Sediment Transport on Nearly-Prismatic Beaches

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SUMMARY

Recent progress in the quantitative modelling of the undertow has stimulated the modelling of cross-shore sediment transport. More so than before it seems now possible to attempt the dynamic modelling of beach profile development. Also, integration of dynamic cross-shore sediment transport formulations in horizontally two-dimensional models for water-motion and sediment transport has recently been suggested. This seems to be a first step of integrating depth-averaged 2DH-modelling with 2DV-profile-modelling. Here an overview is given of these developments and the understanding gained sofar of the several current systems and the induced sediment transport and morphology that are found in the situation of random waves normally and obliquely incident on beaches which vary not or only slowly alongshore.

1. INTRODUCTION

Alongshore variations of bottom topography and/or the incident wave field (amplitude or phase) are the cause of mean nearshore currents in the form of circulation cells, nonuniform longshore currents or mixtures of these. Using the radiation stress concept many dynamics of these depth-averaged nearshore currents may be modelled (Bowen, 1969, and many others since; see Battjes, 1988, for a review).

In the case of no alongshore variations in topography and (obliquely) incident wave field the same concept yields a uniform longshore current. Here our interest is into this type of situation: viz. that of obliquely incident, horizontally uniform waves on beaches of which the alongshore variation of topography can be neglected for the horizontal current dynamics. It should be noted that in reality longshore periodicities in bottom topography and flow are generated without obvious externally imposed periodicities, particularly if the waves are of near-normal incidence (Sonu, 1972). However, we choose our longshore length scale such that these periodicities are averaged out. In this situation there are no distinct horizontal circulation cells with a depth-averaged nonzero flow, only a nearly-uniform longshore current.

In addition to this depth-averaged longshore current system we here consider the vertical circulation system resulting from the vertical imbalances of the mass and momentum fluxes, even though the depth-integrated flux balances are satisfied (cf. Stive and De Vriend, 1987, SDV furtheron). This vertical circulation system induces a near-bottom current field which in general significantly differs from the depth-averaged field.

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In the described case of oblique wave incidence we will find a vertically nonuniform (both in magnitude and direction) current field nearshore. In the upper half of the water column the mean velocity vector more or less follows the depth-integrated longshore current vector but close to the bottom the mean velocity vector is deviated towards the offshore due to the undertow (De Vriend and Stive, 1987, DVS furtheron and Svendsen and Lorenz, 1988). Obviously for the sediment transport the lower layers are the most relevant. Here, it is the sediment transport and eventually the resulting morphology that forms our main reason of interest in the watermotions.

In addition to these mean flow current systems the waves induce oscillatory currents in their propagation direction. Apart from their interaction with mean currents which may lead to net flows as well they may also be the cause of net sediment transport due to their asymmetrical features (cf. Bowen, 1980).

The purpose of this paper is to discuss some recent developments with respect to the understanding of the several current systems (section 2) and the induced sediment transport and morphology (section 3) that are found in the situation of random waves normally and obliquely incident on a nearly-prismatic beach.

2. CROSS-SHORE AND LONGSHORE WATERMOTIONS

2.1 Cross-shore flow due to normally incident waves

The first situation considered is that of a random wave field (with directionality and wave group effects neglected; see Section 5 Discussion) normally incident on a prismatic (i.e. alongshore homogeneous) coast of arbitrary cross-shore profile. In this situation we distinguish the following water motions which are both in cross-shore direction:
- a depth varying oscillatory flow, in phase with the random wave surface variations, and
- a depth varying mean flow, steady on the scale of many waves, which in the surfzone and below the wave trough level mainly consists of the undertow.

The oscillatory flow

For the present interest it appears furtheron that the importance of the oscillatory flow may be expressed by the following low order flow moments (cf. Bowen, 1980):
- the even moments $\langle|u|^3\rangle$ and $\langle|u|^5\rangle$, which are nonzero for symmetric velocities,
- the odd moments $\langle u|u|^2\rangle$ and $\langle u|u|^3\rangle$, which are zero for symmetric velocities.

The even moments are relatively easy to evaluate. Unlike the odd moments they do not depend on the wave asymmetry features. The theoretical estimate by Guza and Thornton (1985) for a random, linear sea (Gaussian model) gives statisfactory results. However, also a bichromatic model yields statisfactory results (Roelvink and Stive, 1988b).

