CHAPTER 69

A Steady-State Wave Model for Coastal Applications

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A steady-state spectral model is presented. This model produces a solution equivalent to a full time-stepping spectral model, but at much reduced computational times. Comparisons shown here demonstrate that the spectral model provides a good representation of shallow-water wave propagation phenomena and that wind effects can significantly influence near-coast wave conditions.

Introduction

Over the last twenty years a wide range of numerical models have been developed to assist coastal engineers, planners, and scientists in the estimation of near coastal wave conditions. Early models were based on monochromatic wave theory and ray tracing methods. Subsequently, models have developed along two lines, monochromatic and spectral. The major distinction between the two approaches pertains to the degree of randomness assumed in the physical system. If waves are primarily deterministic, variations in the phase function must be considered and nonlinear behavior of the type discussed by Yuen and Lake (1975) or Berkhoff (1976) can result. On the other hand, if waves are considered to be a random superposition of essentially linear components, nonlinear behavior must be treated as independent of phase. Nonlinearities of this kind relate to wave-wave interactions of the type discussed by Hasselmann (1962) and result in energy fluxes among the linear components which can become very important in shallow water (Resio, 1987, 1988). Also, as will be discussed later, diffusive energy fluxes due to spatial variations in energy can also be treated independent of consideration of the phase function.

In deep water, spectral models have gained a clear advantage in terms of being able to represent important aspects of wave generation, propagation and decay. In near coastal environments, some persist in using linear, monochromatic models (refraction diagrams, shoaling coefficients, monochromatic diagrams, etc.), however most

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serious applications have now moved on to either spectral models or nonlinear models of combined refraction-diffraction of the type described by Berkhoff (1976) and Ebersole et al. (1986). Unfortunately, both of these classes of models require considerable computer resources for their execution. The purpose of this paper is to introduce a variation of spectral modeling which is much more efficient in terms of computer time and storage requirements than previous methods. The accuracy of this model and its potential application in various situations is then dicussed.

Theoretical Considerations

Overview

Although all of the equations used below can be generalized to include the effects of currents, for simplicity and in order to express these equations in terms of energy densities rather than action densities, it is assumed here that currents are negligible. In this case in the absence of energy sources and sinks, we have

$$F(\underline{k}) = \text{constant} \tag{1}$$

along a wave range where $\underline{k}$ is the wave number vector and $F(\underline{k})$ is the energy density at that point. Or, if energy sources and/or sinks are significant we have, along a ray $\underline{s}$,

$$\frac{\partial F(k)}{\partial s} = \Sigma S_i'(\underline{k}) \frac{\partial s}{\partial t} \tag{2}$$

where S' represents a single source/sink term and t is time. In conventional spectral models, a spatial grid such as seen in Figure 1 is laid out. At each point in such a grid, an equation equivalent to (2) is solved

$$\frac{\partial E(f,\theta^{x,y,t})}{\partial t} = \frac{\partial c c_g E(f,\theta^{x,y,t})}{\partial x} c_{g_x} + \frac{2 c c_g E(f,\theta^{x,y,t})}{\partial y} c_{g_y} + \Sigma S_i(f,\theta,x,y,t) \tag{3}$$

where $E(f,\theta^{x,y,t})$ is the conventional spectral energy density in frequency-direction space, c is the phase velocity and c_g the group velocity, respectively, of waves with frequency f, x and y are orthogonal spatial axis, subscripts "x" and "y" denote component values in those directions, and $S_i(f,\theta)$ represents the effects of the i^{th} source term. The five-dimensional nature of (3) readily in infers one of the major problems associated with its computer solution. The dependence of energy densities and sources on x,y,t have been explicitly written in equation (3) in order to point out this problem. In subsequent usage, dependencies on space and time will be assumed but not written; thus, $E(f,\theta,x,y,t)$ will now only be written as $E(f,\theta)$. At each point in the grid, equation (3) must be solved for each frequency-direction component over each time stop. A "typical" number of frequencies and directions is about 300. If we use as an example a grid covering 10 miles by 10 miles with a spatial resolution of 1/4 mile, equation (3) must be solved about 80 million times in a simulation in order to achieve steady-state conditions (i.e., in order to perform the appropriate wave transformation).

