CHAPTER 85

INITIAL MOTION IN COMBINED WAVE AND CURRENT FLOWS

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Abstract

Measurements are presented of the conditions for the initial motion of sediment under combined steady and oscillatory flow. The measurements were made in a steady flow flume with an oscillating tray set into its bed. The direction of oscillation of the tray was at right angles to the axis of the steady flow flume. Four different grades of sand were tested.

It is found that the critical condition for the initiation of motion is reasonably represented by a critical value of the vector sum of the component shear stresses assuming no nonlinear interaction between the steady and oscillatory flows. The resultant bed shear stress was also calculated with the aid of several combined wave-current models. The results of these various approaches are compared with Shields curve.

Introduction

The conditions under which sediment first begins to move due to a flow of fluid are of great importance in many engineering problems. There have consequently been numerous studies of the initial motion condition in both pure steady flow and in pure oscillatory flow (see, for example, Sleath 1984). On the other hand, there have been surprisingly few investigations of the initial motion condition for combined steady and oscillatory flow. Hammond and Collins (1979) studied the case where the steady and oscillatory flows are collinear. More recently, Katori et al (1984) have made measurements of steady and oscillatory flows at right angles to each other. The aim of the present paper is to study the second of these two cases.

Experimental Layout

The experimental apparatus is shown schematically in Fig. 1. The sediment is contained in a tray set into the bed of the flume. The tray oscillates (see Sleath, 1976,

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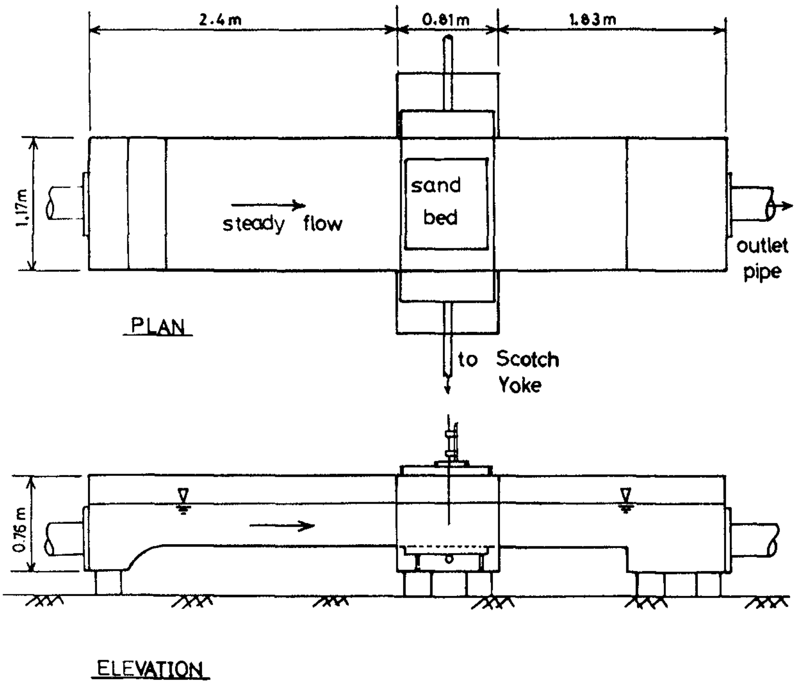

Fig. 1. Schematic drawing of the combined steady and oscillatory flow flume

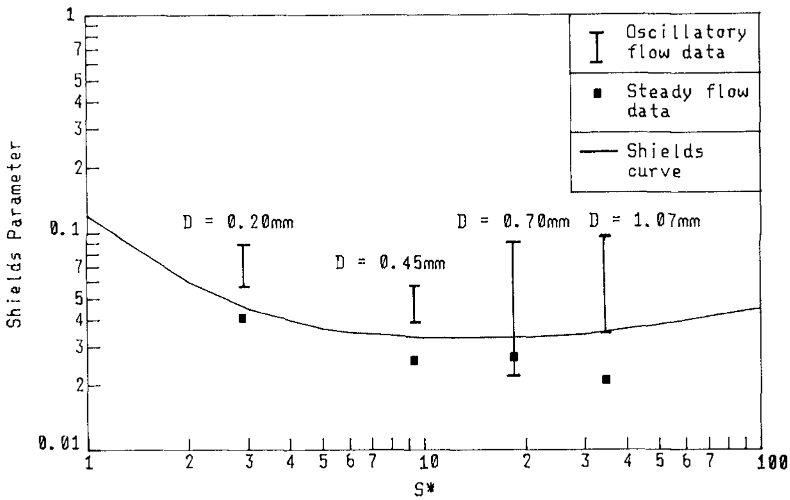

Fig. 2. Results of initial motion experiments under steady flow alone and oscillatory flow alone

