

CHAPTER 152

RELIABILITY ANALYSIS OF RUBBLE-MOUND BREAKWATERS

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To verify the effect of wave period on the motion of concrete blocks in rubble-mound breakwaters, simple physical models are employed and their motion is investigated by the numerical simulation. Finally, risk or reliability are calculated and the weight of concrete blocks for given physical condition is discussed by them.

Introduction

In Japan, many kinds of concrete blocks have been used to protect coastal structures. The weight of concrete blocks is computed by Hudson's formula for given design wave height, density of concrete blocks and breakwater slope. However, the weak points of Hudson's formula are described as follows:

1. It does not contain the effect of wave period;
2. It includes an uncertain parameter K_D ;
3. Incident wave height is not constant, but probabilistic; and
4. Reliability or risk are not considered with this formula. It says only safe or not.

Especially, the item 1 has been pointed out by field engineers for a long time. To study the above points, we introduce simple physical models to express the incipient motion of concrete blocks and analyze the motion for given parameters and physical conditions which are governed by appropriate probabilistic laws. Finally, the risk is defined by the ratio of the number of concrete blocks moved to the total number of trials.

Stability Analysis of Concrete Blocks

The incipient motions of concrete blocks in the rubble-mound breakwater which frequently occur are classified into three: They are sliding, rocking and lift-up. To analyze the sliding motion of a concrete block, the sliding force f_s

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and the resistance force f_R are expressed by referring Fig.1(a).

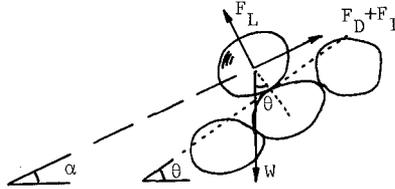


Figure 1(a). Motion of a Concrete Block

$$f_s = \epsilon_1[(F_D+F_I)\cos(\alpha-\theta)+F_L\sin(\theta-\alpha)] - \frac{\rho_s-\rho_w}{\rho_s} W \sin \theta \tag{1}$$

$$f_R = \left[\frac{\rho_s-\rho_w}{\rho_s} W \cos \theta - \epsilon_1 F_L \cos(\alpha-\theta) + \epsilon_1 (F_D+F_I) \sin(\theta-\alpha) \right] \mu \tag{2}$$

in which $F_D = \rho_w C_D (c_2 \ell^2) u |u| / 2$, (drag force); $F_I = \rho_w C_M (c_3 \ell^3) du/dt$, (inertia force); $F_L = \rho_w C_L (c_2 \ell^2) u^2 / 2$, (lift force); C_D = the drag coefficient; C_L = the lift coefficient; C_M = the virtual-mass coefficient; ρ_w = the water density; ρ_s = the density of a concrete block; θ = the repose angle of a concrete block; α = the angle of the breakwater slope makes with the horizontal; ℓ = a characteristic length of a concrete block; $c_1 \ell$ = the distance from the point of support of the center of gravity of the block; $c_2 \ell^2$ = the projected cross-sectional area of the block on a plane perpendicular to the direction of the velocity; $c_3 \ell^3$ = the volume of the block; μ = the friction coefficient; ϵ_1 = the sheltering coefficient for sliding motion; and W = the weight of the concrete block. The reason why the direction of the drag and the inertia forces on concrete blocks is upward along the slope is explained as follows: Since the force due to the interlocking effect among concrete blocks is not able to be easily computed, we assume that concrete blocks do not slide down against the interlocking effect. The dropping movement of concrete blocks is observed to be caused soon after the rocking, sliding or lift-up motions of concrete blocks by the experiments. Most of the movement of concrete blocks is produced by the rocking motion. In the rocking motion (Fig.(b)), the sliding and the resistance motion M_s

