

CHAPTER 111

EFFECTS OF A VERTICAL SEAWALL ON PROFILE RESPONSE

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ABSTRACT

An attempt is made to determine beach profile response due to the presence of a vertical seawall placed in various cross-shore positions, and to examine the differences between natural beaches and seawall-backed beaches in response to normally incident wave attack. The investigation was mainly restricted to two-dimensional profile response under erosive wave conditions, with beach recovery response monitored to a limited extent. Spatial and temporal profile response was investigated by examining time-series profile configuration, volumetric changes, sediment transport patterns, and quasi-equilibrium profile configuration. Additionally, dominant profile features such as the break point and reflection bars (as well as scour at the toe of the seawall-backed profiles) were observed and quantified.

INTRODUCTION

An emerging concern by coastal communities and engineers is the effect of seawall placement along beach-dune systems. Since seawalls are most likely to be placed along eroding coastlines, any added erosional pressure due to the presence of the structure is deemed serious.

Important questions raised by interrupting a natural system with a seawall are

- 1) will the presence of the seawall accelerate the erosion process of the beach fronting it;
- 2) what effect does cross-shore placement of the seawall have on profile response;
- 3) what is the resulting profile configuration under erosive wave conditions;
- 4) will profile recovery be impeded by the structure.

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These concerns regarding construction and placement of a seawall necessitate that quantitative and qualitative relationships and trends be investigated. The focus of this study was to examine a fundamental aspect of a seawall-beach system by assessing the influence of a vertical seawall on beach profile response through laboratory experimentation.

Numerous field and laboratory investigations documenting seawall-beach interaction exist, but very few tests have focused on structural influence on profile response. Kraus (1987) provides a detailed literature review of over 70 laboratory, field, theoretical, and conceptual studies; due to space limitations here, the reader is referred to this thorough treatment for a more detailed background on the subject.

OBJECTIVES AND SCOPE

The main objectives of this investigation were to determine beach profile response due to the presence of seawalls, and to examine the differences between natural beaches and beaches fronting seawalls in response to normally incident wave attack. Seawall location was varied throughout the experiments, with each location subjected to the same wave conditions as the "no-seawall" tests. The present investigation differs from previous investigations in that the initial profile shape of $Ax^{2/3}$ has not been examined in detail with a seawall placed along the profile.

The experiments involved monitoring temporal and spatial changes of beach forms on natural and seawalled beaches under various wave conditions and seawall positions. The results were analyzed toward an understanding of:

- 1) characteristic profiles;
- 2) volumetric changes;
- 3) patterns and mechanisms of sediment movement; and
- 4) equilibrium profile configurations.

Profile features of special interest include break point bar formation and the presence of reflection bars. For seawall-backed beaches, the scour trough at the structure toe, the eroded and accreted volumes, and the rates of erosion and recovery were also examined.

DESIGN OF THE EXPERIMENTS

Selection of the initial profile geometry for all tests is based on an empirical relationship proposed by Dean (1977); the same criterion was employed by Kriebel et al. (1986) for tests without a seawall. To simulate natural conditions, similarity criteria are established on the assumption that the energy dissipation per unit volume along a beach profile is uniform, and that the wave properties can be properly scaled by Froude criteria.

Beach Profile Geometry. Each of the model tests conducted in the present study was molded to an initial profile of $Ax^{2/3}$; this profile geometry was determined by Dean (1977). Dean postulated that if the energy dissipation per unit volume in the surf zone is uniform and is a function of sand grain size, D , only, then the following relationship exists:

$$\frac{1}{h} \frac{\partial F}{\partial x} = \zeta_*(D) \quad (1)$$

where F is the energy flux per unit width, h is the water depth, x is a shore-normal coordinate, and $\zeta_*(D)$ is a constant depending on D only. By applying linear wave theory assumptions and shallow water approximations, the following relationship is derived:

$$h(x) = \left\{ \left[\frac{\zeta_*(D)}{\frac{24}{5} \rho g^{1/2} \kappa^2} \right]^{2/3} \right\} x^{2/3} \quad (2)$$

The quantity enclosed by $\{ \}$ is defined as $A(D)$, leaving

$$h(x) = A(D)x^{2/3} \quad (3)$$

This profile shape was examined by Dean and compared to over 500 natural beach profiles along the U.S. Atlantic and Gulf coasts. All initial profiles in the present study utilized the geometry represented by (3).