The odd moments are zero for a symmetric velocity field, but can be nonzero for nonlinear waves such as occur nearshore. Specifically, in the nonbreaking cnoidal type waves the shoreward velocities are stronger and of shorter duration than the offshore flows, leading to nonzero values for the odd moments. On the other hand it appears (Stive, 1986) that the breaking skewtype waves do not contribute to the odd moments, since these waves are symmetrical around the vertical rather than the horizontal plane. An evaluation of the odd moments according to second order Stokes theory (Stive, 1986) gave a first quantitative estimate, but important improvements were obtained by introducing a more general nonlinear theory (Roelvink and Stive, 1988a,b).

The undertow

In the present case of steady (on the time-scale larger than that of the wave groups), vertically two-dimensional motion of normally incident wave groups the depth-integrated total mass and momentum equations yield a depth-mean zero flow and in essence a balance between the radiation stress and the steady wave set-up. However, locally (in the vertical) the mass and momentum fluxes need not be in balance. The potential role of this imbalance driving a seaward directed returnflow or undertow, compensating for the shoreward mass flux above trough level, was first pointed out by Dyhr-Nielsen and Sørensen (1970).

To compensate for the wave-induced non-zero mass flux above wave trough level the depth-averaged net flow below wave trough level is seaward. In the case of non-breaking waves the resulting mean flow profile appears to resemble the socalled conduction solution (Longuet-Higgins, 1953) quite well. This implies that the mean flow in the central fluid region is seaward, but near the bottom it is strongly influenced by the dynamics of the bottom boundary layer, such that a shoreward flow results. In the case of breaking waves, however, the mass flux above trough level is enhanced due to the presence of the surface roller which is one of the manifestations of the breaking process. The effects near the water surface are strong enough to dominate the flow even near the bottom such that an all seaward flow results with its maximum close to the bottom, known as "undertow".

The non-zero mass flux above wave trough level due to the waves alone is theoretically known for periodic waves of constant form relative to an irrotational wave motion below trough level. This theoretical estimate combined with the mass flux in the roller as estimated by Svendsen (1984) may be considered to give an acceptably accurate estimate of the total mass flux above trough level. If we adopt the approach of DVS (1987) for the general case of random breaking waves, characterized by -mutually not interacting- fractions of breaking and non-breaking waves, the total estimate for the total mass flux above wave trough level (m), becomes:

$$m = \left(1 + Q_b \frac{7kh}{2\pi}\right) \frac{E}{c}$$

where the second term represents the empirically derived roller contribution, in which Q_b is the breaking wave fraction.

A crude estimate of the near-bottom undertow may be derived from the above estimate of m by assuming depth uniformity of the flow below wave trough level. A more accurate description including the near-bottom details can be obtained by evaluating the accompanying horizontal momentum balance locally in the vertical, following the above mentioned suggestions of Dyhr-Nielsen and Sørensen (1970). Since 1980 several quantitative evaluations of their concept have been presented in the literature. Key papers in this context are Dally and Dean (1984), who present an internally consistent approach but neglect specific breaking wave effects, Stive and Wind (1986), who indicate the importance of the near-surface layer in breaking waves, and Svendsen et al. (1987), who present an approach for the realistic inclusion of the bottom boundary layer under breaking waves. These three contributions are integrated in a generalized approach by SDV (1987).

A recent finding is (Roelvink and Stive, 1988a,b) that in the region just after the point of initial breaking there is a spatial shift between the maximum gradient of the wave height and that of the undertow. They have found that this is due to the time needed to convert organized kinetic and potential energy into small-scale, dissipative turbulent motion. The underlying reason for this finding is that in the above described undertow model the wave breaking induced shear stress at trough level used to model the effects of the surface layer is proportional to the ratio of wave energy dissipation over wave phase speed (see SDV, 1987). This shear stress is very important in the driving of the undertow and it appears that the lag found in the predictions of the undertow gradients is due to the incorrect assumption that the dissipation source term used in the wave height prediction model is a measure of the actual dissipation. A better interpretation is that this source term is a measure of the actual production of turbulent energy. The actual dissipation may then be derived with the aid of a turbulence model. The correction to the undertow improves the prediction importantly.