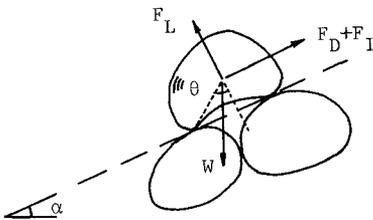


Figure 1(b). Motion of a Concrete Block

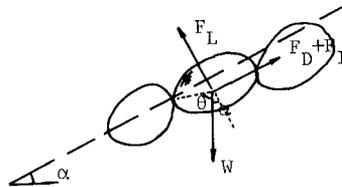


Figure 1(c). Motion of a Concrete Block

and M_R about the contact point of a concrete block for the condition of incipient motion are given (Walton et al. 1981) by,

$$M_R = \frac{\rho_s - \rho_w}{\rho_s} c_1 \ell W \sin\theta \cos\alpha \quad (3)$$

$$M_s = \epsilon_2 [(F_D + F_L) c_1 \ell \cos\theta + F_L c_1 \ell \sin\theta] + \frac{\rho_s - \rho_w}{\rho_s} c_1 \ell W \cos\theta \sin\alpha \quad (4)$$

in which ϵ_2 = the sheltering coefficient for the rocking motion. In the lift-up motion, the lift force f_L and the normal component of the weight to the slope f_w are expressed as L (see Fig.1(c)):

$$f_L = \epsilon_3 [(F_D + F_L) \sin(\alpha + \theta - \frac{\pi}{2}) + F_L \cos(\alpha + \theta - \frac{\pi}{2})] \quad (5)$$

$$f_w = \frac{\rho_s - \rho_w}{\rho_s} W \cos(\frac{\pi}{2} - \theta) \quad (6)$$

in which ϵ_3 = the sheltering coefficient for the lift-up motion.

Movability Condition of Concrete Blocks

It is not certain that the geometrical position of concrete blocks is similar to the proposed ones in the stability analysis. We define a word "movability" when concrete blocks are in the same condition as the proposed state in the stability analysis. To connect the movability condition with the stability analysis, the following assumptions are introduced:

1. The occurrence position of a concrete block which has the movability condition is uniformly distributed from the breakwater bottom to 90% of the significant wave height above the still water level along the breakwater face;
2. The number of the concrete blocks with the movability condition per one wave is k ; and
3. The movability condition of concrete blocks continues for n_i wave cycles for $i=1$ to k .

We consider that the process (2) and (3) are very analogous to service problems in the queueing theory. So the integers k and n_i for $i=1, 2, \dots, k$ are probabilistic variables and described by the Poisson process as follows:

$$p(k) = \frac{\exp(-\lambda) \lambda^k}{k!} \quad (7)$$

$$p(n_i) = \frac{\exp(-\lambda') \lambda'^{n_i}}{n_i!} \quad (8)$$

in which λ and λ' = parameters to govern the Poisson process.

Risk and Reliability

To calculate the risk or the reliability of the rubble-mound breakwater, let us define new variables R_s , R_r and R_ℓ as follows:

$$R_s = f_R - f_S \quad (9)$$

$$R_r = M_R - M_S \quad (10)$$

$$R_\ell = f_w - f_L \quad (11)$$

The variables on the right side in the above equations are derived in the stability analysis. The probability density functions $f(R_s)$, $f(R_r)$ and $f(R_\ell)$ are computed by the Monte Carlo method for given probabilistic variables. The failure probability or the risk of each concrete block is expressed by

$$P_s = \int_{-\infty}^0 f_s(R_s) dR_s \quad (12)$$

$$P_r = \int_{-\infty}^0 f_r(R_r) dR_r \quad (13)$$

$$P_\ell = \int_{-\infty}^0 f_\ell(R_\ell) dR_\ell \quad (14)$$

The reliability is shown as the difference of the risk from 1. Thus, if these three motions are assumed to be independent, the occurrence probability of motion for i -th block in the breakwater slope is given by

$$P_e(i) = P_s + P_r + P_\ell - P_s P_r - P_r P_\ell - P_\ell P_s + P_s P_r P_\ell \quad (15)$$

The whole failure probability of the rubble-mound breakwater P_w is obtained by computing the failure probability of union set for the failure probability of each block $P_e(i)$ as follows:

$$P_w = 1 - \prod_{i=1}^m [1 - P_e(i)] \quad (16)$$

in which m = the number of concrete blocks per unit width. Since the motions treated in the stability analysis are incipient, it is reasonable to assume that the motion of each block is independent in the above derivation.

