CHAPTER 101

PROBABILISTIC ANALYSIS OF SEAFLOOR LIQUEFACTION

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To investigate the reliability of a sandy soil layer in an ocean wave environment a liquefaction model is used in conjunction with a first order reliability method. Thus, sensitivity indices of the soil-water system with respoect to the uncertain strength and input variables are computed, and the relative importance of the various factors defining the problem can be determined. The relationship of this approach with more conventional design methods (deterministic models, risk models) is discussed along with the range of applicability of the different safety measurements.

INTRODUCTION

When ocean waves propagate over a cohesionless oceanfloor, hydrodynamic pressures are continuously being generated at a rate which depends mainly on the permeability of the material. Sandy soils with low permeability coefficients are unable to dissipate the excess pore pressures, which eventually may become equal to the total stresses acting on the soil element. The result is the complete loss of all intergranular effective stress and bearing capacity of the material, a cyclic instability phenomenon known as liquefaction of the sand.

In this paper the liquefaction behavior of seafloor soils is evaluated by means of an analytical model-proposed earlier in the literature-which takes into account the physics of the phenomenon. Based on that model the following steps are taken:

a) Carry out a parametric study to rank the relative importance of the factors governing liquefaction processes (deterministic approach).

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b) Incorporate uncertainties in input/output variables into a fully probabilistic analysis of the problem.

c) Use reliability techniques to compute sensitivity factors and critical values of the random variables.

d) Compare the above methodology with classical risk methods.

DETERMINISTIC APPROACH

For the purposes of this study, a simplified pore pressure analysis of the liquefaction phenomenon, first described by Layas (1982) and Rahman and Jaber (1986), has been chosen. In this model harmonic linear waves are used, with fixed periods, T, and wave lengths, L, related by the formula:

$$T=\sqrt{\frac{2\pi L}{g\tanh(kd)}} \qquad (1)$$

where $k=\frac{2\pi}{L}$ is the wave number and d is the water depth.

Two cases are considered:

a) Undrained model (infinite layer)
b) Drained model (finite layer)

Undrained model

If the permeability of the soil is zero, the pore pressure, u_g, generated at the sand layer (semi-infinite half space) at depth z is derived, according to the following algorithm.

1. Assume a linear pore pressure generation curve, u_g (t):

$$\frac{u_g}{\sigma'_{vo}} \simeq \frac{N}{N_l} = \frac{t}{t_l} \qquad (2)$$

(σ'_{vo}=initial effective stress; N_l=no. of cycles to liquefaction)

2. Assume a liquefaction cyclic strength curve, $\tau_l(N_l)$, of fatigue type:

$$\frac{\tau_l}{\sigma'_{vo} D_r} = a N_l^{-b} \qquad (3)$$

(a,b=constants; D_r=relative density)

3. Compute the hydrodynamic pressure on the sea bed by linear wave theory:

$$p_o(x,t)=\frac{\gamma_w H}{2\cosh(kd)}\cos(kx-\omega t) \qquad (4)$$

(k=wave number; w=wave frequency; d=water depth)

4. Compute the wave-induced shear stresses in the soil by elasticity theory:

$$\tau(x,z,t)=\frac{\gamma_w H K z}{2\cosh(Kd)}e^{-Kz}\cos(Kx-\omega t-\varphi) \qquad (5)$$

5. Evaluate the liquefaction time at depth z, t_l, from the conditions at the onset of liquefaction (equations 2, 3 and 4) written for $z=z_l$, $N=N_l$, and $t=t_l$)=

$$t_l=T\cdot N_l=T\cdot\left[\frac{\gamma' 2a\cdot D_r\cosh(Kd)}{\gamma_w HK}\right]^{\frac{1}{b}}\cdot e^{Kz_l/b} \qquad (6)$$

Then, substituting in eq. (2), yields:

$$u_g=\frac{\gamma' z t}{t_l} \qquad (7)$$

where γ' is the buoyant unitweight of the soil.

Notice that the amplitude of the harmonic expressions 4 and 5 is independent of x, meaning that at the same depth the induced stresses at different horizontal positions have the same maximum values. Therefore neglecting the differences in phase, only one-dimensional responses need to be considered (Seed and Rahman, 1978).