2.2 Cross- and longshore flow due to obliquely incident waves

The second situation considered is that of a random wave field (with directionality and wave group effects still neglected) obliquely incident on a prismatic coast of arbitrary cross-shore profile. In this situation we distinguish the following water motions.

Watermotions which are zero when depth- and time-averaged but nonzero when considered on the timescale of the wave period or when considered locally in the vertical, viz.

- wavepath directed, depth varying oscillatory flow, in phase with the random wave surface variations, and
- depth varying mean flow, steady on the scale of many waves, compensating for the wavepath directed mass flux above wave trough level, which in the surfzone and below the wave trough level mainly consists of the undertow.

In addition to these watermotions there are nonzero, time- and depth-averaged watermotions alongshore, viz.:

- a wave-induced mean longshore current, and (potentially)
- a mean current due to an alongshore pressure gradient (e.g. due to the tide).

The wavepath directed oscillatory watermotions clearly will have components both along- and cross-shore. The evaluation of these watermotions may straightforwardly be derived taken account of the local wave angle with the parallel depth contours. Here we further concentrate on the description of the time-averaged, vertically varying current system.

The traditional depth-integrated, horizontal circulation approach obviously has a too narrow physical basis to describe the in essence three-dimensional current pattern. For this we need models which deal with the flow in three dimensions. Since the three-dimensionality is such that the variations in the horizontal directions are zero or only weak compared to that of the vertical direction, flow models using a profile technique can be useful. A first, practical formulation combining the typical cross-shore vertical flow variations with the weaker horizontal flow variations due to alongshore nonuniformities was given by DVS (1987). Their approach can be considered a natural step between 2D and fully 3D models and is therefore termed quasi-3D approach. Here we discuss this approach and some consequences for the even more simplified situation of alongshore uniformity.

In the terminology of DVS (1987) the nonzero depth-averaged timemean watermotions are termed the primary flow. The locally (in the vertical) nonzero but depth-averaged zero timemean watermotions, for instance the undertow, are called the secondary flows. As the terminology indicates this approach assumes in essence that the secondary flows are weak enough to assume that their effects on the derivation of the primary flow are negligible. Furthermore, they assume that the primary flow profile and related primary bottom shear stress are in a single vertical plane. These assumptions allow a full uncoupling between the primary and secondary flows. This approach seems a reasonable first step, but it should be mentioned that in general the uncoupling will not be permitted, especially if the secondary flows are not much smaller than the primary flows. Interactions will then result, such as for instance described by Van Kesteren and Bakker (1986), who find that a sophisticated boundary layer approach for the combination of waves and currents under an angle shows the generation of a secondary flow due to the interaction between primary flow and the oscillatory wave motion.

If the uncoupling between primary and secondary flow is accepted, following DVS (1987) the primary flow may be determined from the conventional depth- and time-averaged horizontal momentum balance equations, and the secondary flow due to e.g. the surface breaking of the wavefield may be determined from the local balances of mass and horizontal momentum in the plane with its orientation in the direction of wave energy flux propagation.

The result of the suggested modelling approach yields a vertically nonuniform (both in magnitude and direction) current field nearshore. In the upper half of the water column the mean velocity vector more or less follows the depth-integrated longshore current vector but close to the bottom the mean velocity vector is deviated towards the offshore due to the undertow. An example of the resulting (computed) vertically rotating mean velocity vector for Egmond beach is given in Figure 1. A preliminary check with nature on this phenomenon as given in DVS (1987) indicates that it resembles reality, but more definitive comparisons are awaited.

This section on the formulation of the watermotions is concluded with a finding, which has obvious implications for the modelling of both the depth- and time-mean currents and the sediment transport (also see De Vriend and Ribberink, 1988). The principle calculations of SDV (1987) indicate that the magnitude of the secondary bottom shear stress is some 10% to 20% of the magnitude of the primary shear stress. The implication of this for the resulting near-bottom shear stress orientation and closely related near-bottom flow orientation may be indicated with the following approximations.