Wave Conditions

The probability density functions of wave height and period are given by

$$P_{H/\bar{H}} = \frac{\pi}{2} \left(\frac{H}{\bar{H}} \right) \exp \left[-\frac{\pi}{4} \left(\frac{H}{\bar{H}} \right)^2 \right] \quad (17)$$

$$P_{T/\bar{T}} = 2.7 \left(\frac{T}{\bar{T}} \right)^3 \exp \left[-0.675 \left(\frac{T}{\bar{T}} \right)^4 \right] \quad (18)$$

in which \bar{H} = the mean wave height and \bar{T} = the mean wave

period. The cumulative distributions of the wave height and period are obtained by integrating Eqs.(17) and (18). The significant wave height and period are expressed by using the mean wave height and period, respectively.

$$H_{1/3} = 1.60 \bar{H} \quad (19)$$

$$T_{1/3} \approx 1.1 \bar{T} \quad (20)$$

In the sampling process, the waves satisfying the following condition are removed as the breaking waves before they reach the breakwater slope:

$$H/L \geq 0.142 \quad (21)$$

The condition the wave does not break on the breakwater slope is given by

$$\frac{H}{L} \leq \sqrt{\frac{2\alpha}{\pi}} \frac{\sin^2 \alpha}{\pi} \quad (22)$$

For the nonbreaking waves the shallow water theory gives the flow velocity and the acceleration

$$u = \sqrt{\frac{g}{h}} \frac{H}{2} \sin \sigma t \quad (23)$$

$$\frac{du}{dt} = \frac{g\pi H}{L} \cos \sigma t \quad (24)$$

in which H = the wave height; h = the water depth; $\sigma = 2\pi/T$; T = the wave period; and L = the wave length. The exerted force on a concrete block due to the wave breaking p_m is calculated by Goda's formula (Iwagaki & Sawaragi, 1979). The total force P_m exerted on the concrete block is given by

$$P_m = c_2 \ell^2 p_m \quad (25)$$

In this case it is assumed that the drag force F_D has a uniform distribution between 0 to P_m . In the stability analysis, the inertia force is neglected and the lift force F_L is transformed by

$$F_L = \frac{C_L}{C_D} F_D \quad (26)$$

For the broken waves, the velocity u' of the water mass at any location between the SWL and the point of maximum wave runup may be approximated by (Shore Protection Manual, 1977),

$$u' = \sqrt{gh_b} (1 - x_1/x_2) \quad (27)$$

and the wave height H' above the ground surface is written by

$$H' = h_c (1 - x_1/x_2) \quad (28)$$

in which $h_c \approx H_b/2$; h_b = the water depth at the breaking point; x_1 = distance from the still water line to the concrete block; and x_2 = distance from the still water line to the limit of wave runup. For the broken waves, the distance x_2 is suggested by

$$x_2 = 2 H_b \cot \alpha \quad (29)$$

For the nonbreaking waves, we substitute the above equations to get the velocity of the water mass and the wave height above the breakwater surface. But the limit of wave runup is obtained by (Gunbak et al. 1979),

$$\frac{R}{H} = \frac{0.8\xi}{1 + 0.5\xi} \quad (30)$$

in which $\xi = \tan \alpha / \sqrt{H/L}$, surf similarity parameter.

Other Conditions

Several parameters included in the formulation of the stability analysis are assumed to be probabilistic variables. The shape factors of the concrete block c_1 , c_2 and c_3 are supposed to be uniformly distributed between 0 and 4, 0.8 and 1.2, and 0.9 and 1.1, respectively. The value of the drag coefficient C_D , the lift coefficient C_L , and the virtual-mass coefficient C_M are also probabilistic. According to Christensen et al. (1982), in the case of a circular cylinder, these values are normal probability variables of which means and coefficients of variation are 0.60 and 0.24, 0.60 and 0.24, and 1.20 and 0.22, respectively. The friction coefficient of tetrapods μ behaves like the probability variable which is uniformly distributed from 0.5 to 0.6 (Takeda, 1981). The sheltering coefficients ε_1 , ε_2 and ε_3 are also considered as the probability variables. For the sliding, the rocking and the lift-up motions, they are assumed to be uniformly distributed from 0.8 to 1, 0.4 to 0.8, and 0 to 0.3, respectively. The repose angle of the concrete block is also the probability variable which is uniformly distributed from 56 to 63 degrees (Takeda, 1981).