Similarity Criteria. In order for the laboratory model to faithfully represent prototype conditions, similarity criteria should be established and observed. Constructing a moveable bed model requires the horizontal scale, vertical scale, and grain size and specific gravity of the bed material to be specified.

To maintain proper similarity between model and prototype requires the geometry conditions, flow conditions, and sediment transport processes to be similar. By utilizing an undistorted model with sediment identical to the prototype, the transport process can be approximately simulated if

- 1) the Froude criterion is fulfilled for the flow field;
- 2) the sediment fall velocity scale ratio behaves as the square root of the length scale; and
- 3) the sediment is large enough to ensure a turbulent boundary layer and to ensure that the properties of the granular material are maintained.

Empirical Orthogonal Functions. By utilizing empirical orthogonal functions (eigenfunctions) to represent a series of beach profiles, all the profile data in that series may be represented in a compact form by showing spatial and temporal variations of dominant profile features. Profile variations are taken from an elevation, h , measured a distance, x , from a baseline over time intervals of variable spacing. Profile elevation at a given time may be represented by h_{xt} , where x represents an index for the spatial profile coordinate and t represents the temporal index.

The empirical eigenfunction analysis seeks to represent h_{xt} in terms of a linear expansion of the product sum of the spatial and temporal eigenvectors in the form

$$h_{xt} = \sum_{n=1}^N w_n c_{nt} e_{nx} \quad (4)$$

where w_n is the weighting coefficient for each of the N eigenvectors, c_{nt} is the temporal eigenvector, and e_{nx} is the spatial eigenvector.

The data set input into such a form results in a set of empirical eigenfunctions which best fit the data in a least-squares sense. The first set of eigenvectors, for the mode $n = 1$, represents the largest percentage of the total variance of the data set, or the dominant mode of the transport process in this case, while successively higher modes represent successively higher order perturbations. This method has been applied to beach profile field data by Winant et al. (1975), Aubrey (1979), and Kriebel et al. (1986).

As each eigenvector exhibits the property of orthogonality, individual eigenfunctions represent a certain percentage of the variance of the mean square of the data; therefore, each solution is unique in that it is not correlated to any other solution, allowing for an explanation of the variance from the input data set (Kriebel et al., 1986). Hence, large numbers of variables may be expressed by those few empirical functions which describe the major percentage of the mean square value of the data.

APPARATUS, PROCEDURE, AND CONDITIONS

System Components

Wave tank. The laboratory tests were conducted in the "air-sea" tank at the University of Florida Coastal Engineering Laboratory. The usable portion of the tank is 37 m long, 1.2 m high, and partitioned by a 1.9 m high concrete block wall into two sections of equal width, each approximately 0.87 m wide. The section of the tank in which the laboratory tests were conducted has one wall constructed of glass panels to permit visual observation of profile changes. The beach was constructed with the toe 18 m from the wave generator and with an approximate profile length of 17 m. The tank dimensions, along with the location of the profile in the tank, are shown in Figure 1.

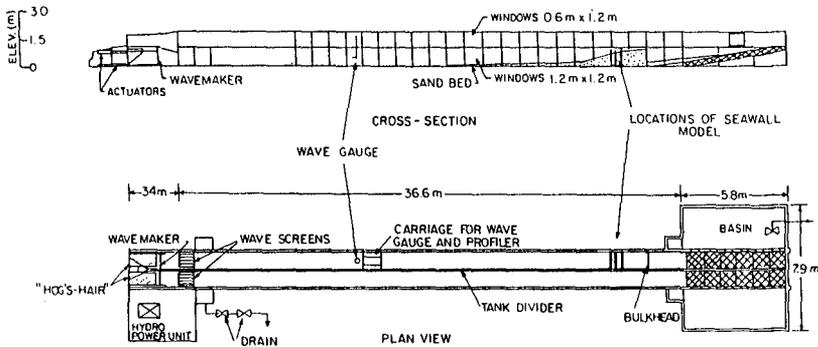


Figure 1. Schematic of Test Facility.

Sediment. The bed material used in the model tests was a fine quartz sand with a median grain diameter of 0.15 mm as determined by sieve analysis of samples obtained at four locations along an eroded equilibrium profile; a settling tube analysis was also performed. Two independent tests were performed on each sample, and the median fall velocity was determined to be 1.77 cm/sec.