In the considered situation of obliquely incident waves to a prismatic coast the primary shearstress direction is alongshore and of approximate magnitude (cf. Longuet-Higgins, 1970):

$$\tau_p(b) = \frac{D_{br}}{c} \sin\theta$$

where D_{br} is the breaking dissipation and c the wave phase speed (also see Figure 2 for notations). The secondary bottom shear stress is in the wave energy flux direction and of approximate magnitude (SDV, 1987):

$$\tau_s(b) \approx 0.15 \left(0.5 + 7 \frac{kh}{2\pi}\right) \frac{D_{br}}{c}$$

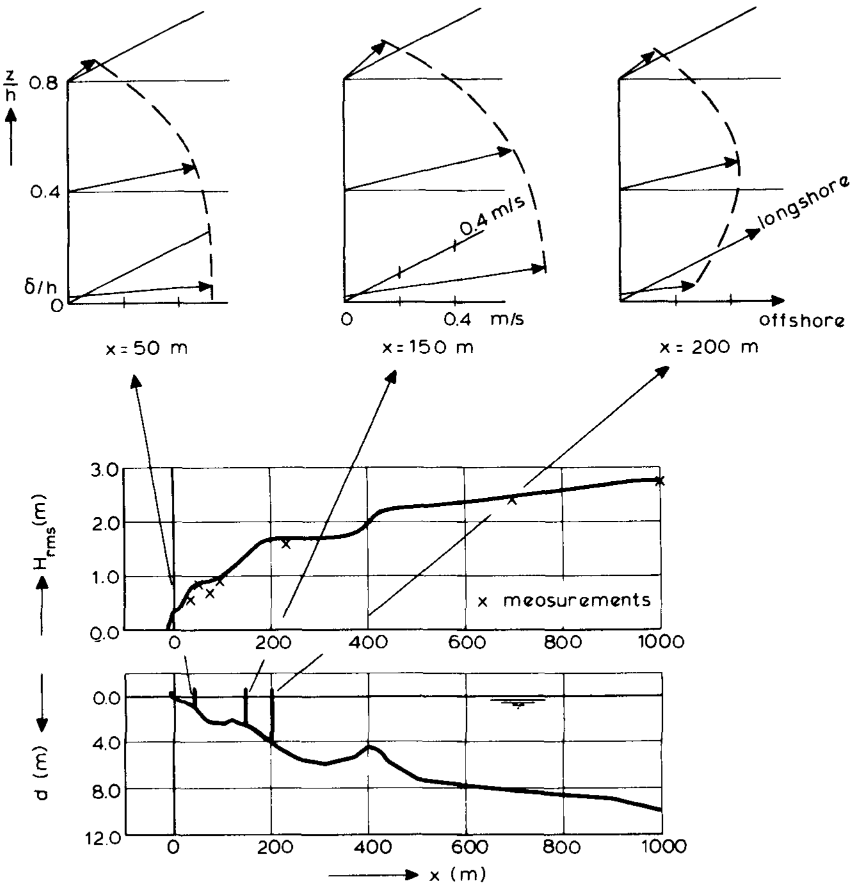

Figure 1 Model predictions of rotating mean velocity vector at Egmond beach (adapted from DVS, 1987)

The angle of the resulting velocity vector a may be derived from these aproximations straightforwardly, taking into account the refraction of the wave field over the parallel bottom contours. As a result it appears that

$$\tan\alpha = \text{function}\left(kh,\ \theta_{inc}\right)$$

which has been evaluated using Snel's law and the dispersion relationship with the result shown in Figure 2. It appears that the deviation towards the offshore direction of the resulting shearstress vector is stronger for the smaller angle of wave incidence. This is as expected since the longshore-current driving forces and resulting currents get weaker in this situation. This is in close correspondence with the field evidence gained and documented by Ingle (1966, pages 55-60), which is based on tracer experiments.