<u>Drained model</u>

For a permeable stratum the equation governing the pore pressure rresponse of the soil, u, becomes:

$$c_v\frac{\partial^2 u}{\partial z^2}=\frac{\partial u}{\partial t}-\frac{\partial u_g}{\partial t} \qquad (8)$$

where c_v is the consolidation coefficient of the sand.

Equation 8 quantifies the balance of the simultaneous generation and dissipation of pore pressures that continuously occurs inside the soil layer. By inserting in eq. 8 the correspondig value of the generation rate term:

$$\frac{\partial u_g}{\partial t}=Cze^{\tau z}\frac{\gamma'}{T}=\left(\frac{\gamma_w\pi}{\gamma' a D_r}\right)^{\frac{1}{b}}\left(\frac{H/L}{\cosh\left(\frac{2\pi d}{L}\right)}\right)^{\frac{1}{b}} z\, e^{\tau z}\frac{\gamma'}{T} \qquad (9)$$

and solving for the appropiate boundary and initial conditions, namely (Fig. 1):

B.C. previous top $\longrightarrow \left[u(z,t)\right]_{z=0}=0$ (10-1)

B.C. impervious bottom $\longrightarrow \left[\frac{\partial u}{\partial z}(z,t)\right]_{z=D}=0$ (10-2)

I.C. $\longrightarrow \left[u(z,t)\right]_{t=0}=0$ (10-3)

the following expression for the excess pore-pressure curves is found:

$$u(z,t)=\frac{C}{c_v}\frac{\gamma'}{T}\left\{w(z)+v(z,t)\right\} \qquad (11)$$

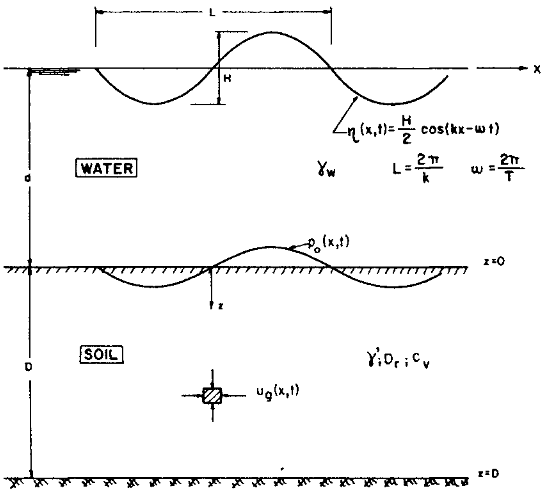


Fig.1. Scheme of the problem.

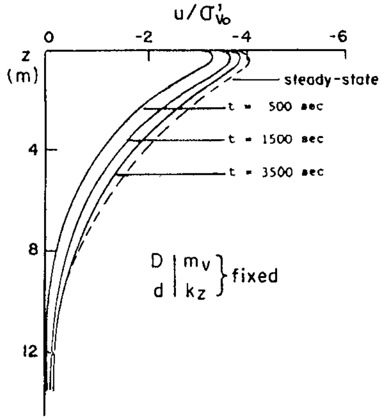


Fig.2. Dynamic isochrones ata different stages of the storm. Drained layer (after Layas,1982)

in which:
$$W(z)=\left(\frac{2}{\tau^3}-\frac{z}{\tau^2}\right)e^{\tau z}+\left(\frac{D}{\tau}-\frac{1}{\tau^2}\right)e^{\tau D}z-\frac{2}{\tau^3} \qquad (12\text{-}1)$$

and
$$v(z,t)=-\frac{2}{D}\sum_{n=0}^{\infty}\left\{h(\alpha_n,D,\tau)\right\}e^{-c_v\alpha_n^2 t}\sin(\alpha_n z) \qquad (12\text{-}2)$$

are, respectively, the steady-state and transient components of the dynamic pore pressure curves.

In the above expressions,

$$\alpha_n = \frac{2n+1}{2}\frac{\pi}{D} \qquad (13\text{-}1)$$

is a geometrical value, whereas

$$\tau = -\frac{K}{b} \qquad (13\text{-}2)$$

is a parameter related to the input storm as well as the mechanical properties of the soil. Obviously, as $t\rightarrow\infty$ the term v(z,t) vanishes, and the limiting (steady-state) isochrone becomes fixed in shape:

$$u(z) \simeq \frac{C\gamma'}{c_v T}\,W(z) \qquad (14)$$

Figure 2 exemplifies this behavior for different times of evolution of a given storm at a liquefiable site.

