming the system state at time t-1 to the state at time t.
$H(t)$ is a transformation matrix, describing the linear relation between the system state and the measured state.
$\underline{w}(t)$ is the white noise, affecting the state $\underline{x}(t)$.
$\underline{v}(t)$ is the white noise, affecting the observations $\underline{z}(t)$.

$F(t)$ and/or $H(t)$ contain the physics of the system. Both $\underline{w}(t)$ and $\underline{v}(t)$ refer to additional noise, generated during one time step.
Using the sytem equation it is possible to estimate the system state $\underline{x}(t\text{-})$ one time step ahead (- denotes an estimate before the measurements are taken into account). The term $\underline{w}(t)$ is added because the transformation through $F(t)$ is not perfect: an error will occur which is not known at every time. Only the statistical characteristics of this error are known or can be estimated. Assuming a Normal distribution of this noise with mean 0 and variance $Q(t)$, this can be denoted as $\underline{w}(t) \approx N(0,Q(t))$. This variance $Q(t)$ is used in the filter

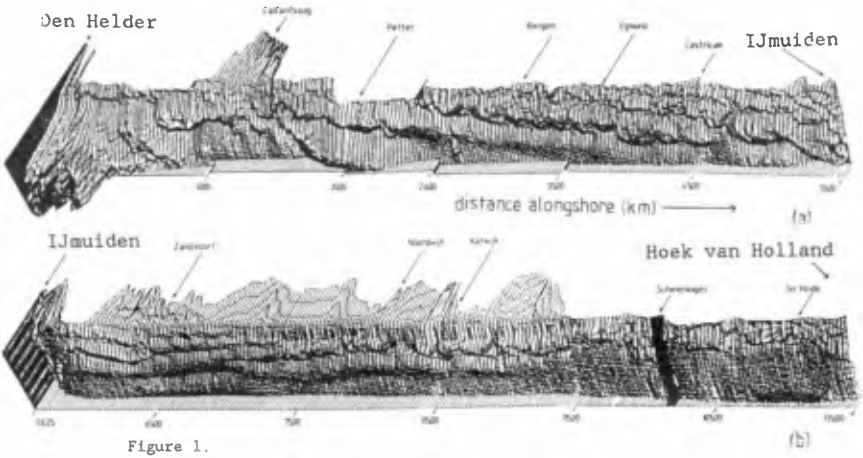

Figure 1.
Three dimensional picture of the coast between
(a) Den Helder and IJmuiden (b) IJmuiden and Hoek van Holland
(see fig 9[a] for situation Holland coast)

Figure 2.
3-D-time diagram of profile between (a) Den Helder and Petten
(b) Petten and IJmuiden (c) IJmuiden and Scheveningen

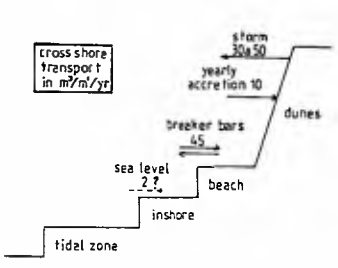

Figure 3.
Estimated cross-shore transport along the central coast of Holland (storm arosion is balancad by on-shore windtransport during quiet conditions)

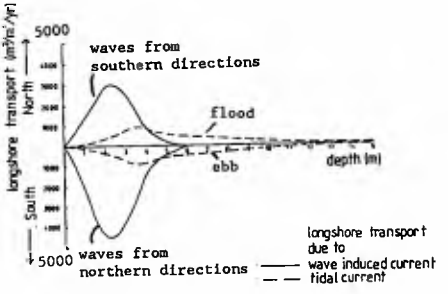

Figure 4.
Longshora transport, along the coast of Holland as a function of depth.

algorithm. Like all other noise terms it is used in the form of a covariance matrix, so it is possible to take into account not only variances but also noise covariances.
Applying the measurement equation (disregarding the noise) results in an expectation of the measured state at the considered time. This equation is also affected by errors, which are characterized by $\underline{v}(t) \approx N(0,R(t))$. The variance R(t) will appear in the filter algorithm. When the measurements $\underline{z}(t)$ actually become available, these are taken into account to get an improved estimate $\underline{x}(t+)$, using the equation:

$$\underline{x}(t+) = \underline{x}(t-) + K(t) \{ \underline{z}(t) - H(t)\, \underline{x}(t-) \} \qquad (3)$$

where $\underline{x}(t+)$ is the optimal estimate.
$\underline{x}(t-)$ is the estimate of the state at time t before the measurements are taken into account.
$\underline{z}(t) - H\, \underline{x}(t-)$ is the difference between the measurements and their expected value.