Seawall model. The model seawall was constructed of 13 mm plywood 0.85 m wide and 0.88 m high coated with fiberglass resin to protect against both warping and abrasion. All experiments which were conducted with a seawall had the model installed vertically and flush with the tank bottom.

Instrumentation. Data acquisition was accomplished by utilizing a capacitance-type wave gauge and a Mark-V Electronic Profile Indicator (capacitance-sensing bottom profiler) mounted on a motorized cart which ran along parallel rails located above the test section.

Procedure

Initial Conditions. In each test, the sand bed was initially molded to an $Ax^{2/3}$ profile shape, with an A value of $0.075 \text{ m}^{1/3}$ and a beach face slope of 1/5 from the point of tangency to an elevation above the expected runup limit (Kriebel, Dally, and Dean, 1986). At this elevation, a 30 cm wide berm was constructed, the landward extent of which served as a control point for all profiles. The wave tank tests conducted by Saville (1957) were selected as the prototype tests; based on calculation of the sediment fall velocities, Kriebel et al. (1986) found the prototype to model time scale to be 3.09, and the length ratio to be 9.6. These values were also adopted for the present study. Once the profile was graded, the tank was filled with fresh water and the sand bed was allowed to soak for 10 to 12 hours. Before beginning a test, monochromatic waves were run against the initial profile for one to two minutes to allow for further bed consolidation; the initial profiles were then recorded by moving the profiler along the entire bed.

Data Acquisition. Each experiment was run a sufficient duration to reach a quasi-equilibrium condition (that is, no appreciable changes in profile configuration over time). Wave data were collected intermittently, and profile data were acquired as a time series to monitor profile evolution.

Further Profile Data Conditioning. Two independent profiles were taken at approximately one-third the cross-tank distance from each of the flume walls at each time interval in an attempt to reduce the error introduced by cross-tank variations. Such variations were also noted by Kriebel et al. (1986); possible causes are: wall boundary layer producing wave refraction, uneven bed compaction, and reflected wave interaction with incident waves.

A short series of experiments was conducted to test the profile recovery characteristics of a seawall-protected beach versus a "natural" (unprotected) beach. To accomplish this, the profiles were first subjected to erosive wave conditions accompanied by a storm surge; after eroding the profile for a fixed length of time, the water level was reduced, and lower steepness waves were produced.

Test Conditions

Figure 2 illustrates the seawall locations, water depths, and initial profile geometry used in the present investigation. Table I lists pertinent test parameters such as water depth, wave height, wave period, and seawall location. The letter designations for each Test Classification listed (A, B, C, and D) refer to tests with identical input wave conditions and water depths. Tests 1-13 were performed under erosive wave conditions, both with and without a 10 cm model scale storm surge (0.96 m full scale). This value was chosen to prevent overtopping of the seawall and, for the natural beach test cases, to prevent overtopping of the berm. Tests 14 and 15 were conducted to determine the rate and extent of profile recovery on one natural and one seawall-backed beach, respectively.

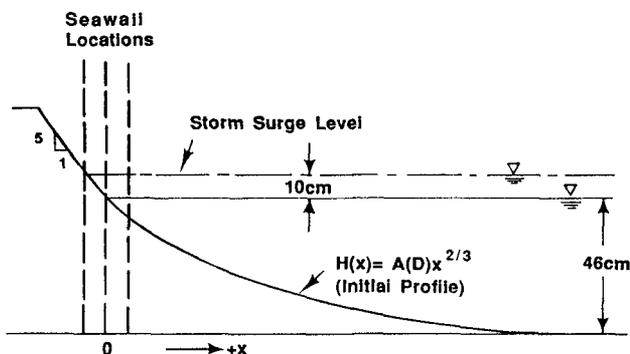


Figure 2. Schematic of Test Conditions.

Table I. Pertinent Test Parameters.