Factor of safety. Parametric study

The model described in the previous section defines the factor of safety against liquefaction at a given time and depth within the layer as:

$$(SF)_d = \frac{u}{\sigma'_{v_0}} \qquad (15)$$

For the particular case of impervious soil, $u=u_g$, and the safety factor becomes:

$$(SF)_u = \frac{u_g}{\sigma'_{v_0}} = \frac{t}{t_l} \qquad (16)$$

Where t_l is a known parameter (eq.6). In both cases the onset of liquefaction (safety factor=1) is defined as the state in which the pore pressure generated by the ocean waves equals the initial effective pressure at some point in the deposit (soil resistance).

Since the parameter SF constitutes a global index of the safety of the deposit against liquefaction under various drainage conditions, its sensitivity to the variables involved is of practical importance.

To investigate further this point a classical parametric study hass been carried out on a digital computer. Table I shows the mean values assumed for all variables (reference state), whereas the corresponding ranges of variation and their effects on $(SF)_u$ and $(SF)_d$ are represented, side by side, on Figures 3 to 11 for a storm of 10 minutes duration. From a comparative analysis of these figures it can be concluded that:

a) The safety factor increases with the wave length and all soil parameters but b (when H and/or b increases the safety factor decreases).

b) Wave heigth and consolidation coefficient affect significantly the liquefaction behavior of the soil.

c) The liquefaction resistance of drained deposits is roughly one order of magnitude (ten times) greater than that of undrained ones.

d) Monotonic variations of the geometrical parameters (d,D) lead to non-monotonic trends for the safety factor. For this reason these parameters are assumed fixed hereafter.

PROBABILISTIC APPROACH

Basic Concepts

In order to evaluate realistically the liquefaction potential of the stratum of sand, the method employed should take into account the probabilistic nature of the problem. Uncertainties in the dynamic bahavior of the soil and/or in its geotechnical parameters are important and cannot be neglected. Furthermore, very often the input load is defined statistically, and so the wave parameters are random in nature.

Again, according to the time span used in the study, two types of models are considered:

a) Reliability models (short term conditions)
b) Risk models (long term conditions)

Reliability model

In this case the wave parameters are those associated to storm conditions at the sea, which in turn depend on the location of the site relative to the zone of wave generation. Then, for a given storm (fixed time), a reliability technique is used to compute the probability of liquefaction of the deposit for the assumed sea states.

A mean-centered, first-order, second-moment reliability technique (FOSM model) has been proposed recently by Layas (1982), but no quantitative results were reported. In this paper a first-order marginal distribution method (Madsen

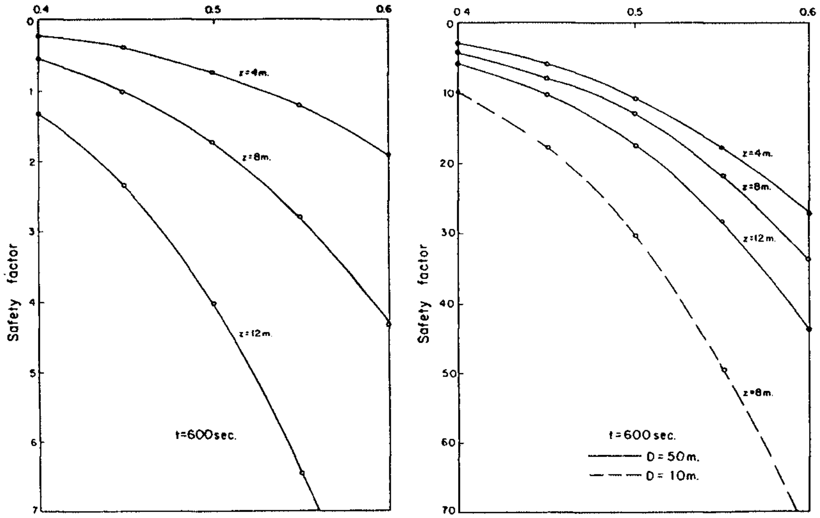


Fig.3. Parametric study.