for a description of the mechanism) with simple harmonic motion in a direction at right angles to the axis of the steady flow flume. The amplitude of oscillation can be varied from 0 to 0.19 m and the working period ranges from 0.5 secs up to about 6.0 secs.

The steady flow is provided by two re-circulating pumps. With the aid of a control valve on the inlet pipe the mean steady current can be varied from 0 to 0.4 m/s. The steady velocity was monitored throughout the tests at a point just upstream of the oscillating tray and at a height of 0.04 m above the bed. In addition, a number of vertical traverses were made which indicated that the velocity profile was logarithmic up to that height at this position.

Four different sands were used in the tests. Their median diameters were 0.20 mm, 0.45 mm, 0.70 mm and 1.07 mm.

The normal test procedure was as follows. At the start of the test the bed of sand was carefully levelled. The bed of sand was then caused to oscillate at a fixed period T and amplitude A_b. Finally, the steady flow velocity was gradually increased until threshold conditions were observed. The criterion adopted for initial motion was Kramer's (1935) weak motion regime.

The depth of water was held equal to 0.17 m for the finest sand and 0.12 m for the other sands.

Experimental Results

As well as performing initial motion experiments under combined flow conditions, initial motion under, (a), pure steady flow and, (b), pure oscillatory flow was also examined. The reason for this was two-fold. First it was necessary to check that the rig worked correctly and produced results, under these conditions, which were comparable with other published values, and secondly it was necessary to find the limits between which we could expect the combined flow results to lie.

The results of experiments under conditions (a) and (b) are shown in Fig. 2. The shear stresses under pure oscillatory flow were found assuming a quadratic drag law in which the wave friction factor was taken from Kamphuis (1975) friction factor curves. It is seen that the data lies close to the Shields curve. Following Madsen and Grant (1976) the Reynolds number has been replaced in Fig. 2 by

$$S_* = \frac{D}{4\nu} \sqrt{\frac{(\rho_s - \rho)}{\rho} g D} \qquad (1)$$

where D is median grain diameter and ρ_s and ρ are, respectively, sediment and fluid density.

The raw results for the critical conditions under which sediment just begins to move in combined steady and oscillatory flow are shown in Figs. 3 - 6. In these figures u_w is the amplitude of the velocity of the oscillating tray and u_c is the depth-averaged steady current just upstream of the test section. In order to guide the eye, curves of the form

$$\left(\frac{u_w}{u_{wo}}\right)^n + \left(\frac{u_c}{u_{co}}\right)^n = 1 \qquad (2)$$

have been drawn through the experimental points. In this equation u_{wo} and u_{co} are the critical values of u_w when the steady current is zero and u_c when the oscillatory velocity is zero, respectively. Values of n between 2 and 4 appear to give the best agreement with the experimental results.

Inertia Effects

One problem with a study of sediment entrainment in an oscillatory tray apparatus is the inertia force acting on the grains of sediment. If we consider spherical particles of diameter D and dry density ρ_s there is an additional force (i.e. in addition to the forces which the particle would experience if it were on a stationary bed in an oscillating flow) equal to

$$(\rho_s - \rho)\frac{\pi D^3}{6}\frac{\partial u}{\partial t}$$

where u is the velocity of the tray.

