CHAPTER 71

STORM STATISTICS IN THE NORTH SEA

By
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1.0 INTRODUCTION

Effective planning of offshore activities requires statistical information detailing storm occurrences and durations (defined as exceedences of an Hs threshold). This information is often referred to as persistence.

The frequency of storm events and the probability associated with a number of successive sea states being above (or below) a given threshold level is often required by engineers to estimate the potential work period and down-time as well as to incorporate the lead and lag times needed to stop and restart interrupted operations due to severe weather conditions.

However, until recently, a lack of sufficiently long data bases has precluded any meaningful investigation of this nature. Unrepresentative or misleading results can follow from data records that were measured over only a limited number of years. Furthermore, significant distortions may be introduced if the data record is not continuous as data gaps interrupt the persistence pattern.

As more recorded data have become available, concerted efforts have been made on this topic; most notably by Houmb and Vik who developed a probabilistic model describing the statistics of storm (and calm) durations and frequencies at varying levels of sea state intensity.

The present study investigates the adequacy of the semi-empirical procedure proposed by Houmb and Vik and also discusses the development of two new modelling techniques. The performance of these models are examined against wave data measured at the BP Forties field in the North Sea. A number of new statistical descriptors relating to the profile and intensity of storms have also been developed.

It is recognised that wave period (Tz) and directional information must ultimately be incorporated in a storm climate model. However, this is beyond the scope of the present discussion.

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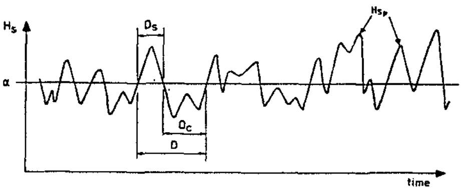
1.1 DEFINITIONS

In the work reported here, the definition of storm events as proposed by Houmb & Vik (1976) is adopted:

A storm is defined as a period of time during which the significant wave height, Hs, exceeds a threshold α (see Figure 1). The onset of a storm condition at α is marked by an upcrossing of the level Hs = α. Measures of storm (D_S), calm (D_C) and storm inter-arrival (D) durations are also indicated.



D_S — STORM DURATION
D_C — CALM DURATION
D — DURATION OF STORM INTER-ARRIVAL
Hs_p — PEAK Hs IN STORM

Figure 1: Definition of storm occurrences and durations.

1.2 DATA

The data used in this study have been measured at the BP Forties Field located in the North Sea.

The original data bank contained measurements extending over the period from June 1974 to December 1981. Measurements of significant wave height based on 20 minute recordings of sea surface elevation were made using Wave Staff and Wave Rider Buoy instruments. Unfortunately, the data is not continuous. In some cases measurements are missing over periods of up to several months. Owing to the importance of a continuous data set for the assessment of storm statistics, the data was subjected to a rigorous treatment and various steps were followed to compensate for data gaps (see Salih, 1987). As a result, it was possible to extract 5 years of almost continuous data covering the period from January 1977 to December 1981.

1.3 RESULTS AND DISCUSSION

1.3.1 Houmb and Vik Model

Initially attention is focused on the semi-empirical procedure proposed by Houmb and Vik (1977) and later developed by Pastene-Beytia (1976) and Dale (1979). The analysis of threshold crossings yields that the storm frequency at a given threshold is expressed as:

$$f_s(\alpha) = \frac{C\ (\alpha - A)^{C-1}}{B^C\ \sqrt{2\pi}}\ \sigma_{\dot{H}s|\alpha}\ \exp[-(\frac{\alpha - A}{B})^C] \qquad (1.1)$$

Where $\sigma_{\dot{H}s|\alpha}$ is the standard deviation of the first time derivative of the Hs time trace at the sea state level α, and A, B and C are the parameters of the Weibull distribution approximating p(Hs).

The expression is derived utilising the main assumptions that:

a) The marginal distribution of Hs can be approximated by a 3-parameter Weibull pdf. The validity of this has been demonstrated by many studies in the field of Long-Term wave statistics (see Burrows et al,1986). The analysis of the Forties data supports the Weibull distribution as an accurate description the probability 'mass' and the extreme tail of the Hs distribution.

