CHAPTER 51

A DIRECTIONAL PREDICTION MODEL OF WAVES WITH VARIABLE WIND

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ABSTRACT.- A model of the response of the direction of the waves to changing winds is presented. First the theoretical results that associate an average direction of energy propagation to every wave component is introduced. From this model, numerical results are presented both by simulations of ideal wind veering situations and compared with field data taken from pitch and roll buoys.

1. INTRODUCTION

The wave generation as a response to action of wind variable in intensity and direction is still an open problem, where either the theoretical models or the numerical methods used for its forecasting and hindcasting require a proper validation based on strictly controlled wind and wave measurements.

This communication presents a theoretical model of the directional wave response, as well as numerical examples in ideal conditions and comparisons with field data taken from the Norwegian ODAP project.

The model allows us to associate an average direction of energy propagation to every spectral component, so that with a little additional computational effort we get a wave prediction method with analogous precision in the two dimensions of the spectrum; frequency and direction.

Numerical results are presented illustrating the general behaviour of the wave directional relaxation. The data available to the author backs up these results.

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2. MODEL

The global mean direction Ω of the wave spectral density function $S(f,\theta)=S(f)\cdot D(f,\theta)$ due to the wind action depends on: The boundary and initial values of the wave field, the change in the directional shape of S, and the direct action of the wind. This effect has, in turn, two components; the energy contribution B_{Δ} , and the change in its directional distribution, J.R.Acinas (1987).

In this communication only the directional variation due to the energy E_{Δ} , is taken into account as a consequence of the wind action in a homogeneous wave field. Starting from the energy balance equations and the global average direction of the wave:

$$dS(f,\theta)/dt = G(f,\theta) \equiv \text{Source Function}$$
(1)
$$\Omega = \arctan\left(\iint_{n=1}^{\infty} \sin\theta \cdot S \cdot df \cdot d\theta / \iint_{n=1}^{2\pi} \cos\theta \cdot S \cdot df \cdot d\theta \right)$$
(2)

Differentiating equation (2) and after some manipulations involving (1) and (2) the result is:

$$\frac{d\Omega}{dt} = \frac{-DI}{A^2 + B^2 E} \frac{1}{dt} \frac{dE_A}{dt}$$
(3)

with:

DI = $(A_{\Delta}B - B_{\Delta}A)\cos \varepsilon - (A_{\Delta}A + B_{\Delta}B)\sin \varepsilon$ A,B and $A_{\Delta}B_{\Delta}$ first order direction estimators of the total wave energy, E, and of the wind contribution, E_{\Delta}. A = $\int f \cos \theta D(f, \theta) df d\theta$; B = $\int f \sin \theta D df d\theta$ $\delta = (\theta u - \Omega)$; B = $\int f S(f, \theta) df d\theta$

If we centre $\mathbb{D}\langle f,\theta\rangle$ around $\Omega,$ we obtain in general

$$\frac{d\Omega}{dt} = \left(\frac{A_{\Delta}}{A} \cdot \sin \delta + \frac{B_{\Delta}}{A} \cdot \cos \delta\right) \cdot \frac{1}{E} \cdot \frac{dE_{\Delta}}{dt}$$
(4)

and doing the same with the energy contribution, $B_{\mbox{\tiny \Delta}}$ we have

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = \frac{A_{\Delta}}{A} \sin \theta \cdot \frac{1}{E} \cdot \frac{\mathrm{d}E_{\Delta}}{\mathrm{d}t}$$
(5)

If we consider, for example, spectral evolution according to the JONSWAP study, we obtain the global average direction of the wave (based on parameters which characterise its growth) by:

$$\frac{d\Omega}{dt} = \frac{A_A}{A} \cdot \sin 8 \cdot \frac{E_A}{E} \cdot \left(-4 \cdot \frac{dLf_P}{dt} + \frac{dL\alpha}{dt} + \frac{dLP}{dt}\right)_A$$
(6)
with: $P = P(\chi, \sigma) = \sqrt{\sum_{i=1}^{\infty} \frac{5}{x^2} \exp(-1, 25/x^4)} \cdot \chi^{Y} dx$

$$y = \exp(-((x-1)/\sigma)^2/2)$$

The response of the wave direction Ω_c , of a spectral band (f_1, f_2) represented by the frecuency, f_c and energy component E_c is similarly given by:

$$\frac{d\Omega_c}{dt} = \frac{A_{Ac}sins}{A_c} \frac{1}{E_c} \frac{dE_{Ac}}{dt}$$
(7)

Systematic application of this formulation is the basis of the development of a directional wave prediction model in which each spectral component has an associated mean direction of energetic propagation.