For the finest sand used in these experiments the additional force due to inertia is negligible compared with the other forces acting on the grains. However, for the coarser sands this is not always the case, particularly at the lowest periods of oscillation. It would be possible to make a correction to the experimental results to allow for this additional force. However, this requires a knowledge of the phase relationships of the forces acting on the particle, the horizontal and vertical directions in which they act and, finally, some estimate of the shape of the grains. When the required correction is large this procedure is not very reliable. Consequently, in the comparison of different models outlined below only those experimental results for which the magnitude of the additional force is less than 30 % of the magnitude of the force which the grains would experience on a stationary bed have been retained. For the purposes of deciding which results to retain the force for a stationary bed has been taken as the vector sum of the forces calculated for oscillatory flow and steady flow separately. The same set of experimental results is used in each of the Figures 8 - 11.

Although the magnitude of the additional force in the retained experimental results may be as much as 30 % of the magnitude of the force for a stationary bed the actual error in the resultant force is generally much less than that.

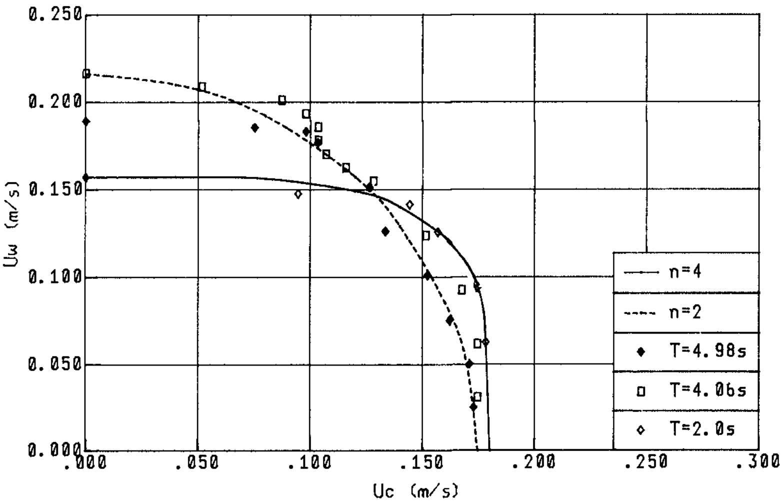

Fig. 3. Critical velocities for D = 0.20 mm

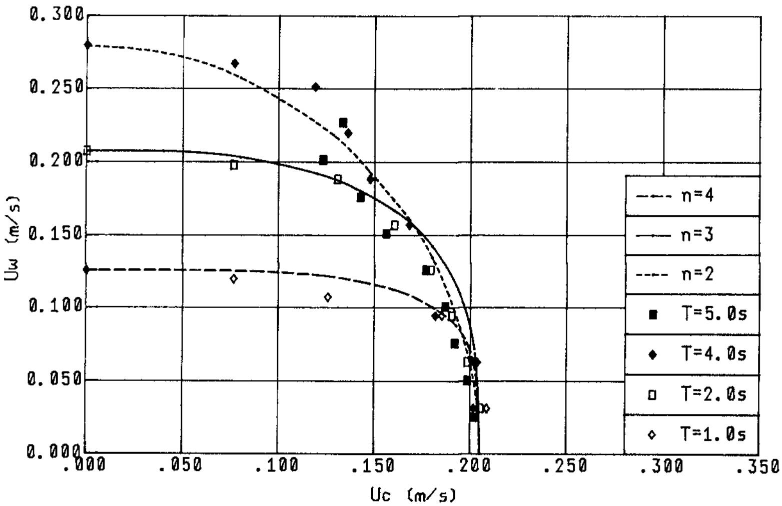

Fig. 4. Critical velocities for D = 0.45 mm

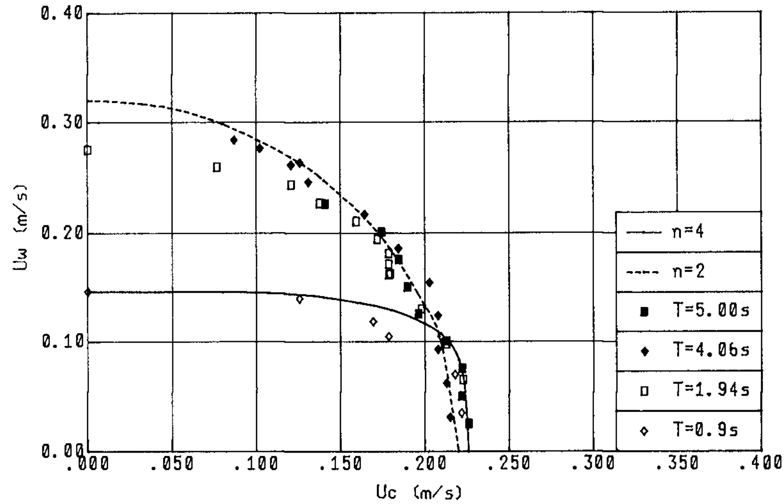

Fig. 5. Critical velocities for D = 0.70 mm

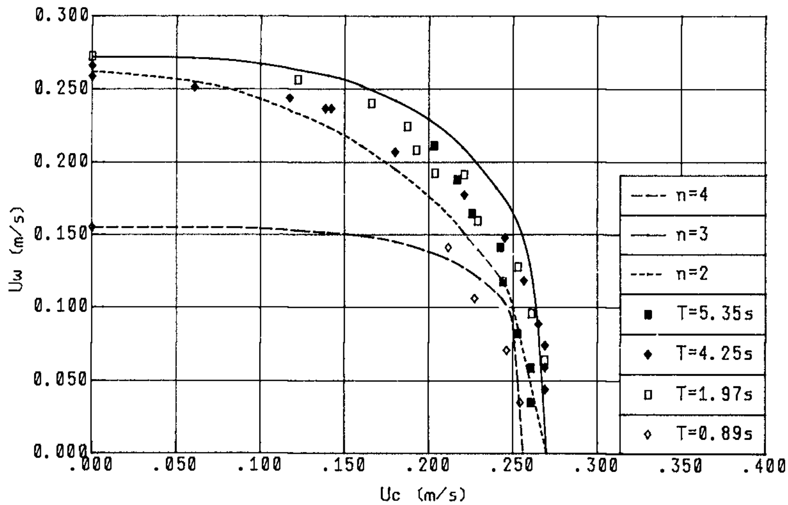

Fig. 6. Critical velocities for D = 1.07 mm

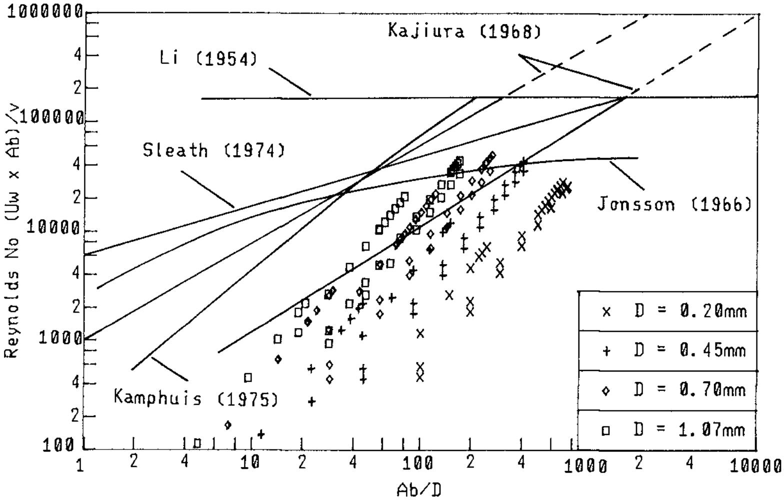

Fig. 7. The oscillatory flow parameters compared with various relationships for transition of an oscillatory flow boundary layer over a flat bed of sand