Nevertheless, the Weibull distribution is often noted for its poor representation of the lower values of Hs. Thus the use of alternative distributions such as the Log-Normal, as proposed by Ochi (1978), and the NMI modified Log-Normal, proposed by the National Maritime Institute (and now British Maritime Technology) (see Fang et al, 1982) has also been assessed against observations. Statistical goodness-of-fit tests, however, has established the Weibull distribution as a superior descriptor of the observed p(Hs) (see Salih, 1987).

b) The second important assumption in the procedure is that the distribution of $\mathring{H}s|Hs$ can be represented by a mean-zero Gaussian distribution with a variance, $\sigma^2_{\mathring{H}s|Hs}$ that linearly varies with level α.

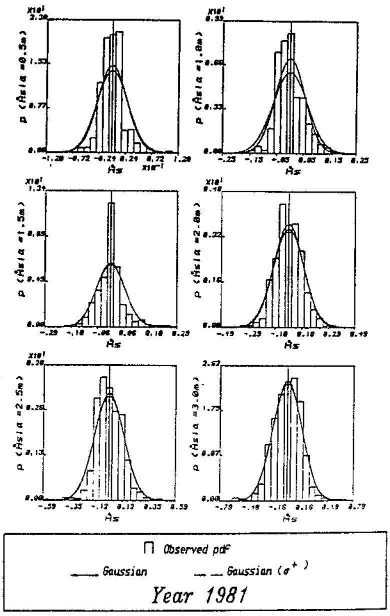


Figure 2:
Probability density functions of Hs with Gaussian pdf superimposed.

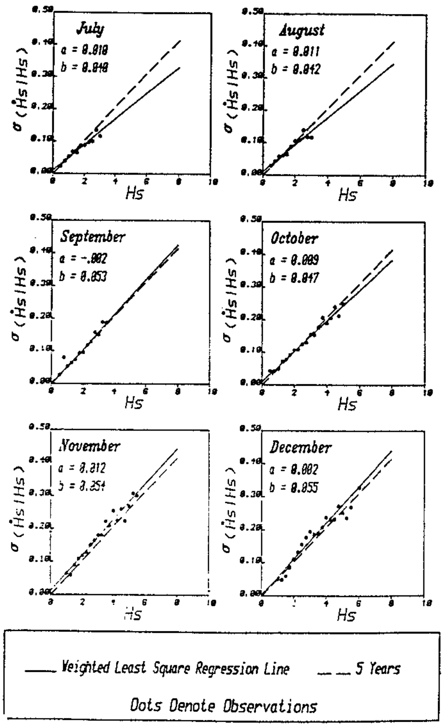


Figure 3:
Variation of standard deviation of $\mathring{H}s|Hs$ with Hs thresholds.

Figure 2 and Figure 3 point to the acceptability of both these assumptions. Houmb & Vik noted no seasonal variation in the $\sigma_{\dot{H}s|\alpha}$ versus α relationship. An alternative procedure for storm modelling proposed by Graham (1983) implies that there is no seasonal variation in the relationship between $\sigma_{\dot{H}s|Q(Hs)}$ and $Q(Hs)=1-P(Hs)$.

The results of this study, however, indicate the existence of a systematic seasonal variation in the relationship in question. It can be noted from Figure 3 that the summer and winter months' linear relationships fall on either sides of the average 'yearly' $\sigma_{\dot{H}s|\alpha}$ versus α relationship.

This systematic variation has been taken as a justification to parameterise the linear relationship (in the form of $\sigma_{\dot{H}s|\alpha}=\kappa\ \alpha$) in terms of Hs statistics. As the main input to the storm model are the Weibull fit parameters, the slope of the straight line, κ, has been regressed against A, B and C and the following least square relationship has been found to provide reasonably accurate estimates:

$$\kappa = 0.0476\ A^{0.201}\ B^{0.116}\ C^{0.185} \qquad \text{...............................(1.2)}$$

The expression of Equation 1.2 has been used to predict frequencies of storm events at varying levels of sea states and have been found to compare favourably with observations as can be seen from Figure 4. The different curves arise from the use of the Weibull and NMI modified Log-Normal distributions for p(Hs) and the average 'yearly' estimates of $\sigma_{\dot{H}s|Hs}$.