3. RESULTS

The model is illustrated both by simulations under ideal wind-veering situations situations (Figs. 1 and 2), and compared with measurements from the ODAP data analyzed by T.Audunson et al. (1981), Fig. 3.

Initial wind and wave conditions are assumed to be homogeneous and are used as input to a parametrical wave prediction model, (Acinas 1981).



- Fig.1 Response of wave direction to a sudden change ∆9, in wind direction. Wind velocity U, constant.
 - a) Global mean wave direction response Ω , to $\Delta \theta_{u}=10^{\circ}$, 22.5° and 45°. Initial peak frecuency $f_{\rm Di}=0.2$ Hz.
 - b) As in a) but for $\Delta \theta_{=}$ 45° and $f_{pi}=.15$, .2, and .3Hz. Response $\Omega_a(3/2f_p)$ and $\Omega_b(2/3f_p)$; $f_{pi}=0.15$ Hz.

Computation is carried out along characteristic lines and for easier comparison of results, $A_{\Delta}/A = 3.5$ has been used for all cases. This number comes from a emperical best fit with the ODAP data.

Inspection of the equations and results shows the following behaviour:

a) For the same initial wave field, the veering rate of Ω grows according to $\Delta\theta u.$

b) For the same variation $\Delta \theta_{\omega}$, Ω grows faster with the largest $f_{\rm P}$ of the initial state.

c) For a given wave field, the direction Ω_{n} , of the high frequencies, fa>fp, responds in a quasiinstantaneous way to the variations of $\theta_{i,i}$, and at a faster rate than the global direction.

On the other hand, low frequencies $fb \leq fp$ show a temporal delay in the response with lower response rate than the global direction.



Fig.2 Response of global, high and low frecuency directions
 Ω, Ω_a and Ω_b, to changing wind (U, Q); f_{pi}= .2Hz.
 a) (U constant, θ_u variable) b) (U variable, θ_u variable)

d) As the wave field grows towards FDW (fully developed wave spectrum) levels, the direction at all frequencies tends to align with that of the wind.

These results are in analogous with the theoretical-experimental results suggested previously by several authors; D.E. Hasselmann et al. (1980), H.Gunther et al (1981), T.Audunson et al (1981), J.H.Allender et al (1983).

From the ODAP data available to the author(Fig. 3.a) the prediction model has been evaluated for the period in which there is a growing sea, so that we can expect quasi-homogeneous conditions. A rapidly growing wind dominantes this period. The results are given in Fig. 3.b.

4. CONCLUSIONS

The response of wave directions, due only to the energy variation, under changing wind direction conditions can be summarized according to our results in three stages: 12) The high frecuencies respond quasiinstantaneously, and quickly adjust to wind direction then remaining saturated in energy.

During this first stage the low frequencies do not react to the change in wind direction. This may be explained because its energy B_{Δ} , does not increased.

 $2^{\underline{a}}$) The low frequencies then begin to increase in energy, with a subsequent very rapid variation in its direction Ωb , that is very strong and has an inflexion point.

32) On approaching FDW for a given wind all the components tend asymtotically to the wind direction in the same way.

These three stages can be totally or partially completed, but are always in order and with the same characteristics. This behaviour is clear both in the measurements as well as in the numerical calculations for the ideal situations (Figs. 1 and 2).

It is worth mentioning that the theoretical directional model does fit well to the available data (Fig. 3).

As a consequence, it's logical to begin to use the directional results of the model in the engineering evaluations of wave forecasting and hindcasting.

The directional measurements currently available are very limited. An international data base is therefore proposed which would greatly assist future theoretical work.



Fig.3 Response of wave direction Ω, to a changing wind (U,θ).
a) Analysis of field observations by T. Audunson,

- S. Barstow and H. Krogstad, 1981.
- b) Imput (U, θ_i) values and numerical present results $\Omega(t)$ compared with the response obtained from a) field data.

REFERENCES

Acinas J.R. 1981. Previsión del oleaje en los grandes puertos del Cantábrico MOPU.

1987. Relación direccional oleaje-viento. Análisis teórico. CEDEX en edición.

Allender J.H.,J.Albrecht and G.Hamilton 1983. Observation of directional relaxation of wind sea espectra.J. of Physical Oceangraphy, vol 13.

Audunson T., S:F. Barstow and H.B. Krogstad 1981. Analysis of wave directionality from H.P.R. Buoy operated Offshore Norway.Wave wind direct.Paris 1981.

Holthuijsen L.H., B.Monsselman and A.J.Kuik 1984, A model for the response of wave directions to changes in wind direction. Symp. on description and modelling of directional seas. T.U. Denmark.