Test Number and Classification	Period (sec)	Water depth (cm)	Wave height (cm)	Seawall location, (m from 46 cm SWL)
1 A	1.81	46	11.75	N/A
2 A	1.81	46	11.75	-0.3
3 A	1.81	46	11.75	0.0
4 A	1.81	46	11.75	+0.3
5 B	1.30	46	8.80	N/A
6 B	1.30	46	8.80	-0.3
7 B	1.30	46	8.80	0.0
8 B	1.30	46	8.80	+0.3
9 C	1.81	56	11.75	N/A
10 C	1.81	56	11.75	N/A
11 C	1.81	56	11.75	-0.3
12 C	1.81	56	11.75	0.0
13 C	1.81	56	11.75	+0.3
14 D	1.81	46	4.00	N/A
15 D	1.81	46	4.00	-0.3

RESULTS AND DISCUSSION

Composite Profiles

Wave data were obtained periodically between profile data-taking intervals; wave height records were acquired by sampling with the capacitance gauge immediately seaward of the profile toe. Reflection envelopes were obtained by moving the gauge horizontally in the direction of wave propagation at a fixed rate of speed.

For tests with a 46 cm water depth, profile configurations both with and without seawalls were remarkably similar in overall planform. With the exception of local effects due to the presence of the seawall, the main bar-trough features existed for all profiles. The presence of the seawall on the profile shifted the location of the break point bar and also created a scour trough at the toe of the structure. For an elevated water level (10 cm storm surge), on the other hand, the profiles backed by a vertical wall were markedly dissimilar in final planform to those profiles without a seawall. This is apparently due to the artificially reduced surf zone width and increased wave reflection created by the wall. Reflection bars were more evident for the walled profiles than the "natural" beach profiles for the storm surge conditions; reflection coefficients were higher for the seawall-backed profiles.

Test Classification A experiments exhibited a similar trend in the formation and migration of the break point bar. Breaking waves rapidly formed a small break point bar formation at a distance of 4.3 m to 5.3 m from the baseline (intersection of 46 cm SWL with the initial profile). Bar migration in all tests in this category ($H = 11.75$ cm, $T = 1.81$ s) was observed to be offshore, with the break point location becoming fairly well-stabilized between 3.23 hours and 4.2 hours model time, as seen in Figures 3 and 4; once this occurred, bar width decreased and bar height increased. Sediment transport seaward of the break point bar then became predominantly offshore, forming a large trough 3.1 m to 3.5 m in length, with an offshore bar at the profile toe.

Test Classification B experiments also exhibited a consistent unidirectional mode of offshore sediment transport. Break point location ranged from 1.7 m (Test 7, seawall at SWL) to 2.8 m ("natural" beach) from the baseline.

Laboratory experiments conducted to simulate a 10 cm storm surge with the same wave conditions as Tests 1 through 4 displayed rapid bar formation. Bar migration for the "natural beach" tests was observed to progress offshore, with bar location stabilizing at 3.5 m from the baseline (see Figure 5). Test 10 was conducted as both a confirmation of the repeatability of the test program (Test 9) and as a means to establish initial conditions for the "natural" recovery test (Test 14).

Classification C experiments which investigated the effects of varying seawall location (Tests 11 through 13) established break point bar formation within the first 0.5 hours of testing. Bar migration for all tests was minimal due to the "fixed" shoreline created by the wall; as shown by figure 6, this, coupled with a comparatively high reflection

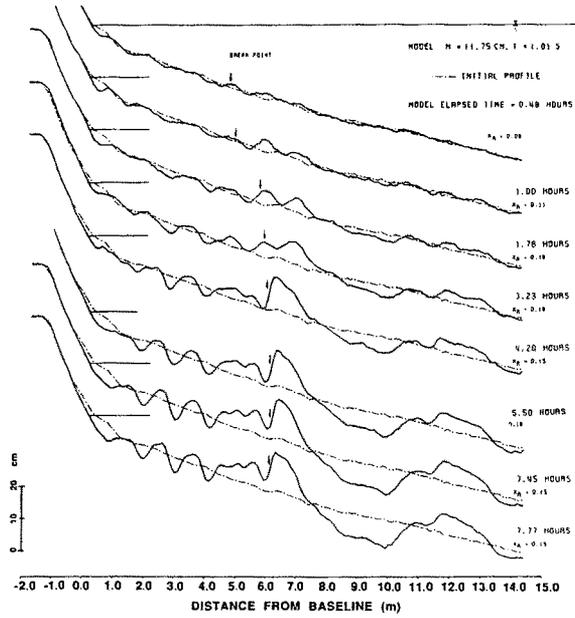


Figure 3. Time-series Profile Evolution, Test 1.