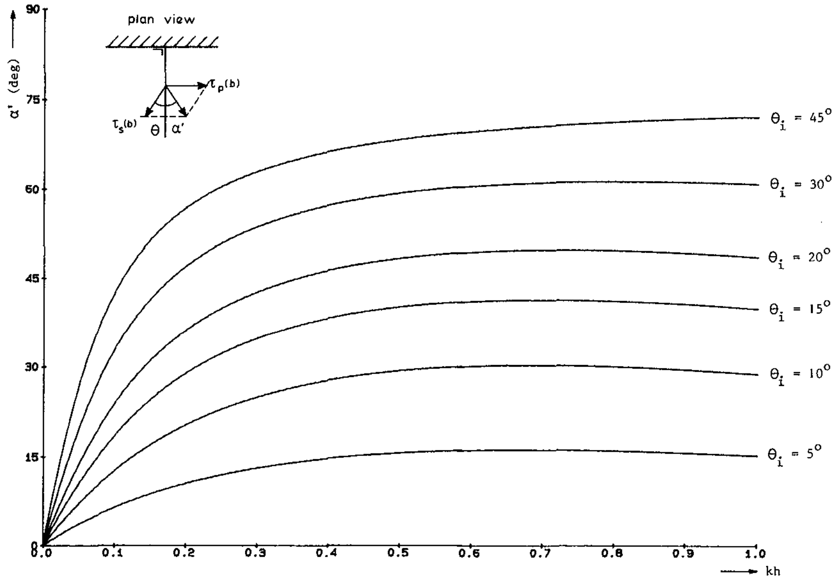

Figure 2 Increase of rotation towards the offshore direction of the resulting bottom shearstress for a decreasing angle of deep water wave incidence as a function of the relative waterdepth

3. SEDIMENT TRANSPORT AND CROSS-SHORE MORPHOLOGY

3.1 Instantaneous sediment transport formulation

Until quite recently nearshore sediment transport computations -in any rate those involving the alongshore horizontal dimension- were based on the concept of "waves or waves plus current stirr sediment up and the mean current transports the sediment". The existing formulations are usually based on an adapted uniform (river) flow formula, for instance the total bed and suspended load description after Bijker (1968) with a vertical sediment concentration description which is multiplied by the mean flow profile.

From the foregoing section it may be clear that the concept of "only the mean current transports" is considered insufficient: asymmetry-induced and wave-grouping induced effects can only be included in a formulation which is based on an instantaneous sediment transport description. The state-of-the-art in this respect is limited and with respect to accuracy not at the stage at which the above formulations are currently. It is considered essential though for the present problem. A reasonable first approximation seems to be the formulation of Bailard (1981), which is an extension of Bowen (1980) and Bailard and Inman (1981). In this formulation a distinction is made between bedload transport in a granular-fluid shear layer and suspended load transport in a layer of greater thickness, typically in the order of several centimeters. These transports are calculated as vertically integrated, but still instantaneous quantities.

Without going into the detail of the derivations and the formulations it may be schematically said that the instantaneous sediment transport is assumed to respond in a quasi-steady manner to the time-varying flow on the one hand and to the downslope gravity force on the other hand, as follows:

$$q(t) = \text{const}.u(t)\ |u(t)|^n + \text{const}\ |u(t)|^m.\partial z_b/\partial x$$

with n = 2,3, m = 3,5 and $\partial z_b/\partial x$ is the local bed slope. Note that as expected for any suitable time-varying formulation the sediment transport rate, q, contains terms proportional to some power of the near-bottom, time-varying cross-shore flow, u(t).

Field information of the last few years leads us to believe that the quasi-steady response assumption is valid for most natural beaches, but also in the laboratory it appears that under random waves the surf zone with sediments of 100 to 200 µm median grain diameter creates prevailing sheet flow conditions in the regions of significant sediment transport. Where the formulation fails is in the relatively protected areas in the trough behind breaker bars or somewhat more offshore, where one often may encounter rippled beds. For these regions a possible extension to the approach is to introduce non-steady response, for which it becomes important to include flow acceleration effects, resulting in terms with odd and even moments of the time-varying acceleration.

A further restriction to Bailards approach is the vertical integration that is performed, which assumes that the -known- vertical flow distribution is in a single plane. In the case of obliquely incident waves it was found in the foregoing that there is a vertical mean flow distribution which shows a rotation from alongshore in the upper water column towards the offshore in the lower part of the column. Clearly, a suitable formulation should also make inclusion of this effect possible.
For the moment we must approximate the rotating velocity vector effect by choosing a representative near-bottom mean velocity, which in our case of obliquely incident waves will have both a cross-shore and an alongshore component.