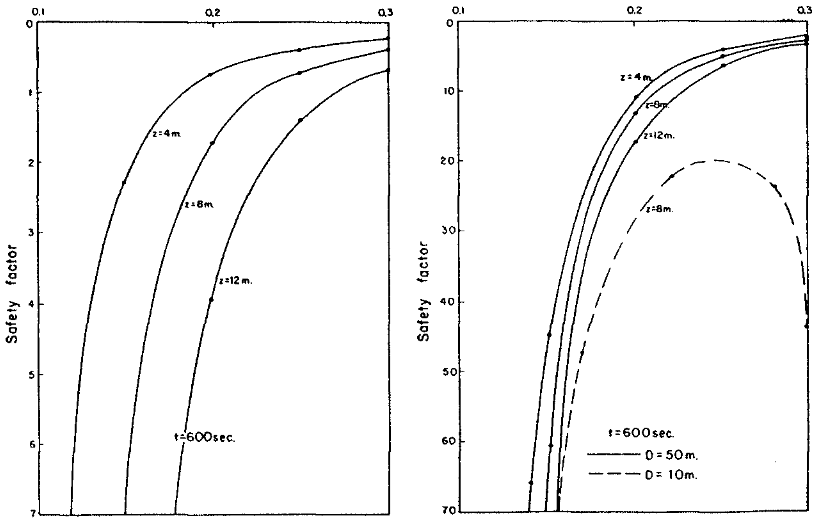


Fig.4. Parametric study (cont.)

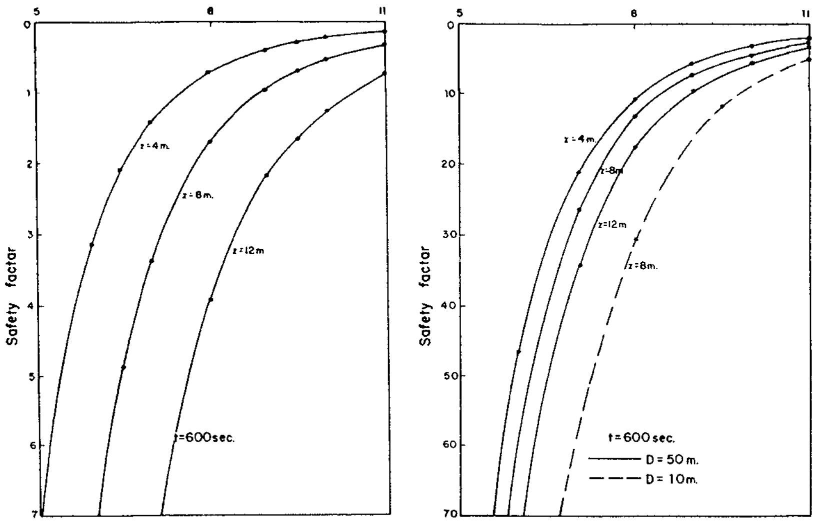


Fig.5. Parametric study (cont.)

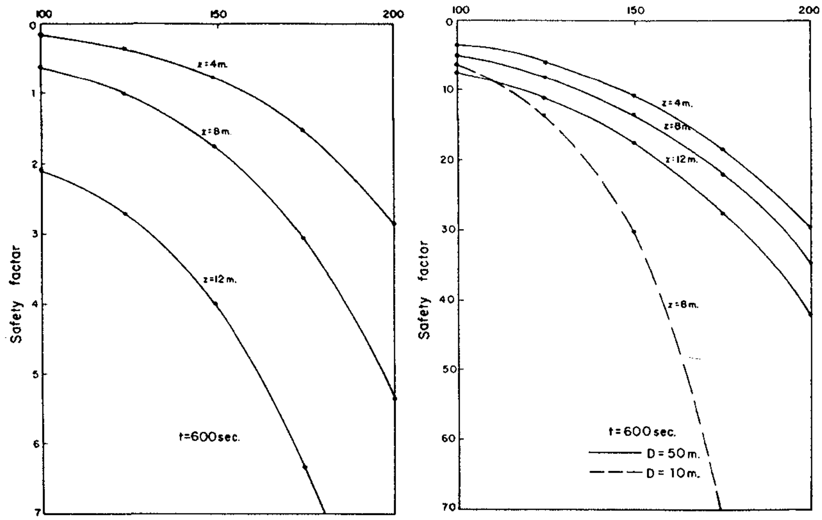


Fig.6. Parametric study (cont.)