Some groups have attempted to use a different approach to solve the problem of predicting near coastal wave spectra (Abernethy, et al., 1977). As seen in Figure 1, wave rays can be "backtracked" out from a point of interest. It is then possible to calculate equation (3) at each point along each of the rays shown there, provided that the form of the source terms in (3) are such that the spectral components can be considered as uncoupled. Unfortunately, the wind source term, wave-wave interactions and wave breaking are not treated adequately in this context. In all existing wave models, the net energy input into frequency components above the spectral peak becomes quite coupled by the normalization methods used to achieve a particular equilibrium range behavior. Also, the net source to the forward face due to wave-wave interactions and the redistribution of energy on the rear face depends on the local specrtral shape. Similarly wave breaking depends on the total energy in the spectrum. Since information of this kind is not calculated, the source term treatment in such a model must necessarily be somewhat arbitrary and imprecise. Thus, if we seek an accurate solution to (3), we are left with the need to perform our calculations over an entire grid, rather than along rays passing irregularly through it. In this way all complete spectral information is available at each grid point and all source terms can be reasonably estimated.

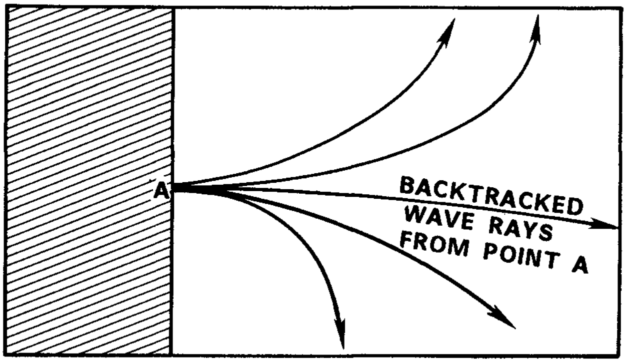

Figure 1. Example of wave rays converging on Point A at the coast.

Let us assume that we are not interested in energy propagating away from a coast. Even if such waves exist they will not affect the incoming waves very much since the peak frequencies in incoming spectra will almost always be very different from the outgoing spectra. In this context, all of the source terms and propagation effects can be seen to reside essentially within a half-plane. This is the key to recognizing a much more efficient solution method for steady state wave transformations.

The coastal wave transformation process can be regarded as one involving internal mechanisms (primarily wave-wave interactions and wave breaking) and external mechanism (primarily wind forcing and possibly some bottom interactions). For a given spectrum at the seaward boundary, if winds are relatively constant over the time span required for the waves to propagate across the grid (typically only 15 minutes to 1 hour for near-coast simulations), a simplified solution to (3) may be employed with essentially no loss of accuracy. In this context let us proceed to a solution of the transformations of the wave transformations from the first column to the second. An adequate solution for this problem can, by induction, be extended to the remainder of the grid.

Propagation

Since (3) is an inhomogeneous partial differential equation, solution techniques usually solve the homogeneous part first (essentially wave propagation) and then solves the inhomogeneous part (the effects of source terms). In order to avoid time consuming calculations, some simplifications are in order. If a ray passing from column 1 to column 2 does not pass through a caustic, Snell's law may be used to specify the angle and height variation from one depth to another

(4) $$\theta_2 = \sin^{-1}(\theta, c_2/c_1)$$

where subscripts "1" and "2" refer to the column locations. For waves which pass through a caustic, an approximate solution is used.

(5) $$\theta_2 = 2\tan^{-1}\left(\tan(\theta_{\frac{1}{2}})\ \exp(-\frac{\partial c}{\partial h}\frac{\partial h}{\partial s}\Delta s/2c_g)\right)$$

where h is the local water depth and Δs is the distance along s from a point in column 1 to a point in column 2. Regardless of whether or not the wave ray passes through a caustic, it is assumed that $F(\underline{k})$ remains constant as it should for a conservative system. In terms of the frequency-direction form for spectral energy densities used in equation (3) this means that

(6) $$cc_gE(f,\theta) = \text{constant}$$

which provides sufficient information for closure of the propagation problem. A modeling system based on this approach can be used to construct ray segments "piecewise" from column to column.