So, for the time being it is suggested to rely on the above approach and we note that this involves basically the evaluation of the odd and even flow moments resulting from time-averaging of the above equation. The contributions to these moments for the purely cross-shore case are identified and analyzed in several publications each extending the foregoing ones (Bowen, 1980; Bailard, 1982; Stive, 1986; Roelvink and Stive, 1988a,b).

In the case of obliquely incident waves the near-bottom mean current will be composed of both the undertow and the primary flow, which then make an angle with the oscillatory current. Bailard's approach is also then quite straightforward. In the case of a small angle of wave incidence no major principle differences are expected in the cross-shore sediment transport distribution compared to the normally incident case.
In general, the effect will be that of additional stirring for the cross-shore terms.

3.2 Cross-shore morphology of a nearly-prismatic beach

With the presented modelling approach for the near-bottom mean and oscillatory flow (section 2) and resulting sediment transport (section 3.1), which is applicable to a prismatic beach of arbitrary profile, we have available the constituents for a cross-shore sediment transport model with which profile deformations can be determined by applying the sediment balance equation.

The practical relevance of this schematized situation of a prismatic beach lies in applications dealing with the larger scale behaviour of an alongshore uniform coastal stretch which is not or only locally influenced by cross-shore structures. We assume then that on the larger -longshore- scale the local circulation systems are averaged out and that the overall behaviour of the coastal stretch may well be described by assuming that the alongshore flows are negligible for normally incident waves or that they are uniform alongshore for obliquely incident waves. Our main interest is then into the development of the cross-shore profile, since it is only in the cross-shore direction that there are sediment transport gradients present resulting in bottom topography changes.

A small extension of this approach is for situations where there is a slow variation of an alongshore feature, such as the coastline direction. This variation is then assumed to be effective on a large timescale only, such that the local coupling of the weakly dynamic alongshore processes with the strongly dynamic cross-shore processes is the only (non-dynamic) interaction that needs to be accounted for. Some consequences of this approach are that locally the primary flow follows the parallel bottom contours, while the cross-shore primary flow is neglected. Examples are weakly curved coastal stretches or weakly curved coastal regions near estuary mouths.

Some first results based on this approach lead to interesting insights into the cross-shore profile development in case there are alongshore transport gradients present. An example is given in Figure 3, where the profile developments are compared for the case of no and a weak alongshore variation in the orientation of a coastal stretch. It appears that the existence of an alongshore gradient in the sediment transport influences bar formation and the mean profile slope in the surf zone.

4 DISCUSSION

An overview is given of the developments and understanding gained sofar of the current systems and the induced sediment transport and morphology that are found in the situation of random waves normally and obliquely incident on beaches which vary not or only slowly alongshore. For a similar overview in case the horizontal variations in alongshore and in cross-shore direction are equally important reference is made to De Vriend and Ribberink (1988) in these proceedings. Some of the aspects that need extra attention from the research point of view are indicated below.

the wave field

With respect to the natural representation of the wave field it should be noted that the aspect of randomness in the wave height decay has been included but sofar the effects of wave grouping and wave directionality have been disregarded. With regard to wave grouping effects reference is made to Roelvink and Stive (1988b), who present a first modelling approach. Their results indicate that outside the surfzone wave grouping reduces wave asymmetry effects, while the opposite is true inside the surfzone. With regard to wave directionality effects reference is made to Guza and Thornton (1986), but their results are not conclusive in this respect. Lastly, it should be mentioned that the approach sofar assumes uncoupling between breaking and nonbreaking waves, using the idea of applying the theory for the separate fractions.