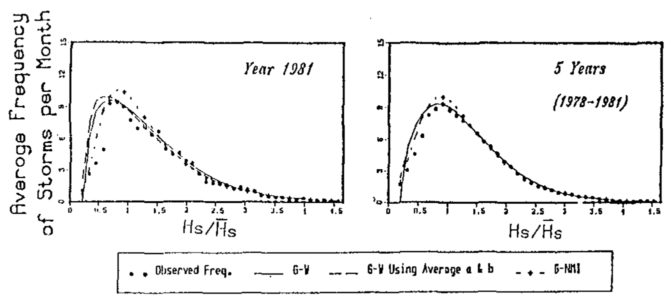


Figure 4: Average frequency of storm occurrences with prediction from Houmb & Vik model.

The average duration of storms, $\bar{D}_s(\alpha)$, can be obtained from Equation 1.1 as

$$\bar{D}_s(\alpha) = \frac{\text{Total time } (Hs > \alpha)}{\text{Total number of storms}} = \frac{1-P(Hs)}{f_s(\alpha)}$$

$$\bar{D}_s(\alpha) = \frac{B^C\ \sqrt{2\pi}}{C(\alpha - A)^{C-1}\ \sigma_{\dot{H}s|\alpha}} \qquad \text{...................................(1.3)}$$

The average durations of calms, defined as non-exceedence of the α level, and the average inter-arrival of storms (and calms) can be expressed as:

$$\bar{D}_c(\alpha) = \frac{P(\alpha)}{f_c(\alpha)} = \frac{1.0}{f_c(\alpha)} - \bar{D}_s(\alpha) \quad \ldots\ldots (1.4)$$

Hence the average duration of storm inter-arrivals at level α becomes:

$$\bar{D}_i(\alpha) = \bar{D}_s(\alpha) + \bar{D}_c(\alpha) = \frac{1.0}{f(\alpha)} \quad \ldots\ldots (1.5)$$

Average durations evaluated from Equation 1.3 to Equation 1.5 are compared with observations in Figure 5; again the modelling procedure closely approximates the observed behaviour. The noted discrepancy between long durations, in part at least, arises from sampling problems and also in how to deal with long sequences which extend beyond the seasonal date windows (ie storms in February, say, terminating in March or starting in January).

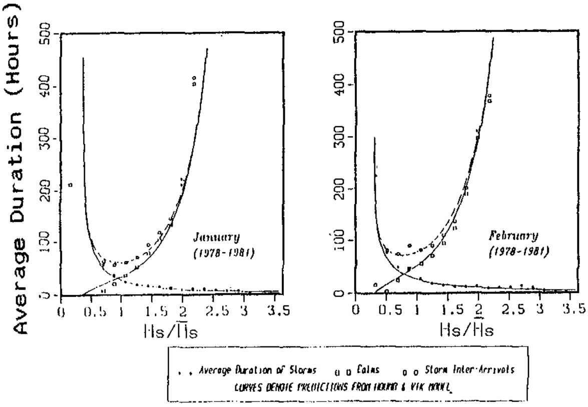


Figure 5: Average Durations of storms, calms and storm inter-arrivals with predictions from Houmb & Vik model.

1.3.1.1 Distribution of Durations

The use of a basic Poisson model for the description of storm events has been argued by Houmb & Vik. The suitability of this model requires that: (a) the arrival rate of storms is stationary, (b) the probability of storm occurrences in a short interval of time is proportional to the length of the interval and (c) the probability of multiple storm events in a short period of time is insignificant compared to a single event.

If the storm events are described by a Poisson model, then distribution of storm durations is exponential. Whilst some of the theoretical justification for the adoption of exponential model for storm durations follows from a Poisson assumption for storm events, as indicated by Houmb and Vik, the underlying assumptions are not fully satisfied in practice. Houmb & Vik, however, found little empirical evidence to support this and instead proposed the use of the more general 2-parameter Weibull distribution. Dale (1979) reports, nevertheless, that the distribution of storm durations at Seven Stones' light vessel off the South West coast of England become exponentially distributed at higher threshold levels of sea state.