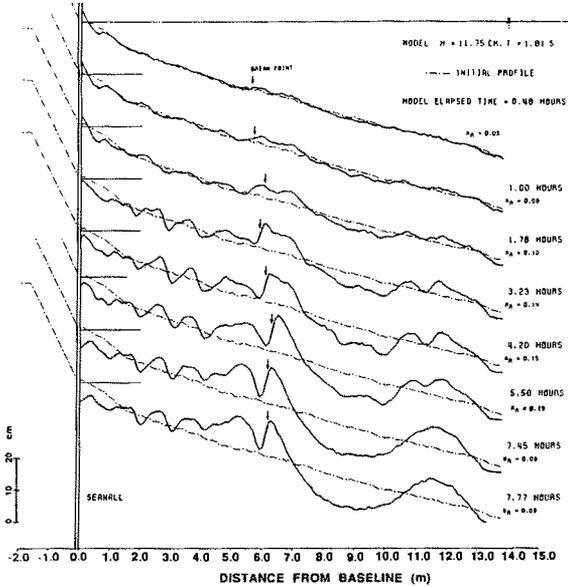


Figure 4. Time-series Profile Evolution, Test 3.

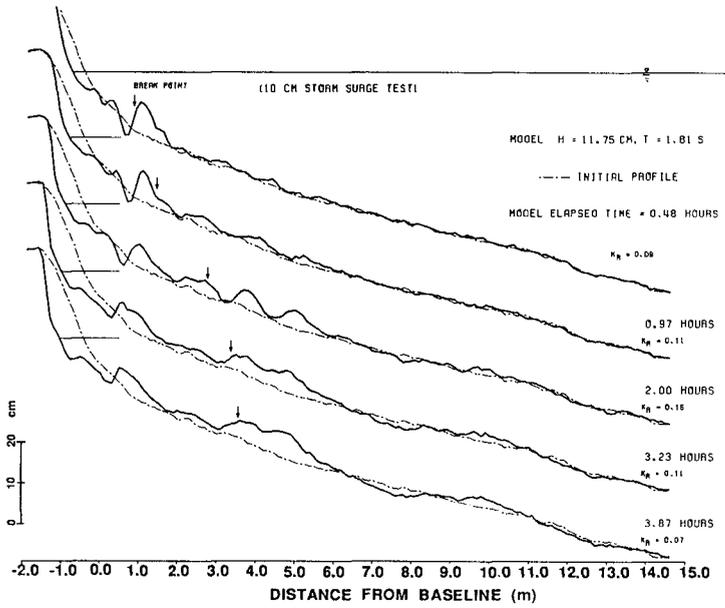


Figure 5. Time-series Profile Evolution, Test 9.

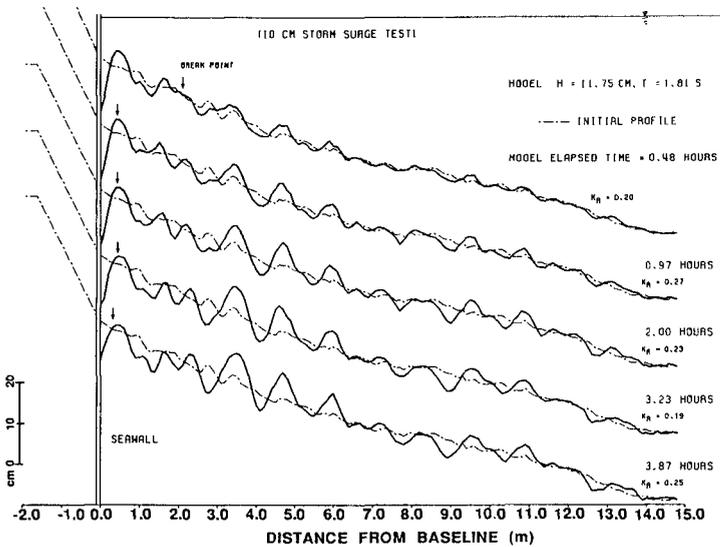


Figure 6. Time-series Profile Evolution, Test 12.
(Same wave parameters as Test 9).

coefficient, led to bar formation in proximity to the structure (0.5 m to 1.0 m). All tests displayed a partial recovery of the toe scour volume after a maximum scour condition at roughly 2.0 hours elapsed model time. This recovered volume was transferred from the break point bar crest to the scour trough. Bedform undulations seaward of the break point showed continuous growth throughout the test, with a dominant bar spacing of 1.2 m to 1.3 m, or approximately one-quarter the incident wavelength. Smaller bar formations randomly spaced along the profile acted to disrupt the dominant reflection bar spacing.