Divergence-Convergence Effects

Equation (6) is the typical form for wave propagation found in most spectral models today. Its validity as seen here follows from the ray-based concept of wave propagation. However, if we examine the situation in Figure 2 as an example of a potential problem, we see waves propagating along the trough of a bathymetric feature. The wave ray traveling straight down the trough will not move to either side since the slope orthogonal to the ray path is zero. Whereas this may actually be true for the infinitesimal-width ray exactly in the center, it is not true for any rays displayed any distance whatsoever to either side. If, for example, all the energy in a spectrum were concentrated in that energy band propagating down the trough, the usual solution method would indicate that there would not be no change in wave height down the trough. It can be shown that, in general, the ray-based solution method does not represent divergence-convergence of wave rays over a grid cell. Thus, for a more accurate solution as may be warranted in sensitive applications, a divergence-convergence term is explicitly added to the solution method here. The form of this term is

$$\frac{\partial E(f,\theta)}{\partial t} = \nabla \cdot \underline{\Gamma}_n \tag{7}$$

where Γ is the net flux of energy across a row boundary of the grid.

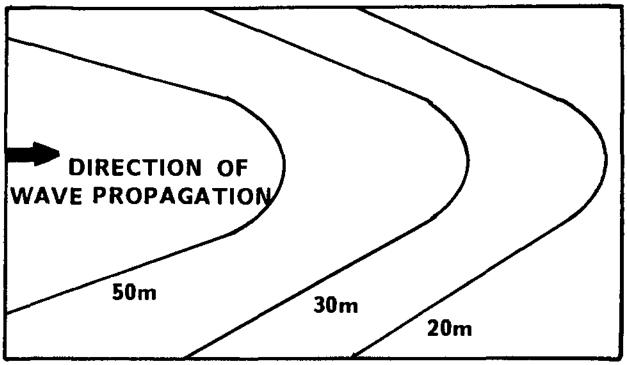

Figure 2. Example bathymetry for waves propagating over a crest.

Consideration of Diffraction in a Spectral Model

A detailed treatment of diffraction within the steady-state spectral model introduced here is considerably beyond the scope of this paper; however, a brief description of the general methodology will be given. First, one must recognize that since spectra are assumed to have random phases, phase relationships for a diffracted wave are only fixed between that wave and the individual component which created it. Thus, one cannot convert directly from a solution to the Fresnel Integrals and presume that the amplitude functions represent freely propagating waves. Instead, it is necessary to treat any phase-dependent amplitude interaction in the context of a bound wave, bound to the spectral component which created it. However, it appears, as will be shown later, that phase-dependent solution techniques may only be required in situations with very steep energy cutoffs and very narrow spectral widths.

If we ignore phase effects, diffraction resembles a diffusion process. Any variation in wave amplitude along a wave creates results in a spatial redistribution and redirection of the wave energy. The following is used to represent the effects of diffraction in areas not affected by steep surface-piercing structures.

$$E_j(f,\theta_i)^{n+1} = \sum_{\ell=-1}^{+1} \sum_{\kappa=-2}^{+2} \varepsilon_{\kappa\ell} E_{j+\kappa}(f,\theta_{i+\ell}) \tag{8}$$

where the subscript j refers to a grid row, i references an angle band, and $\varepsilon_{\kappa\ell}$ is a matrix of normalized multipliers.

Source/Sink Integration

The source terms employed in this model are similar to those discussed in Resio (1987, 1988) There are four source terms in the model:

1. wind inputs;
2. energy fluxes due to wave-wave interactions;
3. wave breaking; and
4. bottom interaction effects.

For all tests of source terms, bottom friction and other bottom interaction effects will be assumed to be negligible compared to nonlinear energy fluxes.