Illustrative Examples

Fig. 2 compares the computed relative damage with the observed relative damage, introducing the queuing theory for different values of λ' and $\lambda = 1$. The value of λ' designates the average number of cycles for a concrete block to continue to have movability condition. From the same figure the value of $\lambda' = 0.3$ seems to be appropriate. In this study these values are used for the numerical calculations. Next, to apply our model to a designed rubble-mound breakwater, let us determine the weight of a concrete block by the following Hudson's formula:

$$W = \frac{\rho_s g H^3}{K_D \left(\frac{\rho_s}{\rho_w} - 1 \right)^3 \cot \alpha} \quad (31)$$

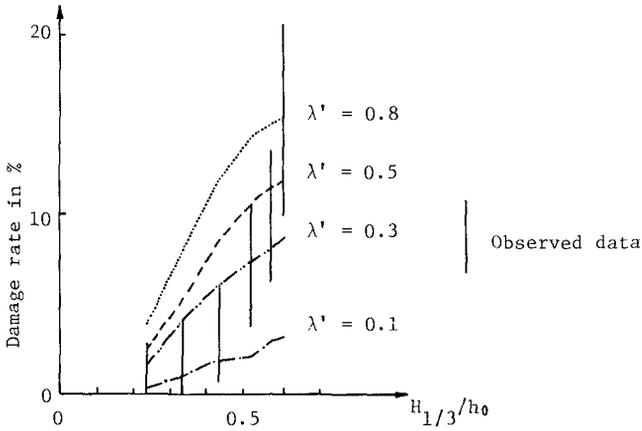


Figure 2. Comparison of Computed Relative Damage with Observed Relative Damage for Different Values of λ'

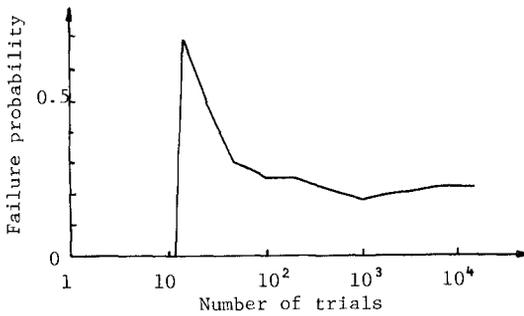


Figure 3. Effect of Number of Trials on Failure Probability

in which H = the design wave height and K_D = a parameter. The value of K_D is selected by the characteristics of a concrete block and the condition of wave breaking. For the values of $\rho_s/\rho_w = 2.23$, $\cot\alpha = 4/3$, $K_D = 8.3$ and $H = 4\text{m}$, the weight of a concrete block of 7.15 ton is computed by Eq. (31). Usually the weight of 8 ton is used by considering easiness of construction method and adding a safety factor. Our model can obtain the reliability or the risk for the arbitrary value of the weight of a concrete block. The weight of a concrete block to correspond to the required value of the reliability or the risk should be selected in the field. Fig.3 represents the effect of the trial number on the failure probability. The increase of the trial number the failure probability converges to a constant value. Fig.4 indicates the effect of the significant wave height on the failure probability. As expected, the fail-