The recovery tests (Test Classification D), which were conducted by reducing the water level to 46 cm at the termination of Tests 10 and 11 for the "natural" and seawalled beaches, respectively, showed an apparent isolation of the seaward half of the profile from any significant bar migration (caused by the reduced water level and wave height). Break point bar location in Test 14 shifted onshore approximately 1.1 m due to water level reduction.

Volumetric Profile Changes

Volume Changes over Profile Length. Break point bar formations within each test classification displayed nearly identical maximum volumes, despite the fact that bar location was dependent on seawall position. For Test Classification A, bar volumes ranged from 110 cm³/cm (seawall located -0.3 m (shoreward) of 46 cm SWL) to 123 cm³/cm for the natural profile. Test Classification B bar volumes ranged from 60 cm³/cm (natural profile) to 74 cm³/cm (seawall +0.3 m). Maximum bar volumes for Test Classification C varied between 62 cm³/cm (seawall + 0.3 m) and 70 cm³/cm (natural profile).

Comparison of Denial Volume to Eroded Toe Volume. The volume change obtained with a natural profile subjected to erosive wave conditions was compared to the additional volume eroded in front of a seawalled beach; regions of interest are shown schematically in Figure 7. The final eroded profiles of the natural beach cases were selected as the baseline profiles for this study; final eroded profiles for the seawalled beaches were then used to calculate volume change. This procedure was performed for tests 1 through 13; the results appear in Figure 8. A linear regression analysis yielded a least-squares fit with a slope of 0.616. The additional volume eroded at the seawall toe was less than the volume denied to the profile upon placement of the seawall on the beach. Only one case (Test 6) exhibited a larger toe scour volume than the volume denied the profile (results plotted in Figure 8 are for final volumes only).

Empirical Eigenfunction Analysis

Standard Analysis. A standard eigenfunction analysis was performed on the laboratory data; as with the results obtained by Winant et al. (1975), the mean beach function was found to represent over 99 percent of the mean square value of the data. Laboratory test results also showed similarity to the second and third eigenfunction data obtained at Clearwater Beach, Florida by Kriebel et al. (1986).

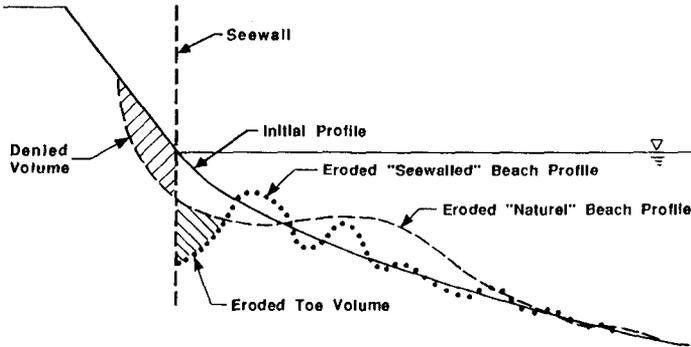


Figure 7. Schematic of Profile Features of Interest.

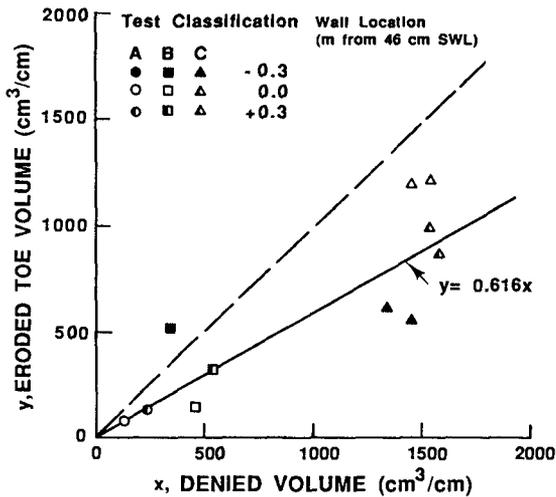


Figure 8. Denied Volume versus Eroded Toe Volume.