Tests of Steady-State Spectral Model

Tests of Propagation Only

A simple case to demonstrate that the spectral representation of refraction and shoaling used here is consistent with theoretical monochromatic results can be found in the propagation of waves with different periods and propagation directions across a slope with parallel depth contours. For this case, the monochromatic waves were approximated by a spectrum with energy in an angular band of one degree centered on the specified approach angle with a frequency band of 0.01 hertz. As seen in Table 1, the computed results show very good agreement with theoretical predictions.

Table 1

Wave Height Comparison

Angles	Depths (ft.)							
	50		20		10		5	
(o)	a	b	a	b	a	b	a	b
0	.96	.96	.92	.92	.98	.99	1.11	1.11
10	.96 9.70	.96 9.70	.92 8.00	.92 7.80	.98 5.70	.98 5.90	1.11 4.60	1.10 4.30
20	.96 19.30	.95 19.40	.91 15.50	.91 15.40	.97 11.60	.96 11.70	1.09 8.30	1.08 8.60
30	.95 29.20	.95 29.00	.89 22.90	.89 22.90	.94 17.20	.94 17.20	1.05 12.60	1.05 12.50
40	.95 38.40	.95 38.50	.86 29.80	.86 29.90	.90 22.30	.90 22.40	.99 16.00	.99 16.20
50	.94 48.10	.94 47.90	.82 36.70	.82 36.50	.84 26.90	.84 27.00	.92 19.50	.92 19.40
60	.92 57.30	.92 57.10	.76 42.40	.76 42.30	.75 30.90	.75 30.80	.82 21.80	.82 22.10
70	.87 65.70	.87 65.60	.65 47.00	.65 46.80	.63 33.80	.63 33.80	.68 24.10	.68 24.00
80	.73 72.80	.73 72.60	.48 49.80	.48 49.90	.45 35.50	.46 35.60	.49 25.20	.49 25.30

NOTE: a = theoretical result
b = model result

To test this model in a situation in which linear models are expected to be inaccurate, the model was run for the elliptical shoal, monochromatic test case described in Vincent and Briggs (1988), which is patterned after that of Berkhoff et al. (1982). The bathymetry for this case is shown in Figure 3. Figure 4 shows a comparison of the waves predicted by this model to those measured in the laboratory. Also shown in that figure are the modeled results of a finite difference solution to the "mild slope" equation (Ebersole et al., 1986). As seen there, the spectral model can reproduce the monochromatic, even for this nonlinear case, results quite well.

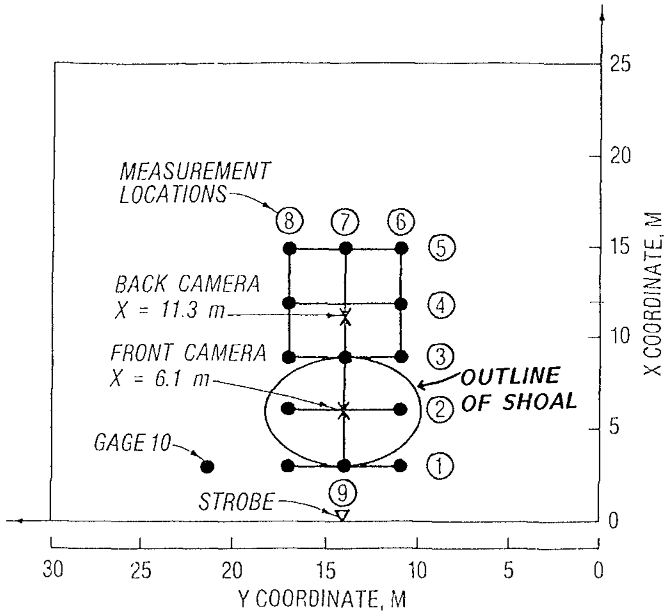

Figure 3. Laboratory set-up for waves over an elliptical shoal (from: Vincent and Briggs, 1988).