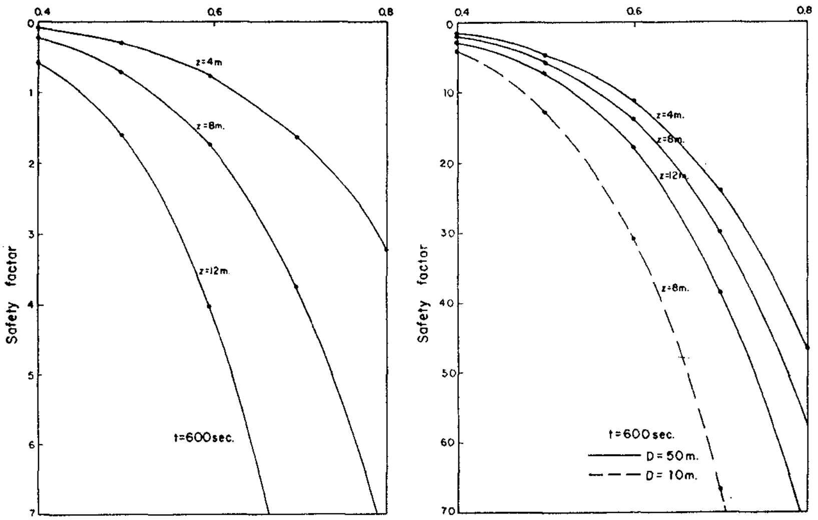


Fig.7. Parametric study (cont.)

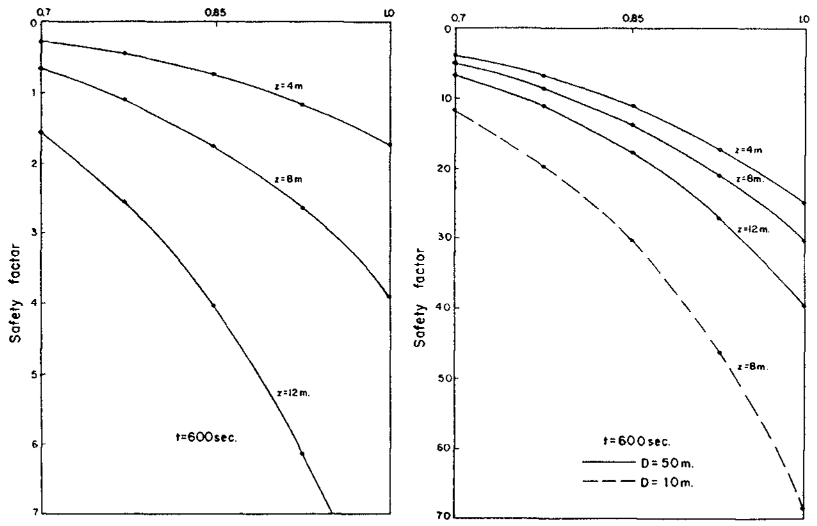
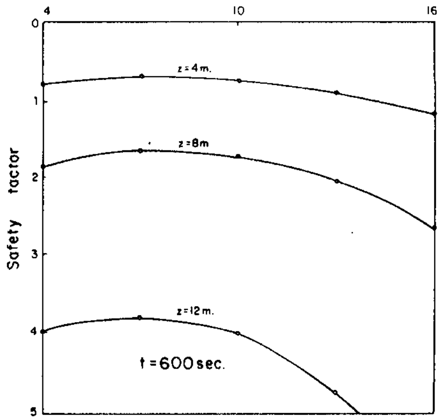
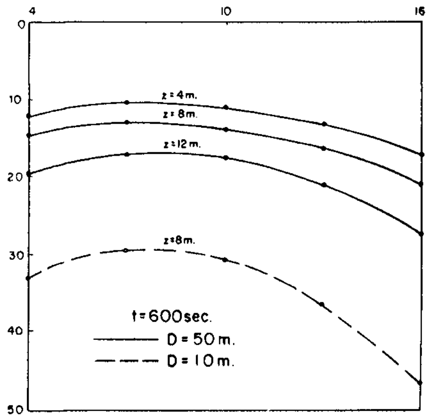


Fig.8. Parametric study (cont.)



d

Fig.9 Parametric study (cont.)



d

Fig.9 Parametric study (cont.)