Modified Eigenfunction Analysis. Based on further interpretation of the remaining data, the influence of the initial profile on the mean profile configuration (first eigenfunction) was found to be highly dominant. Since the field studies cited previously dealt with spatial and temporal changes over long periods of time, no true "initial" profile existed in these investigations. Hence, a modified eigenfunction analysis was undertaken in which the method of analysis was unchanged, but the input data set was reformatted such that the initial profile acted as a baseline configuration. All profiles within a test set were subtracted from the initial profile, and the eigenfunction analysis performed on this modified data; typical results are illustrated in Figures 9 and 10. Additionally, the temporal

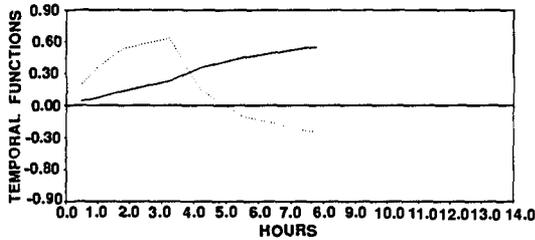
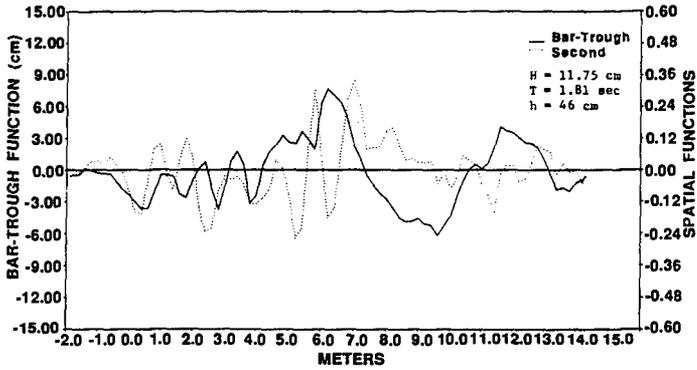


Figure 9. Modified Eigenfunction Analysis, Test 1.

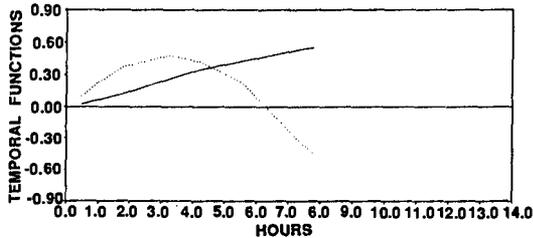
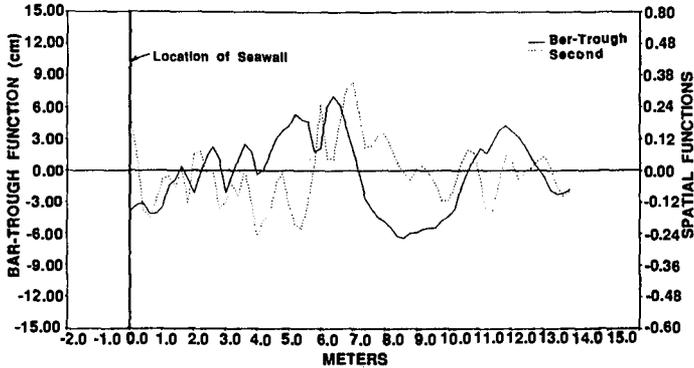


Figure 10. Modified Eigenfunction Analysis, Test 3.

functions were unified such that all values originated as positive, with an erosion from the modified mean profile assigned a negative spatial value, and an accretion a positive spatial value.

The first three eigenfunctions represent over 97.5 percent of the variance from the mean in all test cases. The first eigenfunction, referred to as the "bar-trough" function, explains 79.88 to 95.91 percent of the variance from the mean square value of the data. The second eigenfunction represents 60.0 to 86.8 percent of the variation from the first eigenfunction, with 6.2 to 29.0 percent of the variance explained by the third eigenfunction. Due to the complex behavior exhibited by the third eigenfunction, only the first and second eigenfunctions were analyzed.

The first eigenfunction appears to represent a "bar-trough" behavior along the profile length. The solid line for the spatial function (shown in Figures 9 and 10) displays the location and magnitude of the bar and trough features present in each profile as a weighted average of the input data; by coupling the first temporal function with the first spatial function in each plot, the time rate of change of the profile features could be interpreted.

Tests 1 through 3 showed maxima on either side of the break point bar crest which dominated the first 3.23 hours of the test, then diminished in effect with further profile evolution. By comparing these features with Figures 3 and 4, the dual-crested bar was observed to initially steepen, then become subdominant upon emergence of single-peaked, highly skewed break point bars. The maxima which appear shoreward of the break point correspond to the skewness of the berm erosion or toe scour trough and bar feature (depending on whether a seawall was located on the profile).