Figure 6 points to the general acceptability of the Weibull model and also confirms Dale's observation that the distributions, towards the higher thresholds, become exponential. This is perhaps an indication that the assumptions inherent in the Poisson model become more valid at these thresholds.

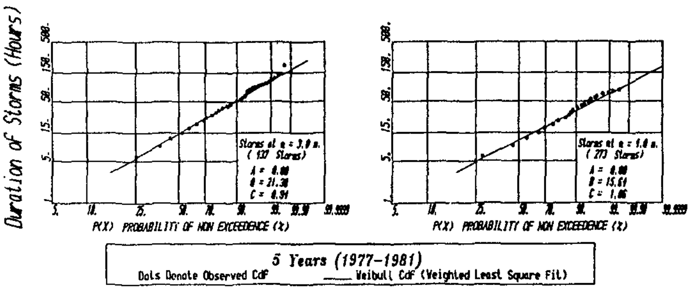


Figure 6: <u>Distribution of storm durations with 2 parameter Weibull model superimposed</u>.

1.3.2 Gaussian Transformation Model

It was anticipated that the technique of transforming a general set of correlated random variables into the Gaussian form would enable the utilisation of the wealth of theory available for Gaussian processes.

The time series of Hs, Hs(t), modelled by a Weibull distribution can be transformed into a mean-zero Gaussian process, X(t), of unit variance by applying the approximate formulae for Gaussian transformation given by Abramowitz & Stegun (1964),

$$X(t) = f - \frac{c_0 + c_1 f + c_2 f^2}{1 + d_1 f + d_2 f^2 + d_3 f^3} + \text{error} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.6)$$

where |error| < 0.00045, and

$$f = \sqrt{\ln\left(\frac{1}{Q(Hs(t))}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.7)$$

Q(Hs(t)) is the exceedence probability of Hs(t) ($\int_{Hs(t)}^{\infty} p(Hs)\, dHs$).

The basis of this procedure is illustrated diagrammatically in Figure 7. The original process, Hs(t), and its Gaussian transform peak jointly and crossings of the level $Hs=\alpha$ corresponds to crossings of $X=\beta$ in the transformed domain.

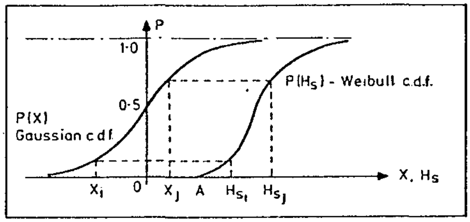


Figure 7: Gaussian transformation of random variables using distribution functions.

The Gaussian transform of Hs(t) is characterised by a wide band power spectrum. Estimates of the spectral band width parameter, ε, derived from duration statistics, are found to be approximately 0.950 for summer months and 0.965 for winter months.

Various storm characteristics can be expressed in terms of the standard deviation of X, which is unity by virtue of Equation 1.6, and that of the first time derivative of X, $\dot{X}$, using results of the classical mathematics associated with stationary Gaussian processes. The frequency of storm occurrences is equivalent to the frequency of up-crossings of the level β,

$$N^{+}(\beta) = \frac{1}{2\pi} \frac{\sigma_{\dot{x}}}{\sigma_{x}} \exp[-0.5 (\frac{\beta}{\sigma_{x}})^{2}] \quad \ldots\ldots (1.8)$$

The frequency of up-crossing by Hs(t) of the sea state threshold level of α can be obtained as

$$N^{+}(\beta) = N^{+}(\alpha) \quad \ldots\ldots (1.9)$$

The average durations can be deduced by the same arguments followed to obtain Equation 1.3 through Equation 1.5

In order to justify the application of the theory of stationary Gaussian processes, the problem of non-stationarity has been avoided herein by considering monthly periods. This implies that the statistics for monthly periods are assumed stationary from year to year.