K(t) is the "gain matrix", the essence of the Kalman filter. It has two functions: 1: it re-transforms the correction on $\underline{z}$ to a correction on $\underline{x}$, and 2: it gives a weight to the importance of the correction, varying from zero (perfect theory not to be disturbed by sorded measurements) to one (What You See is What You Get). The expressions for K(t), depending on H(t), Q(t) and R(t) are given in the Appendix, which may be written more or less symbolically as:

$$K(t) = \frac{1}{H(t) \{ 1 + \frac{\text{noise in measurements}}{\text{noise in latest estimate}} \}} \qquad (4)$$

The method has been applied to parts of the Dutch coast, using as physical model the line-theory and estimating the model coefficients. The physical conception involved is the existing of a (set of) coastal constant(s) $\underline{x}$, stationary in time. This means that F(t) in (1) should be the unit (identity) matrix. Improved estimates of $\underline{x}$ are made by checking a calculated coastline (or for the two-line theory: lines of beach and inshore) against measured values $\underline{z}(t)$. This application is best demonstrated by the following example, based on the single-line model which is described in the next chapter, see equation (8). The successive steps of the procedure are shown in fig 5.
In order to find the coastal constant s in (8) the system equation (1) becomes:

$$s(t) = s(t\text{-}1) \qquad (5)$$

with inaccuracy characterized by $w(t) \approx N(0,Q)$

Due to the noise w(t) this "constant" is given the opportunity to change a bit each time step. This noise is thought to have a Normal distribution (Gaussian) with mean 0 and variance Q. In this particular case Q is assumed to remain stationary and uniform along the coast.
Using the Jarkus dataset "measured coastlines" can be determined in each year, giving in each range the mean of the positions of the various contourlines between NAP -5m and NAP +15m (fig 8)(NAP≈MSL). The measurement equation (2) provides the link between s(t) and these measured coastlines, using an explicit difference scheme for the Pelnard-Considère equation (8). This can be rewritten as

$$y_i(t) = y_i(t\text{-}1) + B \{ y_{i-1}(t\text{-}1) - 2y_i(t\text{-}1) + y_{i+1}(t\text{-}1) \}\, s \qquad (6)$$

where y denotes the seaward coordinate of a coastline (fig 8).
i denotes the distance along the coast, t the time.
$B = \frac{1}{h}\frac{\Delta t}{\Delta x^2}$; Δx = distance between two adjacent ranges.

Thus for the one-line theory the measurement equation (2) becomes:

$$\begin{bmatrix} y_2 \\ y_3 \\ . \\ . \\ y_{n-1} \end{bmatrix}_t = \begin{bmatrix} B\,(\,y_1 - 2y_2 + y_3\,) & y_2 \\ B\,(\,y_2 - 2y_3 + y_4\,) & y_3 \\ . & . \\ . & . \\ B\,(\,y_{n-2} - 2y_{n-1} + y_n\,) & y_{n-1} \end{bmatrix}_{t-1} \begin{bmatrix} s \\ 1 \end{bmatrix}_t \qquad (7)$$

with inaccuracy characterized by $\underline{v}(t) \approx N(0,R)$

H(t) (the matrix $[...]_{t-1}$ in (7), see equation (2)) is derived from the measured coastlines at time t-1.
$\underline{v}(t)$ is the Gaussian noise with mean 0 and variance R, in this particular case containing two sources of error: measurement errors and inaccuracy of the model. It appears that it is difficult to determine this variance on physical grounds. So this noise is estimated after study of the measurements and assumed to remain constant with time. This variance can be of magnitude of several tens of m^2 to several hundreds of m^2.

If the matrices Q and R and an initial estimate of s are known the filter procedure can be started, resulting in a "time history" for the observed model coefficient s. The result shown in fig 6 (s = 1 a 1.2 * 10^6 m^3/year) is based on measurements for the coast between IJmuiden and Scheveningen. Results for the two-line model and the use of those coastal constants for making hindcasts and predictions will be exposed by Bakker et al. in another paper (1988).