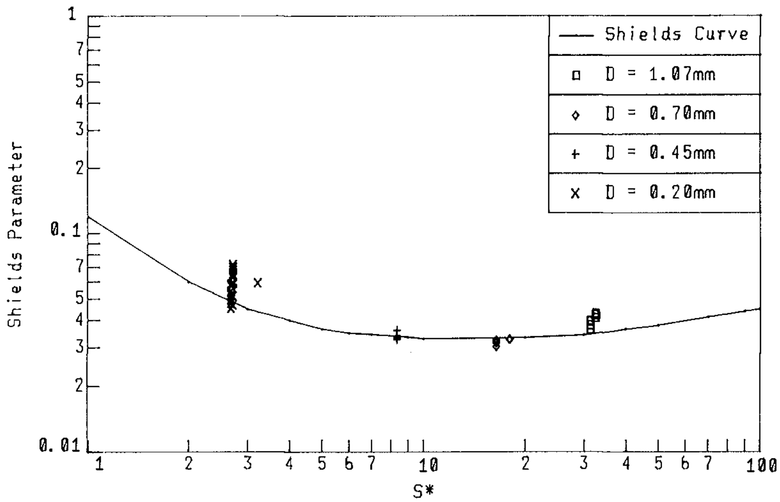

Fig. 8. Initial motion data assuming no non-linear interactions

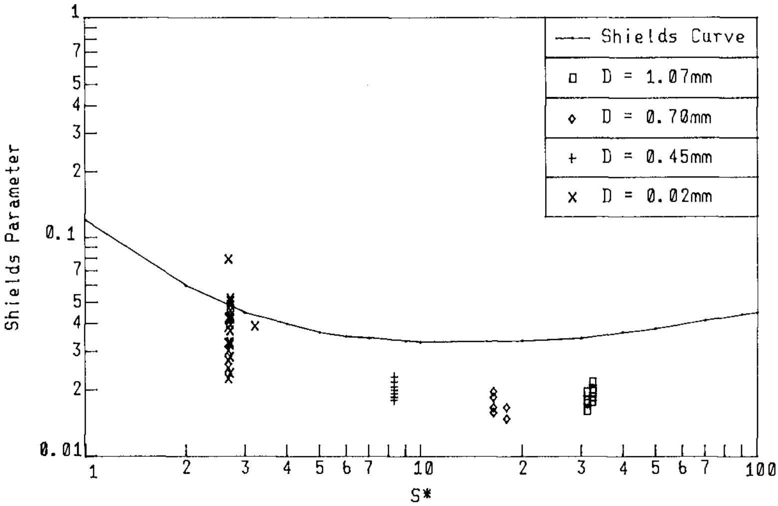

Fig. 9. Initial motion data as analysed by Bijker's (1967) model

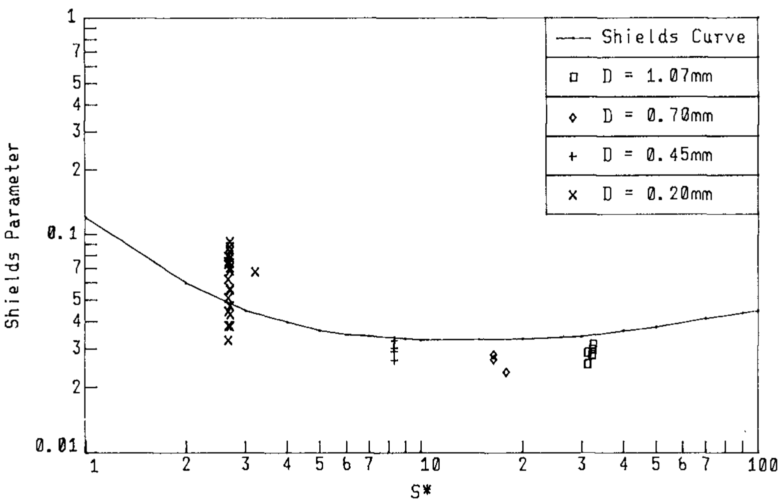

Fig. 10. Initial motion data as analysed by Grant and Madsen's (1979) model

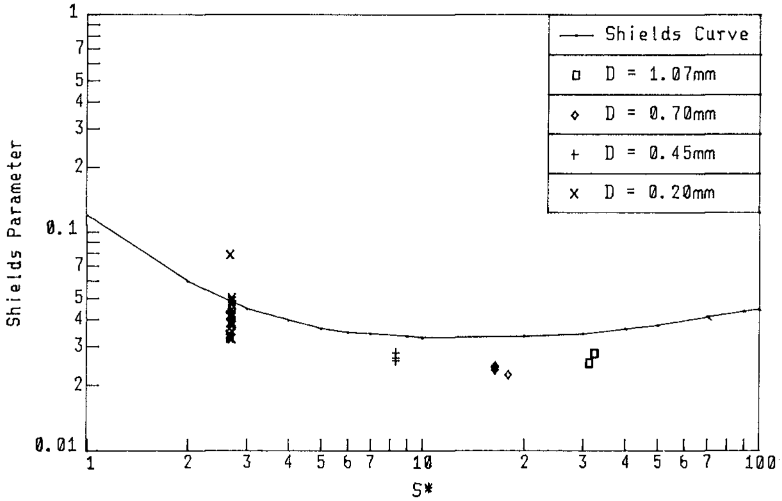

Fig. 11. Initial motion data as analysed by Christoffersen and Jonsson's (1985) model

This is because the additional force is in quadrature with the velocity of the tray and is consequently nearly 90° out of phase with the resultant force on the grains. Also, when the steady current is large this additional force is at a significant angle to the direction of the resultant force which further reduces the overall error. In order to test this conclusion, the comparisons shown in Figs. 8 - 11 were carried out both with a correction for the additional force and also without one. There appeared to be no significant difference between the two sets of results. Consequently, only the uncorrected results are shown in Figs. 8 - 11.

Comparison of Different Models for Combined Shear Stress

It is probably reasonable to assume as a first approximation that the initial motion condition corresponds to a critical value of the bed shear stress. To conform with usual practice we use the term shear stress to mean the horizontal force on the grains per unit area of bed. Lift forces are not entirely negligible but will be ignored for the moment. The problem is how to evaluate the shear stress in a combined flow.

The simplest assumption which we can make is that the resultant shear stress is the vector sum of the shear stress τ_w for the oscillatory flow assuming no effect of the steady current, and that for the steady flow τ_c, assuming no effect of the oscillation, i.e.

$$\tau = (\tau_w{}^2 + \tau_c{}^2)^{\frac{1}{2}} \tag{3}$$

We note, in passing, that shear stress proportional to velocity squared would lead to $n = 4$ in Eq. (2) whereas $n = 2$ would correspond to shear stress directly proportional to velocity.