The predictions of average storm, calms and inter-arrival duration arising from the use of this technique are found to compare favourably with the observed behaviour as can be noted from Figure 8. The predictions are also presented in Table 1 alongside estimates obtained from the Houmb and Vik procedure. It is apparent the transformation technique provides, on the whole, the more accurate predictions.

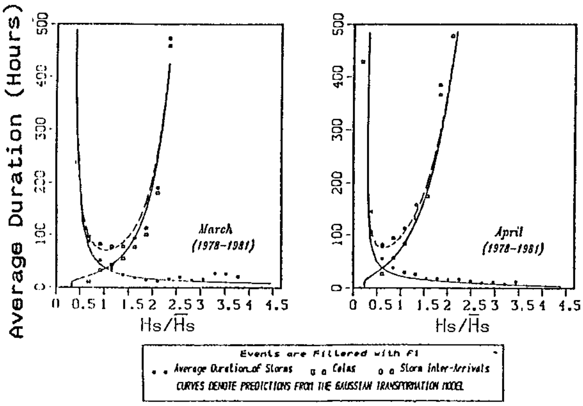


Figure 8: Average Durations of storms, calms and storm inter-arrivals with predictions from Gaussian transformation model.

α/H̄s	Observed	Houmb & Vik	Gaussian Transf.	Markov Weibull	Markov log-Norm.	
0.5	271.5	106.4	153.3	69.9	134.2	
1.0	30.8	24.7	29.2	21.9	22.7	January
1.5	17.5	12.4	16.9	14.2	13.0	
2.0	10.4	7.8	12.3	10.3	9.7	

α/H̄s	Observed	Houmb & Vik	Gaussian Transf.	Markov Weibull	Markov log-Norm.	
0.5	153.3	105.4	108.6	57.2	123.2	
1.0	41.9	30.1	36.0	26.0	28.3	October
1.5	23.9	14.8	21.4	16.4	17.1	
2.0	9.8	9.0	15.3	9.0	13.0	

Table 1. Average duration of storms and inter-arrivals with predictions from Houmb & Vik, Gaussian Transformation and Markov models

The main advantage of the transformation technique is that additional statistical descriptors of 'storm shape' can be established from theory. In particular, the distribution of peak amplitudes of a wide-banded Gaussian process can be used to predict the distribution of peak Hs attained within the duration of a storm. This aspect is to be discussed in a later section.

Furthermore, the technique enables more complex measures of storm statistics to be developed; the theory relating to Gaussian processes can be utilised to establish probability distribution of storm duration. Price and Bishop (1974) derive a distribution for exceedence duration by following a method originally proposed by Rice (1944,1945) However, the resulting expression is inherently complex and requires the evaluation of various higher order spectral moments. In the absence of a long stationary data base, such spectral moments cannot be reliably determined and in subsequence this aspect has not been pursued in the present study.

1.3.3 Markov Chain Model

The use of the first order Markov chain theory to model the time series of Hs requires that only successive sea states, Hs_i and Hs_{i+1}, are statistically correlated. In reality, however, sea states several time steps apart may exhibit appreciable statistical correlation. Thus, at first the validity of the Markov theory, defined to the first order, may be suspect.

Nevertheless, it has been established from the study of the Houmb & Vik procedure that $\sigma_{Hs|Hs}$ increases with the threshold level of Hs. Hence, lesser correlation between successive larger sea states can be anticipated since the linear correlation coefficient is inversely related to the variance. Therefore, the acceptability of the Markov model, at least at these higher levels of Hs, is likely to be enhanced.

The Markov model used in the context of storm modelling requires the identification of two states of the process Hs(t), namely that of exceedence and non exceedence of the level α. The transition probabilities (see Kimura, 1980 and Longuet-Higgins, 1984) associated with these two states can be represented as:

$$p_+ = \int_\alpha^\infty \int_\alpha^\infty p(Hs_{i+1}, Hs_i)\, dHs_i\, dHs_{i+1} \Big/ \int_0^\infty \int_\alpha^\infty p(Hs_i, Hs_{i+1})\, dHs_i\, dHs_{i+1}$$

$$p_- = \int_0^\alpha \int_0^\alpha p(Hs_{i+1}, Hs_i)\, dHs_i\, dHs_{i+1} \Big/ \int_0^\infty \int_0^\alpha p(Hs_i, Hs_{i+1})\, dHs_i\, dHs_{i+1} \quad \ldots\ldots(1.10)$$

Where $p(Hs_i, Hs_{i+1})$ represents the joint probability density function of successive sea states.