IV. 1000 YEARS

Fig 7 shows the accretion and erosion of the coast of Holland, as follows from ancient registrations of the site of the dune foot in the last 100 years according to Edelman (1961). Edelman considers the coast as principally accreting, however with "sinks" at the Northern and Southern boundary. A possible explanation for the coastal behaviour, as indicated already by Edelman and Eggink (1963) is based upon the conception of Pelnard-Considère (1954). This involves that a concave coast accretes and a convex coast erodes. The ruling equation is:

$$\frac{\partial y}{\partial t} = \frac{s}{h}\frac{\partial^2 y}{\partial x^2} \qquad (8)$$

where x,y are the coordinates of the coastline (x in longshore direction, y in seaward direction), t is the time, h is the thickness of the layer of the coast, which is supposed to participate in the process of accretion and erosion (fig 8) and s is the rate of change of coastal transport (m^3/year/radian) as function of the coastal direction. Equation (8) is based on the continuity equation and a transport equation, which read respectively:

$$h\frac{\partial y}{\partial t} + \frac{\partial S}{\partial x} = 0 \qquad (9) \qquad\qquad S = S_0 - s\frac{\partial y}{\partial x} \qquad (10)$$

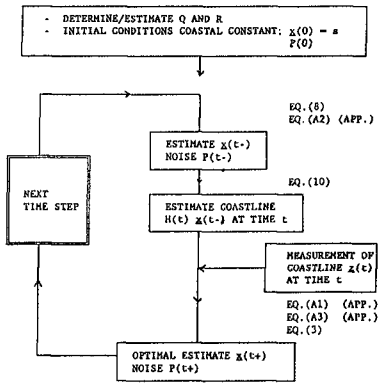

Figure 5.
Procedure Kalmanfiltering.

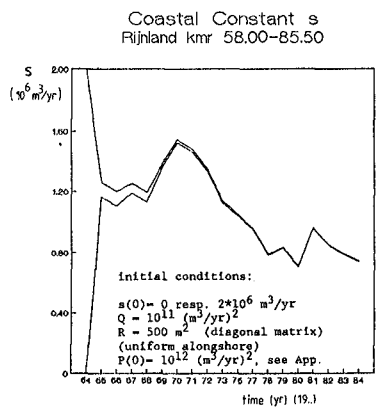

Figure 6.
Value of coastal constant s,
found by Kalman filtering

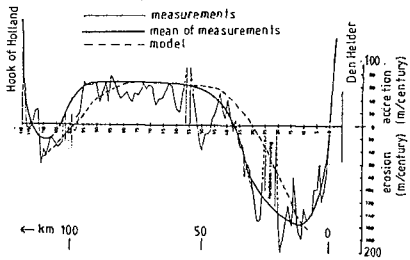

Figure 7.
Accretion and erosion of the coast of Holland
during the last 100 years, after Edelman (1961)

