CHAPTER 45

Solitary waves passing over submerged breakwaters

Mark Cooker *
Howell Peregrine †

Abstract:
A method is described for the computation of the two-dimensional unsteady motion of a solitary wave passing over submerged breakwaters. Far from the breakwater the fluid is assumed static and the sea bed is level. The fluid motion is assumed to be irrotational, incompressible and inviscid. The exact boundary conditions at the free surface and the impermeable bed are satisfied. Laplace's equation for the velocity potential is solved using a boundary integral method.
Numerical results are reported which show the variety of ways in which solitary waves are distorted when they encounter submerged breakwaters.

This work is part of a program of study to provide more understanding of the hydrodynamics of steep waves when they encounter coastal structures. We have developed a computer program which models the motion of waves in 2D, irrotational, inviscid flows. The method is not restricted to any particular free surface motion but we choose here to examine solitary waves. This class of waves includes the largest 2D irrotational, inviscid wave which can steadily propagate on a fixed depth. Solitary waves are everywhere elevated above the undisturbed water level, so it was thought likely that these model the waves most damaging to structures. We can accurately reproduce solitary waves with heights up to 96% of the highest wave.

The numerical method employs a boundary integral technique to solve Laplace's equation for the velocity potential. Bernoulli's equation is used as one boundary condition at the free surface. We ignore surface tension, and take the atmospheric pressure above the liquid surface to be constant. The second boundary condition is to assume that fluid particles on the free surface stay on the surface. All rigid boundaries are assumed to be impermeable. Details of the method are given by Dold & Peregrine (1986).

The method is accurate, stable and efficient. A solitary wave of height 0.5 of the depth can propagate over a horizontal distance of 50 depths with less than 0.1% change in height. The method does not suffer from "sawtooth" instabilities.

The program can be run on an IBM-compatible AT personal computer: in 60 minutes elapsed time we can calculate the motion of a wave passing over a breakwater.

The method also uses conformal mappings to transform flow domains

*Research Assistant †Professor of Applied Maths
School of Maths, University of Bristol,
Bristol BS8 1TW , England.

with irregular beds into regions with a flat horizontal bed. This is done to make Laplace's equation easier to solve. For the examples used here the conformal mapping has a bottom comprising a semicircle (radius R) on an otherwise flat horizontal bed. See figure 1.

The solitary wave is started with its crest far enough from the semicircle for the wave to be unaffected by it in the first few time steps. The wave moves left towards the cylinder. The initial data describing an incident solitary wave is calculated using the method of Tanaka (1986).

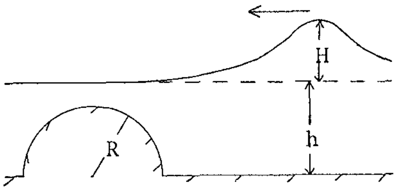

Figure 1 : Solitary wave approaching submerged semicircular breakwater.

Let the undisturbed fluid depth h = 1. Then H, the incident wave height, and cylinder radius R, are dimensionless parameters. The H - R parameter space is surprisingly rich: see figure 2. We expected all waves to steepen and break on the cylinder, but this only happens for the largest waves ($H > 0.6$) and the largest cylinders ($R > 0.8$) and even these extreme waves tend to break downstream of the breakwater.

Small cylinders ($R < 0.3$) cause very large, linearly unstable waves ($H > 0.8$) to eventually break, long after they have passed over the breakwater. See figure 3. Tanaka et al. (1987) show that unstable solitary waves do not necessarily break. The growth in time of the breakwater instability closely follows that discussed by Tanaka et al. (1987). Waves with height $H < 0.77$ develop a train of small dispersive waves in their wake, after they have passed over the breakwater when $R < 0.4$. See figure 4.

A very common but unexpected phenomenon occurs for many waves for cylinders with radius in the range 0.5 to 0.9 . As the wave approaches the cylinder a second crest grows on the opposite side of the obstacle. The second crest grows and soon dominates the first which meanwhile decays. The new crest propagates away from the cylinder. This exchange of crests across the cylinder is reminiscent of a large solitary wave catching up a smaller one. See figure 5.

Most surprising of all are those waves of height between 0.3 and 0.6, passing over cylinders of radius 0.7 to 0.9 . The wave passes over the breakwater and having cleared it a second, stationary wave forms above the left-hand margin of the breakwater. This second wave steepens enough to break over backwards, onto the cylinder. See figure 6.

For the example shown in figure 6 if we increase R from 0.7 to 0.8 the transmitted wave breaks forward before the second waves breaks backward. See figure 7. We are unable to continue the computations beyond the time at which a wave breaks, but experiments have indicated that breaking can occur for both of the waves. The breaking is simultaneous when $R=0.77$ for $H = 0.58$.

Wave tank experiments at Santander University in Spain by Cézar Vidal confirm the backwards breaker. Some measurements of depth as a function of time, at fixed stations, also agree well with prediction.