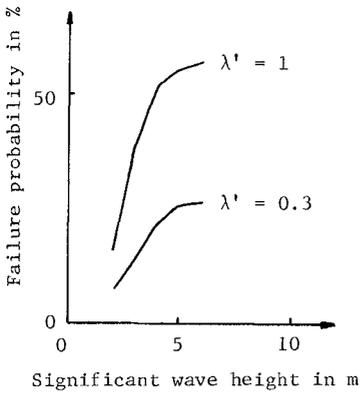


Figure 4. Effect of Significant Wave Height on Failure Probability

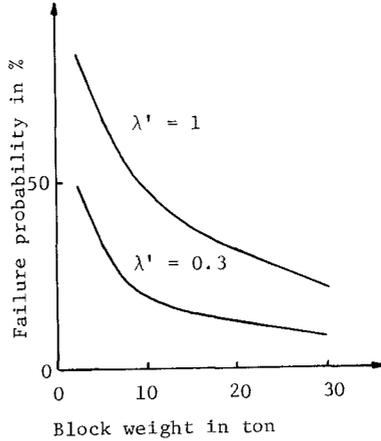


Figure 5. Effect of Concrete-block Weight on Failure Probability

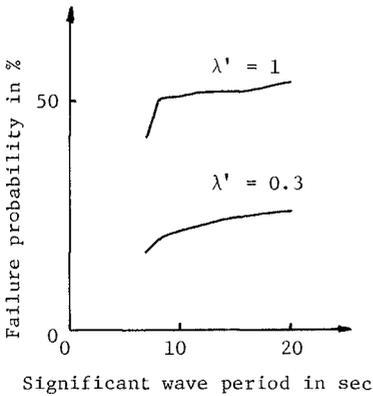


Figure 6. Effect of Significant Wave Period on Failure Probability

ure probability increases as the significant wave height becomes large. The rate of the increase becomes small after 5m. It is explained that some incident waves break before they reach the breakwater slope. Fig.5 shows the effect of the weight of concrete blocks on the failure probability. By increasing the weight of concrete blocks, the failure probability decreases. It corresponds to the increasing of the weight of concrete blocks in the field. Fig.6 represents the effect of the significant wave period on the failure probability. The influence of the wave period, which is not included in Hudson's formula, is found. Thus, the longer wave period has a high risk. Fig.7

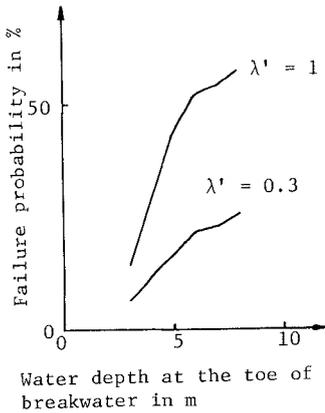


Figure 7. Effect of Water Depth at the Toe of Breakwater on Failure Probability

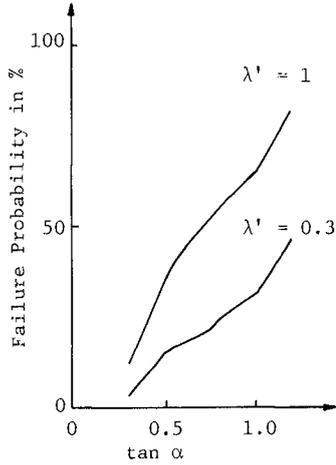


Figure 8. Effect of Breakwater Slope on Failure Probability

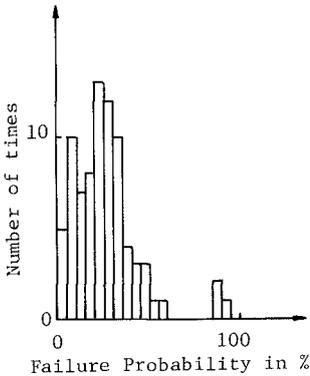


Figure 9. Failure Probability of Destroyed Rubble-mound Breakwaters

describes the effect of the water depth at the toe of the breakwater on the failure probability. Increase of the water depth gives high failure probability or risk. For the higher water level, more waves with larger wave height come on the breakwater slope. Fig.8 indicates the effect of the angle of the breakwater slope from the horizontal on the failure probability. The steep slope induces a high risk. Our model cannot consider the interlocking effect among blocks. Fig.9 shows the failure probability of destroyed rubble-mound breakwaters in Japan introducing the queuing theory when $\lambda = 1$ and $\lambda' = 0.3$. The failure probability using the weight of concrete blocks computed by Hudson's formula is between 10 % to 30 %. The destruction for the failure probability which is greater than 30 % is caused by unexpected design conditions. The destruction for the failure

probability which is less than 10 % is caused by phenomena except our model.

Concluding Remarks

Classifying the motion of concrete blocks into three kinds, the failure probability or the risk of the rubble-mound breakwater are obtained. The reliability is defined by the difference of the failure probability from 1. In a result, the influence of the wave period on the failure probability becomes remarkable. That is to say, attacking of waves with longer period induces high risk to the rubble-mound breakwater as expected. For the other design variables of rubble-mound breakwaters, the computed result is reasonable and does not contradict the field observation. The relative damage computed from the failure probability appropriately predicts the observed one. Since the motion of concrete blocks is concretely investigated, we can develop a new type of concrete blocks to increase the reliability of the rubble-mound breakwaters. Finally, it is found to be rational for us to discuss the failure probability, the risk, or the reliability than K_D value included in the Hudson's formula, when one designs the rubble-mound breakwaters.

Appendix 1.--References

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