It can be shown that the probability of Js sea states consecutively exceeding the level α is given by:

$$p(Js) = p_+^{Js-1}\,(1-p_+) \quad \ldots\ldots(1.11)$$

and it follows that the mean number of sea states exceeding α is:

$$\overline{Js} = \sum_{1}^{\infty} Js \times p(Js) = \frac{1}{(1-p_+)} \quad \ldots\ldots(1.12)$$

Similarly, the distribution of storm inter-arrivals can be formulated as:

$$p(Ji) = (1 - p_+)(1 - p_-)(p_+^{Ji-1} - p_-^{Ji-1})/(p_+ - p_-) \quad \text{...........(1.13)}$$

and the mean number of sea states between storm inter-arrivals follows as:

$$\overline{Ji} = \sum_{2}^{\infty} Ji \times p(Ji) = \frac{1}{1-p_+} + \frac{1}{1-p_-} \quad \text{...........................(1.14)}$$

The only problem remains to choose a suitable probability distribution to model $p(Hs_i,Hs_{i+1})$: An obvious choice may be the bi-variate Weibull distribution originally developed by Rice (1944,1945) and Ulhebeck (1943) and applied more recently by Kimura (1981) to model the joint probability distribution of individual wave heights and periods, p(H,Tz). However, it has been found that the conditional statistics of the observed $p(Hs_i,Hs_{i+1})$ is characteristically different from that inherent in bi-variate Weibull function (see Salih, 1986).

The use of the bi-variate Log-Normal probability distribution, used by Ochi (1987) to model the long-term joint distribution of Hs and average zero crossing periods has also been investigated; more accurate representation of the conditional probabilities is obtained than with bi-variate Weibull model. It must be noted, however, that this choice does not affect the relative merits of either the Log-Normal or Weibull functions to describe the marginal distribution of Hs, p(Hs).

Using both of these bi-variate distributions, the average durations of storms are predicted from the Markov model and the results are presented in Table 1. Also included in this table are the corresponding estimates for the Houmb & Vik and the Gaussian transformation procedures to enable the direct comparison of the various models.

It is apparent that the Markov model consistently underestimates the average durations at the lower thresholds. This arises mainly from the fact that successive sea states are strongly correlated at these levels and thus the assumptions inherent in the Markov model are more severely violated. In line with this reasoning, the predictions become in closer agreement with observed averages towards the higher sea state levels. The accuracy of the Markov model can be of the same order, and occasionally marginally better, as that noted for the Houmb and Vik procedure.

This property renders the Markov model particularly useful for engineering applications where the synthesis of storm statistics at these higher thresholds can be of particular importance.

Furthermore, the Markov model enables the modelling of the distribution of storm durations without the need to invoke any extra assumptions. Figure 9 shows that the Markov predictions are in close agreement with the observed distributions for storms at higher thresholds.

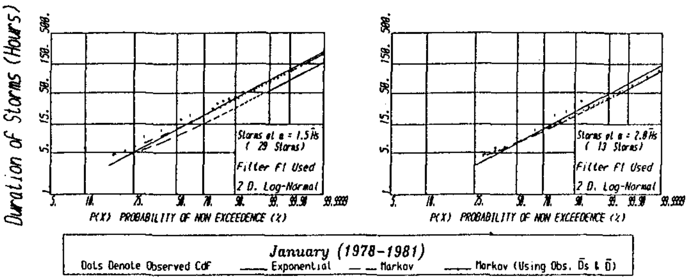


Figure 9: Distribution of storm durations with predictions from Markov model.