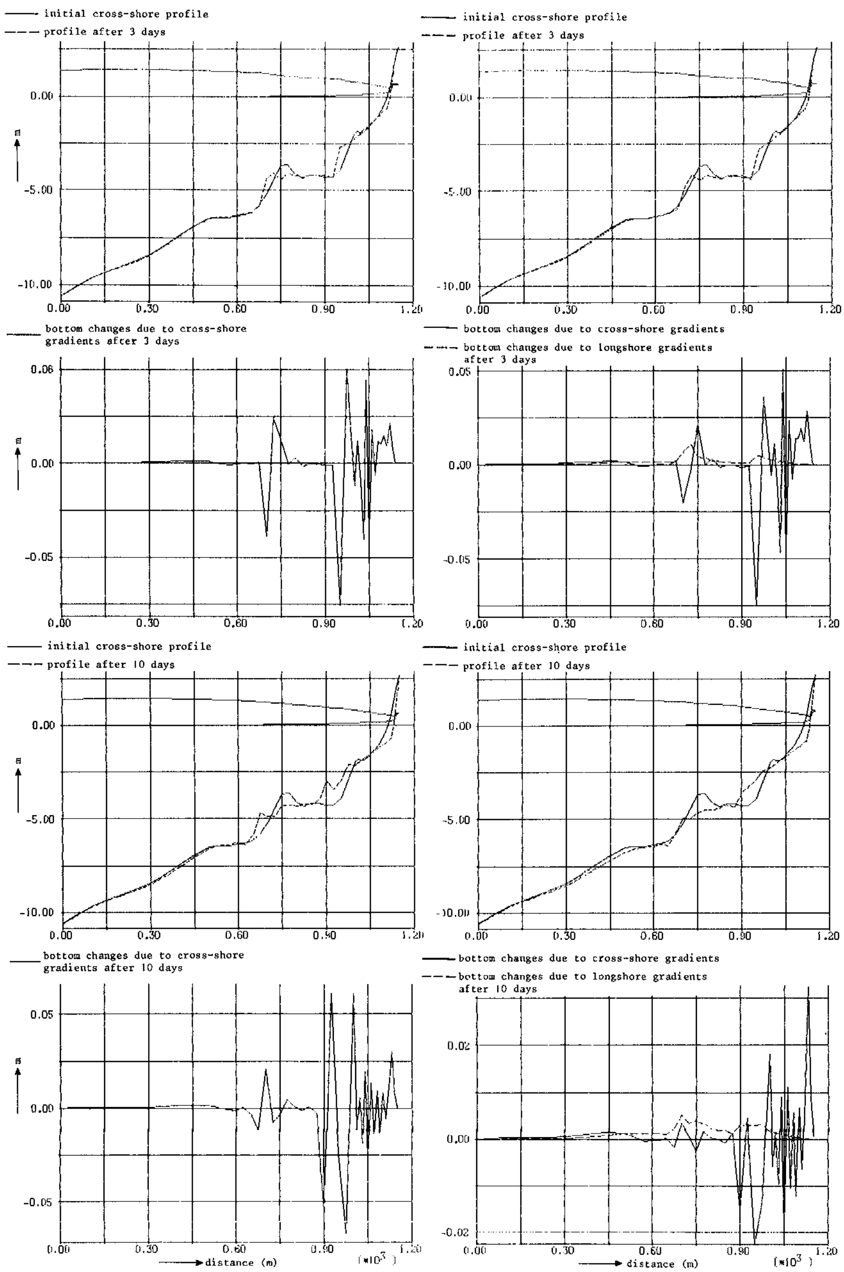

Figure 3 Cross-shore profile development and bottom changes per timestep (Δt = 0.1 days) in case of no (left) and a weak (right; radius of coastal curve of 2000 m) alongshore sediment transport variation (incident conditions H_{rms} = 2 m, T_p = 8 s, θ_{inc} = 10°)

mean flows

With respect to the cross-shore flows it is expected that the present modelling stage is satisfactory. Progress is needed with respect to the modelling of the combined longshore and cross-shore flows for obliquely incident waves. A first approach has been suggested with the quasi-3D formulation, but there are some important assumptions made. Of these the uncoupling between primary and secondary flow calculation should be mentioned here.

sediment transport

With respect to the sediment transport it is essential to have available an instantaneous transport description. An attractive first approach has been elaborated by Bailard (1981), but shortcomings are the quasi-steady response assumption and the depth-integrated approach.

morphology

With respect to the morphological evaluation it should be mentioned that even though the constituent formulations (for water and sediment motion) seem accurate this is not necessarily true when dealing with the morphological development. Progress with cross-shore morphodynamics has been made, but the predictive potential and the aspect of equilibrium profiles need further development. The addition of a longshore flow and sediment transport component needs further evaluation. Preliminary results indicate that longshore gradients can influence the mechanism of bar formation and the mean profile slope.

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