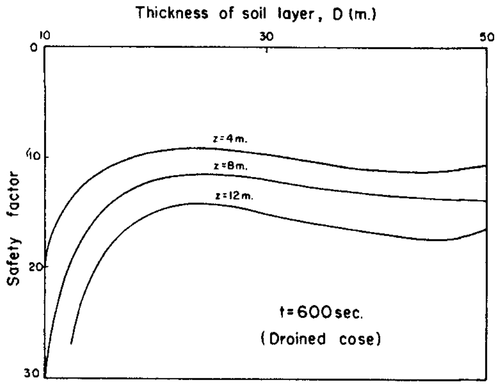


Fig. 10. Parametric study (cont.)

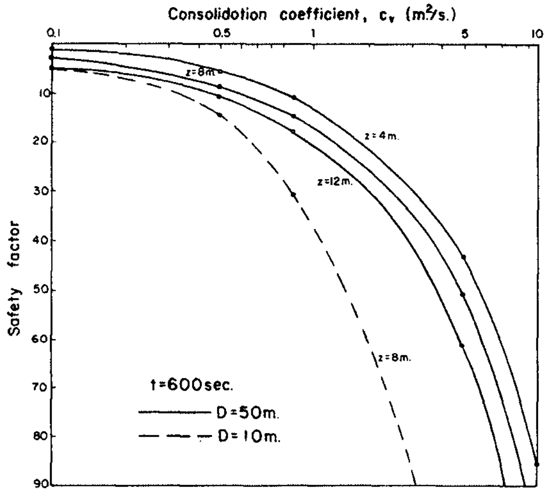


Fig. 11. Parametric study (cont.)

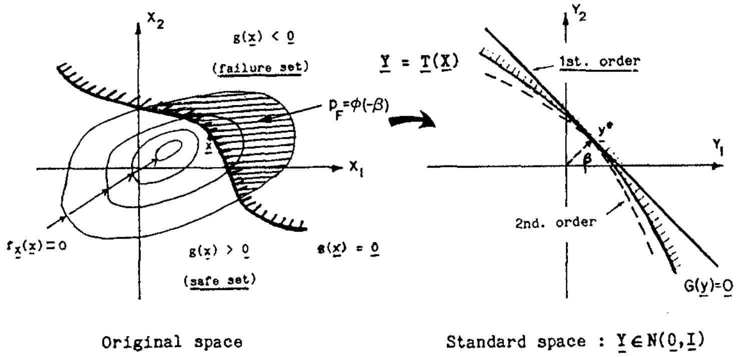


Fig.12. Reliability analysis. Computational procedure.

et al, 1986; Blâzquez and Der Kiureghian, 1987) has been applied to compute the liquefaction susceptibility of the layer (unimodal system). Within this contest, the value of the actual short term probability of failure, p_l, is computed as:

$$p_l = \int_{g(\underline{x})<\underline{0}} f_{\underline{x}}(\underline{x})\, d\underline{x} \qquad (17)$$

in which $g(\underline{x})=0$ is the limit state function of the layer in the liquefaction mode of failure and $f_{\underline{x}}(\underline{x})$ denotes the pdf of the set of joint random variables, $\underline{x}$, describing the state fo the layer. Figure 12 depicts schematically the way to proceed in order to compute p_l for a specific case, namely $\underline{x}=(x_1, x_2)$. Details on the computation procedure may become quite involved and are given elsewhere (Der Kiureghian and Liu, 1986).