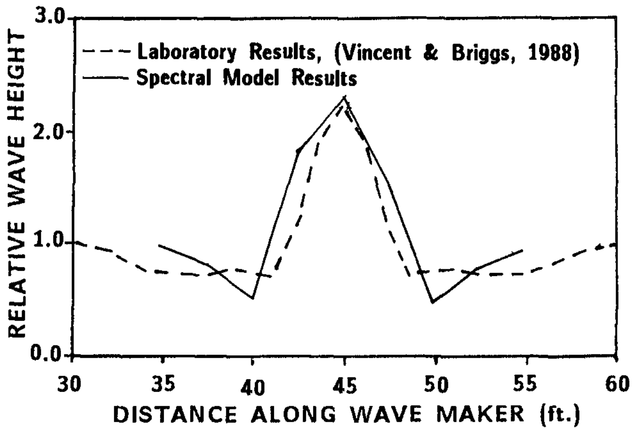

Figure 4. Comparisons of results from spectral model to laboratory measurements for monochromatic case.

Our next test drops the artificial constraint of monochromatic waves and examines model behavior for more realistic cases with reasonable wave spectra. Vincent and Briggs (1988) set up a series of laboratory tests on bathymetry shown in Figure 3 in the directional wave basin located at the U.S. Army Engineer Coastal Engineering Research Center. The input spectrum tested here is shown in Figures 5 and 6. Waves were measured along a cross-section located about 17 feet behind the crest of the shoal as seen in Figure 3. Figure 7 gives a comparison of the steady-state spectral model's performance compared to the measured waves, along with the "equivalent" monochromatic solution (i.e. the solution obtained by treating these wave spectra as equivalent monochromatic waves). Clearly, the influence of spectral shape cannot be neglected. In fact, for waves typical of moderate storm conditions, the monochromatic approximation overpredicts wave heights by about 120%. Even for the case of a spectrum typical of swell, the monochromatic predictions are too high by about 100%. In neither of these test cases were waves observed to be undergoing wave breaking over the shoal; thus, the results shown in Figure 7 should be a fair examination of propagation effects only for realistic wave spectra.

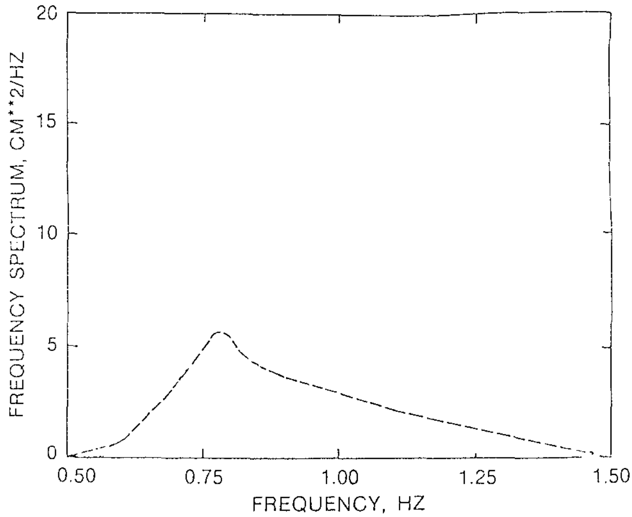

Figure 5. Input frequency spectrum for test case.

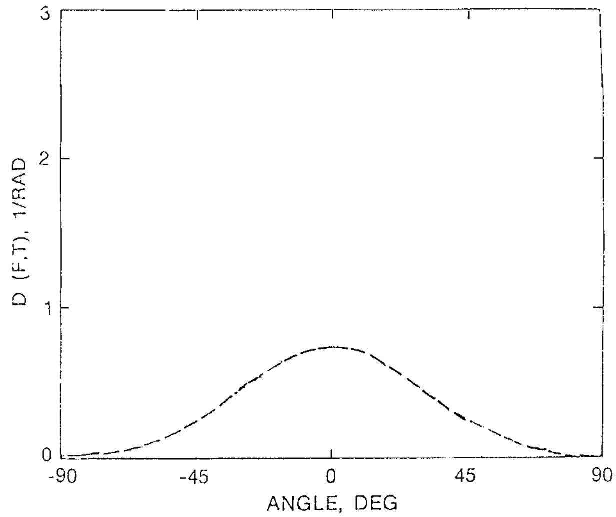

Figure 6. Angular distribution of energy for test case.