Eq. (3) relies on the assumption that there is no non-linear interaction between the steady and oscillatory components of the flow. This assumption is more likely to be correct if the flow is laminar rather than turbulent. Fig. 7 shows how the critical values of $u_w A_b/\nu$ and A_b/D for the present experiments compare with the criteria for transition to turbulence proposed for oscillatory flow by various investigators. The fact that so many of the points lie below or only slightly above Kajiura's (1968) curve for the initiation of turbulence is an indication that turbulence generation by the oscillatory flow would have been negligible in most of the tests. On the other hand, the steady flow Reynolds numbers are well above the limit for transition in almost all cases. We conclude that the flow would have been turbulent in almost all of the tests but not strongly so in the immediate vicinity of the bed.

In view of this conclusion about turbulence, and consequently of the improbability of there being no non-linear interaction between the steady and oscillatory components of the flow, it is rather surprising to find in Fig. 8 that the experimental values of Shields parameter calculated with the aid of Eq. (3) all lie close to Shields curve. In Eq. (3), τ_w was calculated from Kamphuis (1975) curves for friction factor and τ_c from the assumption of a logarithmic velocity profile up to the reference point just upstream of the oscillatory tray.

The other methods used to calculate the resultant shear stress are those proposed by Bijker (1967), Grant and Madsen (1979) and Christoffersen and Jonsson (1985). The corresponding Shields diagram plots of the experimental results are shown in Figs. 9, 10, 11. We see that the closest agreement between the experimental results and Shields curve is provided by the methods of Grant and Madsen and of Christoffersen and Jonsson with, perhaps, slightly less scatter using the latter method. Surprisingly, the scatter with all three of these methods is greater than that obtained from Eq. (3).

Conclusions

The results of this experimental study suggest that Shields curve may be used to predict initial motion of sediment in combined steady and oscillatory flow. A simple model, in which the resultant shear stress is calculated on the assumption that there is no non-linear interaction between the steady and oscillatory flows, gave the closest agreement between Shields curve and the measurements but good agreement was also shown by the models of Grant and Madsen (1979) and Christoffersen and Jonsson (1985).

Appendix

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