For the storm surge tests, the seawalled profiles revealed numerous reflection bars. The main incongruity between the storm surge and normal water level tests with erosive wave conditions was the behavior of the second temporal function. A rapid decrease from an initial peak value was noted in all storm surge tests, which corresponds to a temporal decrease in the scour trough skewness. The no-seawall tests showed a peak spatial function value at the shoreward dune erosion limit, which corresponded to the temporal steepening of the dune face.

The recovery tests displayed extremely "noisy" second spatial functions due to the presence of well-established reflection bars which exhibited minimal spatial change over the seaward half of the profile (owing primarily to a decreased wave height); numerous maxima and minima over the shoreward portions of the profile were also revealed. Temporal changes differed greatly between the two test cases, with the seawalled profile demonstrating a more stable trend of the skewness of the surf zone bar formations in the latter 4.0 hours of testing than that exhibited by the no-wall case. The seawalled profile did not exhibit a relative minimum value of the bar skewness in the vicinity of the break point bar; however, both profiles exhibited a distinct break point bar-trough feature (due primarily to the reduction in water level) which caused the bed to scour shoreward of the break point.

SUMMARY AND CONCLUSIONS

Summary of Investigation. Due to the recent attention being focused on the effects of seawall placement on eroding coastlines, there exists a need to examine the resulting profile configuration along with the erosion or recovery rates to ascertain whether the presence of the structure is detrimental to the beach-dune system. A laboratory test series was developed which examined the cross-shore effects of seawall placement on profile response. Erosive wave conditions comprised the majority of the experiments; a limited study of beach recovery characteristics was also undertaken. A substantial set of laboratory data was obtained, and the results have been presented in a comparative sense to further emphasize the structure effects. The complete data set and analysis are presented in Barnett (1987).

Important Findings. The test results were examined by time-series profile evolution, volume change over profile length, and empirical eigenfunction analysis. For all cases tested, profile configurations with and without a seawall were remarkably similar in overall planform; this suggests that the major transport process is not significantly influenced by the presence of the seawall.

Under erosive wave conditions, the dominant spatial feature is a bar-trough system, with the bar forming in proximity to the wave breaking point and the trough occurring near the still water shoreline. The presence of the seawall accentuates the trough into a scour hole instead of spanning over the swash zone, as is the case with a natural beach. However, while local scour was noted to be severe in many of the seawalled profiles, the volume of sand retained upland of the structure (which would otherwise be eroded under identical wave conditions without a seawall) was found experimentally to be approximately 60% greater than the additional volume eroded at the toe of the structure.

Wave reflection, often considered to be a major adverse influence on scour in front of a seawall, did not appear to play a significant role. Reflection bars of varying quantity and spacing did occur, but were usually secondary features of a predominantly bar-trough system. Water depth, on the other hand, appeared to be a dominant factor affecting erosion for all cases tested, with higher water levels causing a significant increase in erosion. For the seawalled cases, the break point bar was in proximity to the wall; the natural profiles displayed a more seaward bar location. This affected the recovery process when the water level returned to normal.

The empirical eigenfunction analysis is a useful tool for examining profile evolution. By conducting a modified eigenfunction analysis, the spatial and temporal behavior of the profile evolution process was more readily facilitated. The primary bar-trough system and the secondary reflection bar system were clearly revealed in spatial plots for all erosive test cases. The primary system accounted for more than 80 percent of the variation from the initial profile. A relatively smooth spatial function coupled with a slowly-varying temporal function revealed the stable nature and steady evolution of the bar-trough system.

The "noisy" behavior exhibited by the second spatial function of the recovery tests showed the recovery process to be more unstable. The primary recovery process was the removal of the break point bar and the redistribution of sand both onshore and offshore. The more prominent primary spatial function in the seawalled case revealed a more efficient recovery.

The seawalled beach exhibited a more substantial recovery volume in the vicinity of the structure toe than that observed for the natural profile, which showed only a small berm growth. This is not sufficient evidence to construe that placement of a seawall on an eroded beach will promote recovery; rather, the recovery process appears possible on a seawalled beach provided the water level and wave conditions are capable of transporting sediment to the scour trough created under erosive conditions. Further recovery tests should be conducted to confirm this assumption.

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