1.4 STORM SHAPE DESCRIPTORS

The storm models discussed do not provide information regarding profiles and energy contents of the storm events. A variety of statistics can be utilised for describing the storm intensity or severity at given threshold level of sea state. A complete description of the probability distribution of Hs within the duration of the storm would be an ultimate solution. However, this is expected to be rather complex and difficult to test empirically. An alternative approach, that is of lesser complexity, would be to specify certain statistics that characterise the storm profile and from which the severity of the storm can be inferred.

It is considered that useful insight may be obtained by investigating the distributions of the peak Hs attained in a storm, (Hs_p), the mean Hs within the storm events, ($\bar{H}s_s$), or the distribution of the root mean square Hs, ($\sqrt{\bar{H}s^2}_s$). The latter statistic may be taken as a measure of mean sea state energy per unit time.

Figure 10 plots the probability distribution of peak Hs attained within the duration of a storm normalised by the threshold level α.

The predictions arising from the Gaussian transformation model are super-imposed on Figure 10.a. These are based on the expression for distribution of amplitudes in a wide-banded Gaussian process. The results are found to follow the observed behaviour quite reasonably. However, the match is far from exact and the discrepancy arises from certain assumptions associated with the theoretical model (principally that the theory relates to the description of all 'local' maxima whilst only storm 'global' peaks are of significance to this application).

The 2-parameter Weibull distribution has been found to accurately model the upper half of the distribution of Hs_p as can be noted from Figure 10.b.

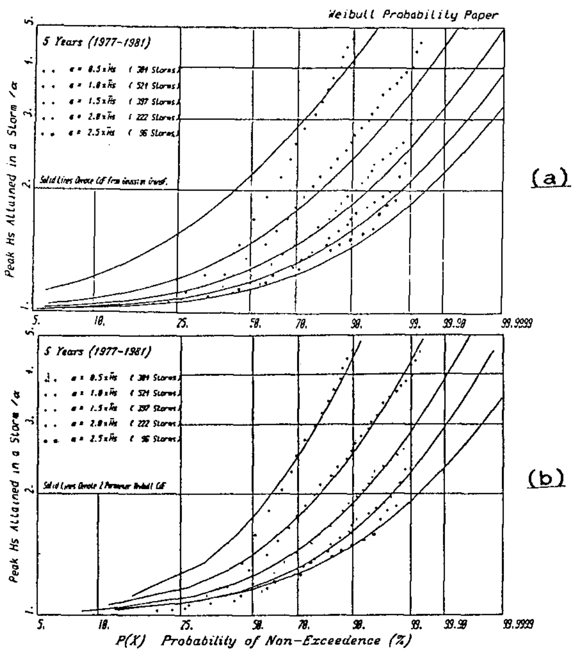


Figure 10: Distribution of peak Hs within a storm.
(a) with predictions from the Gaussian transformation model
(b) with predictions from 2 parameter Weibull model

Figure 11 shows that the average Hs within the duration of a storm can be modelled accurately by a 2-parameter Weibull distribution. The use of other empirical distributions has also been assessed and established that the shifted exponential and Gamma distributions prove generally acceptable for this purpose.

The distribution of root mean square Hs within the storms can be used to provide information relating to the energy content of the storms. Using expressions from the linear wave theory it follows that:

Energy per unit surface area is proportional to $E[Hs^2]$

The results presented in Figure 12 point to the suitability of the 2-parameter Weibull model.

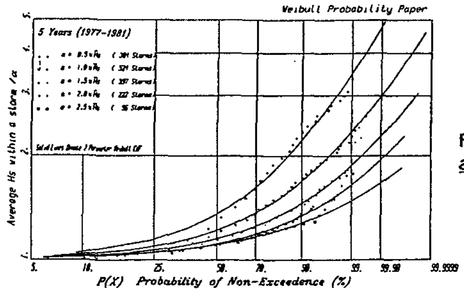


Figure 11: Distribution of average Hs within a storm.

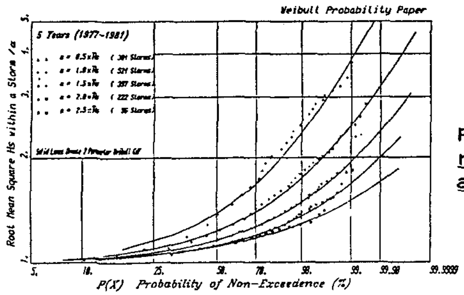


Figure 12: Distribution of root mean square Hs within a storm.