For the liquefaction problem, the limit-state functions are formulated as follows:

Undrained case

$$\{g_{\underline{x}}(\underline{x})=\underline{0}\} \equiv \{u_g - \sigma'_{v_0}=0\} \equiv \left\{\gamma' z \left[1 - \frac{t}{T}\left(\frac{\gamma' 2 a D_r \cosh(Kd)}{\gamma_w H K}\right)^{-\frac{1}{b}} e^{-\frac{Kz}{b}}\right]=0\right\} \qquad (18\text{-}1)$$

Drained case

$$\{g_{\underline{x}}(\underline{x})=\underline{0}\} \equiv \{u - \sigma'_{v_0}=0\} \equiv \left\{\gamma'\left(z - \frac{C}{C_v T}\left[w(z)+v(z,t)\right]\right)=0\right\} \qquad (18\text{-}2)$$

where z,t are parameters and the other symbols stand for random variables. The statistical distribution and first two moments of these variables is given in Table II. Only first order reliability analysis is considered in the present work.

Figure 13 illustrates the results of such an analysis. The probability of liquefaction of either a drained or undrained stratum at a given time and location is computed for a fixed geometry of the problem, specified in terms of water depth and thickness of the layer. A sample calculation (point P; Fig 13) is shown below:

Sample Calculation (point P)

Variables	x_i^*	y_i	Parameters
H	9.61	0.737	t = 600 sec. z = 6 m.
L	164	-0.301	D = 50 m.
D_r	0.50	-0.398	d = 4 m.
γ'	0.75	-0.338	β (First order) = 1.71
a	0.47	-0.298	
b	0.20	0.058	P_l (First order) = 0.0438
c_v	1.34	-0.023	

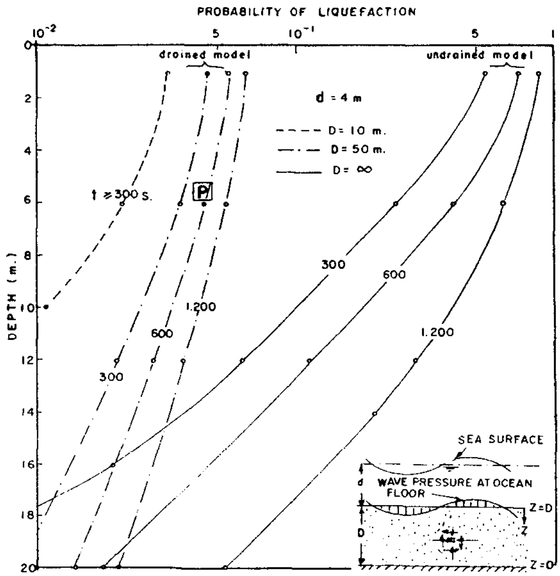


Fig.13. Probability of liquefaction. First-order reliability analysis.

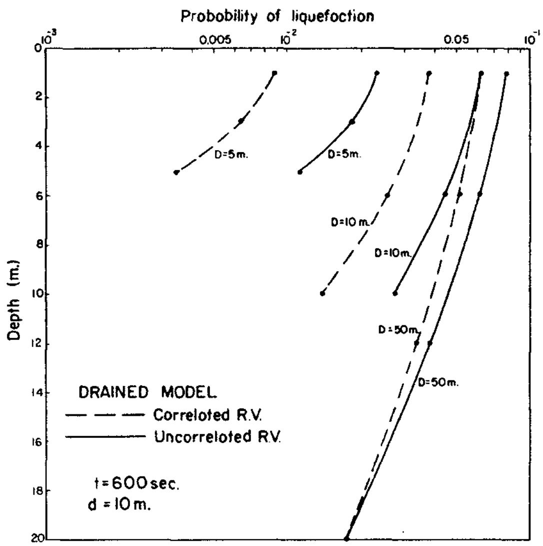


Fig.14. Effects of correlation and layer depth on the probability of liquefaction.

This calculation indicates that, for drained soils, the liquefaction behavior of the soil is mostly influenced (γ_i high) by the wave height and drainage conditions and, to a less degree and with opposite sign, by the relative density of the sand. The same conclusions apply to undrained deposits which, other conditions being equal, are roughly one order of magnitude unsafer than drained ones.

The combined effects of correlations among random variables and layer depth have also been investigated and are displayed graphically in Figure 14. A quick inspection of this figure reveals the probability of liquefaction becomes higher as the degree of correlation decreases and/ or the length of the drainage path increases. Also, as could be expected, deep-sea deposits are less susceptible to liquefaction than surficial seabeds. This effect is more dramatic for thin permeable layers (Figure 15).