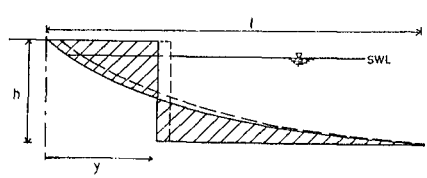

Figure 8.
Line schematization of a coastal profile

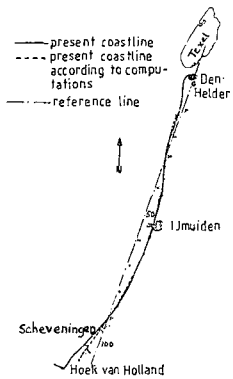

Figure 9[a].
Model-computed coastline,
compared with real coastline

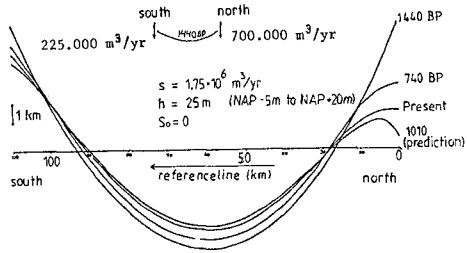

Figure 9[b].
Result line-model and used constants
on a 1000 years time scale

where S is the littoral transport and S_0 is the littoral transport along a coast, parallel to the x-axis.
This physical concept allows for a simplified one-line model concerning the behaviour of the Holland coast in the last 1000 years (between km 10 and km 100) starting from the following initial and boundary conditions (fig 9^b):
a. the initial coast is assumed to have a parabolic shape;
b. at the boundaries the coast is assumed to erode according to $\sqrt{t}$ following the Pelnard-Considère solution for a "negative delta" (1954).
This implies, that the coastal development is schematized according to a parabolic shape, stretching itself, combined with two "negative delta's" on both sides.
Implicit assumptions are a constant subtraction of material at the boundaries, starting more or less abruptly. Physically, this can be justified at the Northern side because of the developing Texel inlet, leading to the Wadden shallows and, at the Southern side, sand subtraction by the emerging outlet of the river Rhine. Development of the Texel inlet was triggered because of the burning and dewatering of existing peat layers (Edelman,1974) which caused the strong development of this inlet, starting about a millenium ago. Since then, the Wadden shallows tend to keep themselves at mean sea level (despite sea level rise). The reason for this is a subtle mechanism of sand, being brought to this shoals during flood tide with rather high flood velocities and coming to rest on this shoals during the ebb with rather low velocities. Already in 1949 Van Bendegom reported about this socalled "sand hunger" of the Wadden shallows.
Fig 9^a shows the present coastline according to the line model and in reality. Fig 9^b gives constants used in the applied model. A value of $s/h = 71 * 10^3$ m^2/year is found, leading with a depth h of 25 m (20 m dune height + 5 m water depth) to $s = 1.75 * 10^6$ m^3/year. The dashed line in fig 7 gives the present coastal erosion/accretion according to the model, compared with the Edelman data. Details are given by Bakker and Smit (1987).
However, the following objection against the model can be made. It appears (Ligtendag en Borger,1987) that the middle part of the dutch coast showed erosion from 1600 A.D. to 1800 A.D. instead of accretion, as found in the last century. Probable cause is wind erosion which is not taken into account in the Pelnard-Considère theory. Only since the 19^{th} century the dune front is protected by planting marram grasses etc.

V. 6000 YEARS

Where for "short" terms longshore transport (only) can provide some explanation of coastal behaviour, for a 6000 year period sea level rise and cross-shore transport become vitally important.
Fig 10 shows the sea level rise in the course of time since 10000 Before Present (BP) according to Jelgersma (1979). Fig 11, from Jelgersma et al. (1985), shows the submerging of the North Sea in the last milennia. Fig 12 shows the development of the Dutch coast according to Jelgersma et al. (1985). Apart from a regression during the period of fast sea level rise a transgression took place between ca. 5500 BP and 3000 BP (van Straaten (1965), Zagwijn (1974), Jelgersma et al. (1985), Roep (1984)). Afterwards, erosion started again.

Is it possible to make a conceptual model of this process of sea level rise? A good model should be able to reproduce as well the coastal transgression from 5500 to 3000 BP as the erosion afterwards.
In the present preliminary stage, it has only been attempted to make a

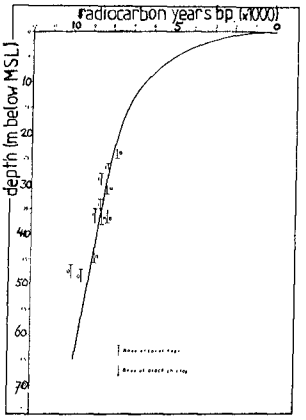

Figure 10.
Sea level rise during
the last milennia,
after Jelgersma (1979)

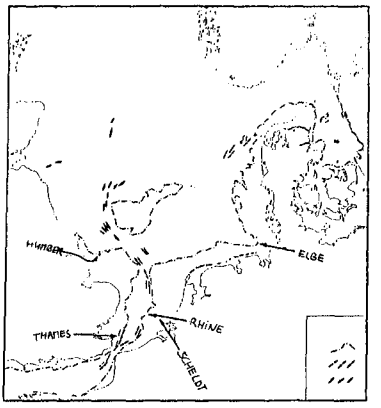

Figure 11.
The submerging of the North Sea
in the last milennia,
after Jelgersma et al.(1985).
-.-.- Coastline 8000 B.P.
sealevel 30m below present
level.
/// Old sediment ridges