1.5 CONCLUSIONS

It is concluded, as judged from the Forties data, that the Gaussian transformation technique provides more accurate predictions of average durations of events than the other two models considered in this study. Furthermore, the potential of utilising the wealth of theory relating to Gaussian processes makes the technique a viable and indeed a superior alternative to the more traditional procedure proposed by Houmb and Vik.

The distribution of maxima of a wide band Gaussian process has been used to predict the distribution of storm peaks and the results of the present study point to the general validity of the resulting distribution particularly towards higher sea state thresholds.

However, further testing and calibration of the transformation model requires access to longer data bases that can be used, for example, to reliably extract the various, higher order, spectral parameters required for the definition of the distribution of durations.

The Markov model proves a useful tool in characterising the storm statistics at higher thresholds of sea states where both the average durations and the distribution of durations are reasonably well represented. It is important to note that the specification of the distribution of durations is an integral part of the Markov approach.

Similar studies are now required on data sets from differing locations and degrees of exposure in order to assess the site specific element of the storm models discussed herein.

1.6 ACKNOWLEDGMENTS

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REFERENCES

1. Abramowitz, M. and Stegun, I.A., - "Handbook of Mathematical Functions", Applied Maths. Series 55, National Bureau of Standard, 1964.

2. Burrows, R. and Salih B.A., - "Statistical Modelling of Long-Term Wave Climates", Proceedings of the 20th Coastal Engineering Conference, Vol I, pp. 42-56, 1986.

3. Dale, J., - "Non-Stationary Statistics of Storms", M.Eng. Thesis, Department of Civil Engineering, University of Liverpool, 1979.

4. Fang, Z.S. and Hogben, N., - "Analysis and Prediction of Long-Term Probability Distributions of Wave Heights and Periods", National Maritime Institute (NMI), R 146, 1982.

5. Graham, C., - "Parameterisation and Prediction of Waveheight and Wind Speed Persistence Statistics for Oil Industry and Planning Purposes", Coastal Engineering, Vol 6, pp. 303-329, 1982.

6. Houmb, O.G., - "On the Duration of Storms in the North Sea", 1st International Conference, P.O.A.C., Trondheim, August 1971.

7. Houmb, O.G. & Vik, I, - "On the Duration of Sea State", Division of Port and Ocean Engineering, Norwegian Institute of Technology, Trondheim, 1977.

8. Kimura, A., - "Joint Distribution of the Wave Heights and Periods of Random Sea Waves", Coastal Engineering in Japan, Vol 24, 1981.

9. Longuet-Higgins, M.S., - "Statistical Properties of Wave Groups in a Random Sea State", Philosophical Transactions of the Royal Society of London, Vol 312 A 1512, pp. 219-250, 1984.

10. Ochi, M.K., - "On Long-Term Statistics for Ocean and Coastal Waves", Proceedings of the 16th Coastal Engineering Conference, Vol I, pp. 59-75, 1978.

11. Pastene-Beytia, J.E., - "The Short and Long-Term Statistics of a Random Sea", M.Eng. Thesis, Department of Civil Enginering, University of Liverpool - 1976.

12. Price, W.G. and Bishop, R.E.D., - "Probablistic Theory of Ship Dynamics", Chapman and Hall - 1974.

13. Rice, S.O., - "The Mathematical Analysis of Random Noise", Bell System Technology Journal, Vol 23 pp. 282-332 and 24 pp. 46-156, 1944 and 1945.

14. Salih, B.A., - "Long-Term Wave Statistics" - Interal Report, Department of Civil Engineering, University of Liverpool, 1986.

15. Salih, B.A., - "Persistence Characteristics of Sea State", Internal Report, Department of Civil Engineering, University of Liverpool, 1987.

16. Uhlenbeck, G.E., - "Theory of Random Processes", M.I.T. Radiation Laboratory Report, No. 454, 1943.