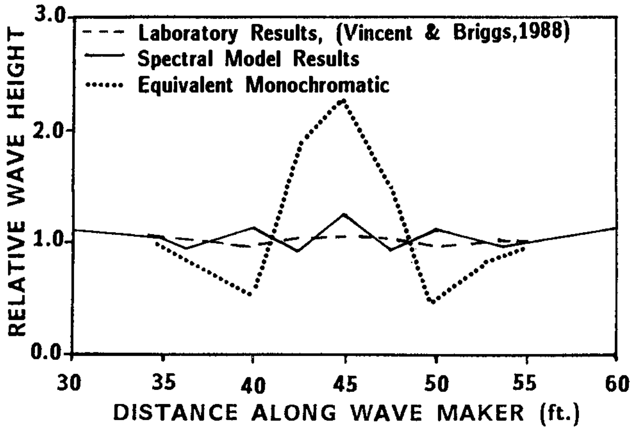

Figure 7. Comparison of results from spectral model to laboratory measurements for broad directional spectrum. Equivalent monochromatic results shown for reference.

Tests of Propagation and Source Terms Together

As some final test cases, let us consider first a situation with a 40-knot wind blowing perpendicular to the coast over a planar shelf with constant slopes of 1/10 and 1/100. The boundary deep-water spectrum is specified to have a JONSWAP shape (δ = 3.3, f_m= 0.10 hz, α = 0.013, σ_a= 0.07, σ_b= 0.09) with a $\cos^{2}\theta$ angular distribution of energy. Table 2 shows the behavior of wave height as a function of distance from shore for the case with the wind source term included and for the case with the wind source neglected.

Next, let us consider a 40-knot wind blowing at an angle of 50^{o} relative to the coast compared to the zero-wind case. Table 3 shows these results. Differences such as seen in Tables 2 and 3 could be very critical on sensitive projects.

Table 2

Comparison of Predicted Wave Heights (Metres) for 40-Knot Wind Blowing Straight Toward Coast and No Wind

Depth	Slope = 1/10		Slope = 1/100	
	No Wind	40-Knot Wind	No Wind	40-Knot Wind
90	6.11	6.11	5.84	5.97
80	6.06	6.07	5.60	5.82
70	5.99	6.01	5.38	5.68
60	5.90	5.94	5.18	5.54
50	5.79	5.83	4.97	5.39
40	5.64	5.69	4.75	5.22
30	5.47	5.53	4.52	5.01
20	5.32	5.38	4.27	4.75
10	5.33	5.35	3.82	4.07

Table 3

Comparison of Predicted Wave Angles (Degrees) for 40-Knot Wind Blowing at 50^{o} Angle to Coast and No Wind

Depth	Slope = 1/10		Slope = 1/100	
	No Wind	40-Knot Wind	No Wind	40-Knot Wind
90	0	0.3	0	0.5
80	0	1.2	0	6.4
70	0	2.5	0	10.3
60	0	3.0	0	15.4
50	0	3.3	0	17.5
40	0	3.1	0	18.2
30	0	2.2	0	16.8
20	0	1.5	0	14.9
10	0	0.9	0	6.3

Conclusions

A steady-state spectral model has been briefly described here. This model has a run time typically over 100 times faster than time-step solutions to comparable problems. Tests of this model support the following conclusions relative to shallow-water wave modeling:

1. The model described here, even neglecting phase-dependent bahavior, can produce reasonable simulations of a wide range of wave conditions, both monochromatic and spectral, even in conditions which significant nonlinear behavior is expected;
2. Equivalent monochromatic methods cannot produce accurate results for realistic wave spectra; and
3. Simulations of areas on the scale of 5 miles or more should consider wind effects on nearshore waves; otherwise, the simulated results can deviate significantly from conditions expected under the action of winds.

Appendix.--References

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