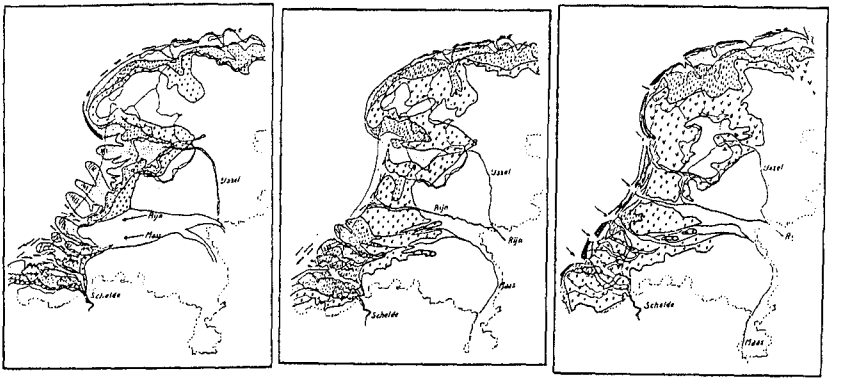

Figure 12.
Development of the Dutch coast according to Jelgersma et al.(1985)

Time (years Before Present)	: (a) 5500	(b) 4600	(c) 2000
Sealevel (m Below Present Level)	: 7	3.5	1

model which gives qualitative agreement, without paying too much attention to quantitative accuracy. Before explaining this model, some literature concerning this subject will be reviewed.
A very simple model for the response of a coast on sea level rise is provided by Bruun ((1983) and earlier papers) (fig 13). He starts from the assumption, that the sea level rise is so slow, that the upper part of the coast - up to a certain socalled "closure depth" - will always be able to adjust itself and will keep a cross-section according to an equilibrium profile. From simple geometry and continuity it shows, that the coastal erosion will be equal to the cotan of the mean slope (1/h in fig 13) times the mean sea level rise a. Here h denotes the socalled "closure depth", i.e. the depth up to which eroded material settles itself.
According to the authors it will always cost time to attain the equilibrium profile. They agree with Wright et al. (1984), that short-term processes take place with a small closure depth, where a larger part of the profile will be involved in long-term processes; the time necessary to reshape the profile to an equilibrium profile will be longer. Fig 14, after Wright et al. elucidates this idea.
The time-dependent adapting of a coastal profile to its equilibrium profile is simulated mathematically by Bakker (1968) and Swart (1974). These authors schematized the coastal profile essentially to two areas, a beach and an inshore, each represented by a line in plan view (fig 15). Beach and inshore are separated by a horizontal flat between. The region seaward of the inshore is assumed to be horizontal; here no cross-shore transport takes place. According to their schematization and according to experiments of Swart, if longshore transport plays no role, the line of beach and inshore will tend to reach an equilibrium distance W in a negative-exponential way.
The model starts from the assumption, that the cross-shore transport S_y depends on the overall coastal slope:

$$S_y = s_y \{y_1 - (y_2 - W)\} \qquad (11)$$

where S_y is positive in seaward direction, y_1 and y_2 denote the position of the line of beach and inshore (fig 15, i.e. the mean of the positions of the equidistant contourlines of the beach and the inshore respectively) and W denotes the equilibrium distance.
Basic physical thought behind the model is, that a slope determines the cross-shore transport; that two counteracting mechanisms determine this slope, where one mechanism increases the offshore transport when the slope increases (gravity mostly) and the other determines the onshore transport and increases, when the slope becomes flatter (for instance the asymmetry of the orbital motion). Making a Taylor series of the transport in the vicinity of the equilibrium slope gives a dynamical equation of the kind as given by (11). Implicitely it is assumed that the cross-shore transport is not affected by local changes in slope of beach and inshore separately, as long as the mean position of the beach, c.q. the inshore remains the same. Some motivation for this assumption can be found from fig 2: like a huge rake the breaker bars correct differences in large-scale slope every three to four years and the local slope is a very short-term feature, effected by the breaker bars and has no essential importance.
This philosophy also more or less determines the lower edge of the inshore, which can be considered as a kind of "closure depth" as well. The theory only holds when the profile of beach and inshore respectively can adapt itself much faster to changes than at the time scale of the coastal process to be schematized by the two-line theory. Looking to fig 3 it might be expected that cross-shore transport below the level, where the breaker bars decay (say 5 m below MSL) will be

much smaller than above this level. Therefore, this provides a logical closure depth. The characteristic time scale T_0 for exponential decay according to the two-line theory equals:

$$T_0 = (h_1 h_2)/(s_y h) \quad (12)$$

where h_1 and h_2 are the thickness of the layer of beach and inshore respectively and h equals $h_1 + h_2$.
In the case of sea level rise, cross-shore transport will take place between the inshore and the layer below, however at a larger time scale (slower) than between beach and inshore. Furthermore, it might be expected that the slope between the inshore and this lower layer to a great extent determines the cross-shore transport between those layers. In other words: the local slope will be more of importance than the overall slope and therefore a two-line approach will not be sufficient. The ideas of Wright (variable closure depth) and of Bakker and Swart (negative-exponential tending to an equilibrium profile) can be combined in the following way. To simulate sea level rise, a 6-line program has been made, starting from 6 layers with seaward distance with respect to a reference point of y_1, y_2,... y_6, each layer having a height of 3 m; the cross-shore transport S_{y1}, S_{y2}... S_{y5} between the first and the second, the second and the third.. etc. layer is related to y_1-y_2, y_2-y_3,... in the following way:

$$S_{yn} = s_{yn} ((y_n - L_n) - (y_{n+1} - L_{n+1})) \quad (13)$$

where n = 1 to 5 and L_n denote the position of y_n in an equilibrium position. The characteristic time T_n is defined accordingly:

$$T_n = (h_n h_{n+1})/s_{yn} \quad (14)$$

Fig 16 shows the schematization and the assumed time scales T_n. The values of T_4 and T_5 are pure guesses: for some justification of the values of T_1 to T_3 is referred to Bakker and Delver (1986).
It is assumed, that the present profile is an equilibrium profile (fig 17^a) and that 6000 BP the rising of the sea level was so fast, that no time was available to bring the profile in an equilibrium shape; the profile of 6000 years ago is assumed to be the present profile, in which the water level is 6 m lowered (fig 17^c); the water level at 6000 BP was 6 m lower than the present level (fig 17^b). This "sudden" exposure of the present coastal area to the waves is caused also for a large part by the degradation and erosion of former coastal barriers in the North Sea, which became submerged. With respect to this conception is referred to Niederoda et al. (1985) for the behaviour of coastal barriers in general and to van Straaten (1985) for the behaviour of the Holland coast. Fig 18^a (in which the third dimension denotes the time) shows how, according to the model, the coast would have been accreted, if the sea level had not risen since 6000 BP; in fig 18^b this sea level rise has been taken into account. In fig 18^a the coast accretes, because the profile of fig 17^c is much flatter than the equilibrium profile; in fig 18^b erosion starts again because the upper part of the profile has formed nearly an equilibrium profile and erodes according to the Bruun conception (see above). For details concerning the way of computation is referred to Smit (1987).
Thus it shows, that the model gives qualitative agreement with the features, found in nature.

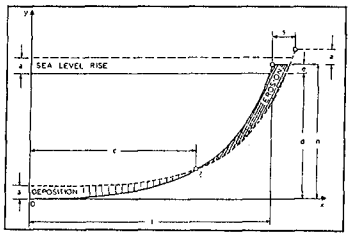

Figure 13.
The Bruun model, translation of coastal profile due to sea level rise.

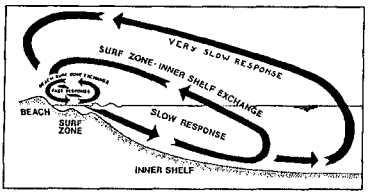

Figure 14.
Interaction of different parts of the profile, after Wright et al. (1984)

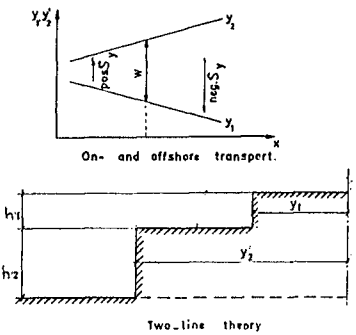

Figure 15.
Schematization in beach and inshore according to Bakker and Swart.

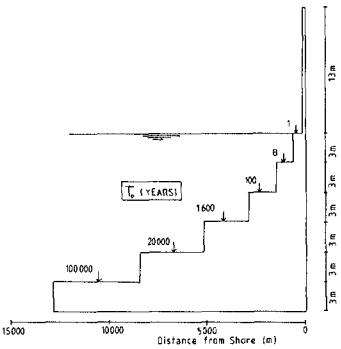

Figure 16.
Schematization n-line model and time scales.