Risk model

In this case the joint distribucion function of the set $\underline{X}$ of random variables (eq. 17) is no longer employed. Wave input and liquefaction response random variables are split up, and the annual (long term) probability of liquefaction of a given soil layer is written as:

$$(p_l)_{D_r} = \int_0^H (p_l \mid H, D_r)\, f_H(H)\, dH \qquad (19)$$

Where $(p_l \mid H, D_r)$ = the probability of liquefaction at the site for specified values of the significant wave height and the relative density of the soil, and $f_H(H)$ = the density function of the sea state H. The last function is independent of the geomechanical properties of the site and can be derived from the long-term statistics of the severity of the sea, a topic that has been throuroughly discussed in the literature (Ochi, 1982). On the contrary, the conditional local probability of liquefaction for a given wave climate at the site, $(p_l \mid H, D_r)$, is not well known yet and depends solely on the uncertainties in the liquefaction model.

As an example, Figure 16 shows the risk of liquefaction at a depth of 1 m for loose and dense sand deposits located at two well different sites in the spanish shoreline, namely Bilbao (Biscay Gulf) and Alicante (Mediterranean Sea). The dramatic effect of the wave climate in the expected liquefaction behavior of the soil is self-evident.

Conclusions

Application of probabilistic techniques to the liquefaction analysis of sandy soil layers in an ocean

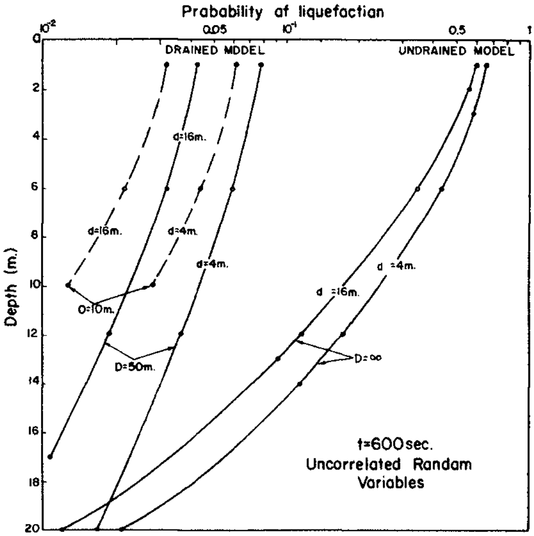


Fig. 15. Effect of water depth on the probability of liquefaction.

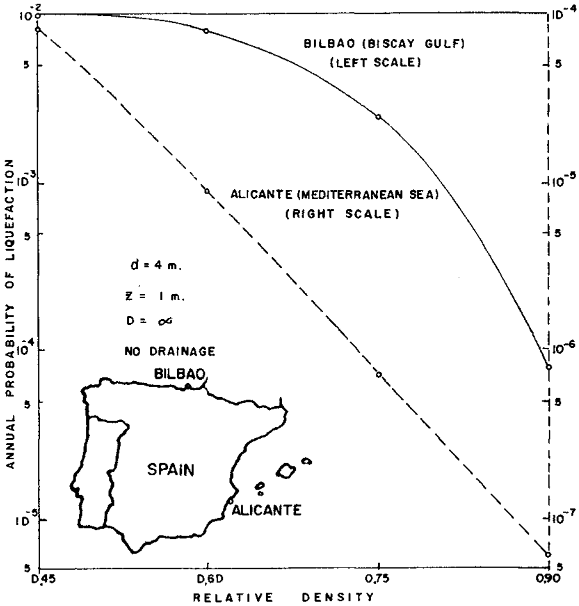


Fig.16. Risk of liquefaction at two sites on the Spanish coast.

wave environment is reviewed. The following conclusions can be drawn:

* Seafloor liquefaction failure is mostly influenced by drainage conditions, and to a less degree by wave height and relative density of the sand.

* Drained soil deposits are roughly one order of magnitude safer than undrained ones.

* Uncorrelated models of liquefaction reliability are overconservative, especially for thin soil layers.

* The probability of liquefaction increases with the thickness of the layer (⇒slower rate of consolidation) and shallow water depth (⇒less confinement).

* Risk liquefaction models-in which wave action and liquefaction resistance are uncoupled-can be easily derived from reliability models for long-term wave conditions.

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