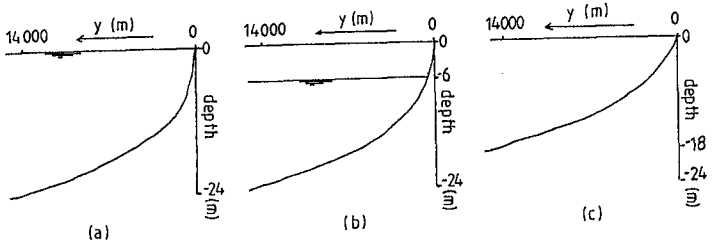

Figure 17.
Intial profile of the coast.
(a) Present profile
(c) Assumed profile of 6000 BP

VI. CONCLUSION

If coastal behaviour is considered at increasing time scales, beginning at a time scale of decennia and going to milennia, the effect of sea level rise and cross-shore transport becomes more and more pronounced. Line models may provide (a rough) insight at every time scale.

For the coast of Holland, coastal constants derived on one hand from data concerning the last 15 years (with the aid of Kalman filtering) and on the other hand from data with regard to the last 100 to 1000 years are not contradictory. For larger time scales, a schematization of the coast in one or two layers (one- or two-line model) will not be sufficient. Small amounts of cross-shore transport from deeper water to the inshore, as well as the local slope of these deeper regions become important at this time scale. In this paper, for a case like this the use of for instance a six-line model is proposed. This simulates as well a closure depth dependent on the time-scale of the feature, corresponding to the measurements of Wright (1984) as a negative-exponential trend to an equilibrium profile according to ideas and experiments of Bakker (1968) and Swart (1974). The method is more sophisticated than the Bruun rule (1983). Accretion of the Holland coast between 4000 years and the Roman times and erosion later on can be simulated this way.

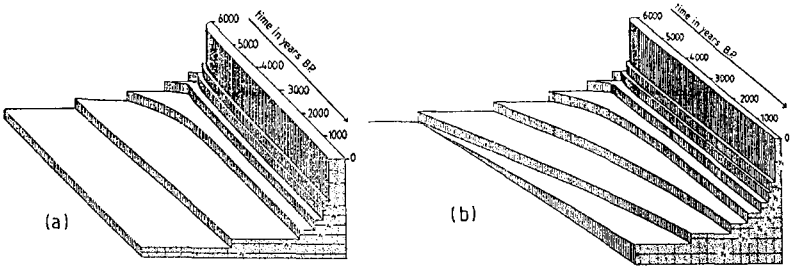

Figure 18.
Development of the profile of 6000 BP,
(a) if no sea level rise is taken into account.
(b) taking sea level rise into account.

APPENDIX

The equation for the Kalman gain matrix K(t) is

$$K(t) = P(t-)\, H^T(t)\, \{\, H(t)\, P(t-)\, H^T(t) + R(t)\, \}^{-1} \qquad (A1)$$

Where

$$P(t-) = F(t)\, P((t-1)+)\, F^T(t) + Q(t) \qquad (A2)$$

is the inaccuracy (variance) belonging to $\underline{x}(t-)$ (The righthand side of (A2) denotes old noise + new noise, so P(t-) involves the total noise as a result of all preceeding time steps)
and

$$P(t+) = \{\, I - K(t)\, H(t)\, \}\, P(t-) \qquad (A3)$$

is the inaccuracy belonging to $\underline{x}(t+)$.
I denotes the unit (identity) matrix.
H^T and F^T denote the transposed of H and F respectively.

By thinking of the matrices as scalars (notation small letter instead of capital) equation (A1) can be rewritten as:

$$k(t) = \frac{1}{h\left(1 + \frac{1}{h^2(t)}\, \frac{r(t)}{p(t-)}\right)} = \frac{1}{h(1 + \text{noise in latest estimate})} \qquad (A4)$$

So the Kalman gain matrix K(t) contains the inverse of the matrix H(t), thus transforming $\underline{z}(t)$ to $\underline{x}(t)$, and a matrix of weight-factors (derived from system- and measurement noise, through P(t-) and R(t) respectively) , determining the weight given to the physics relative to the measurements.
Kalman proved that this algorithm results in an optimal estimate $\underline{x}(t+)$ in least squares sense on time t (Kalman, 1969).
Present estimates are computed in a recursive way, meaning that all previous measurements are taken into account in a sequential way (filtering by a "moving window"). Unlike batch processing of measurements, in this recursive filter there is no need to store past measurements (Gelb,1974). The weight given to the previous measurements depends on the noise characteristics of the model and the measurements.

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