Figure 6 also illustrates the instantaneous streamlines of the flow. The pressure contours can also be found and the total force on the cylinder determined. For example when $H = 0.8$ and $R = 0.9$

the wave breaks on top of the cylinder. See figures 8 and 9. The horizontal and vertical components of hydrodynamic force on the breakwater as the wave approaches are plotted in figure 10. The maximum horizontal component of force occurs during the wave's approach and well before breaking. Both of the maxima in horizontal and vertical components of force are about as large as the hydrostatic force.

This shows how waves can exert large dynamical forces on submerged coastal structures.

Figures 11 and 12 show a sequence of profiles of a solitary wave of height 0.8 breaking on a shoal of elliptical profile. The bed is reminiscent of a gently shelving beach and our method has predicted the classical plunging breaker. The pressure distribution is plotted in figure 13, at a time soon after the front face of the wave has become vertical. The large pressure gradients normal to the front face of the wave and parallel to the bed remind us of the strength of the sea.

REFERENCES

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TANAKA, M. 1986, The stability of solitary waves, Phys. of Fluids, 29, 650-655.

TANAKA, M., J.W.DOLD, M.LEWY, & D.H.PEREGRINE, 1987, Instability and breaking of a solitary wave, J. Fluid Mech., 185, 235-248.

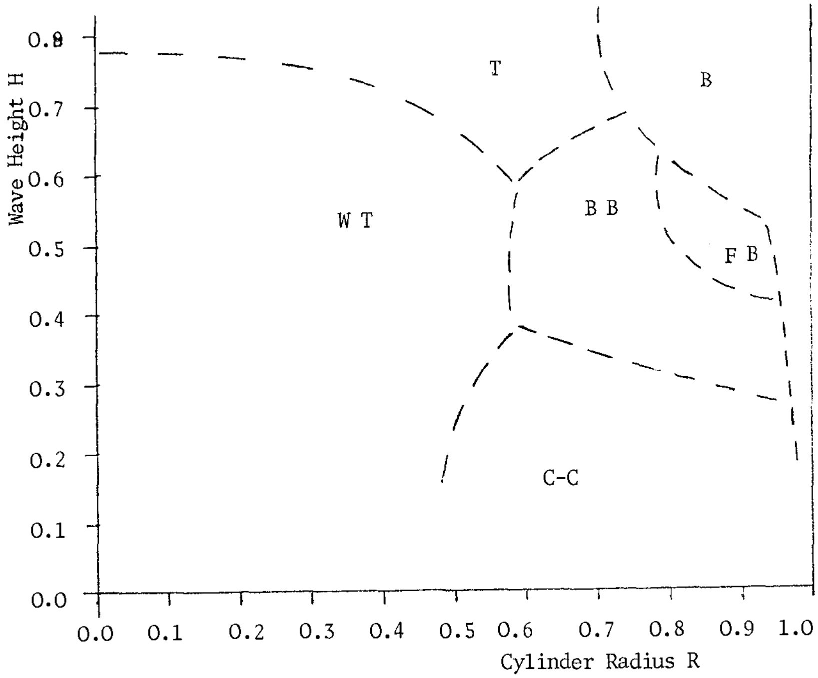

Figure 2 : B: Breaking at cylinder. B B: Backward breaking. C-C: Crest interchange across cylinder. F B: Forward breaking. T: Tanaka-type instability. W T: Wave train behind transmitted wave.

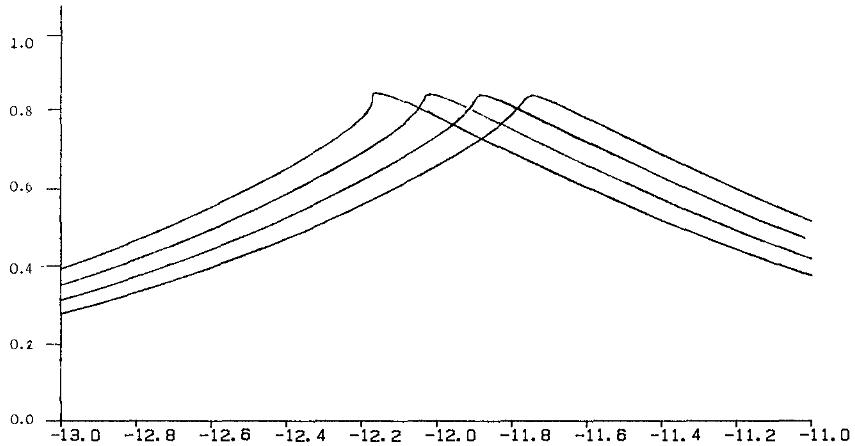

Figure 3: H = 0.8, R = 0.3. Breaking far downstream of cylinder. Natural scaling.

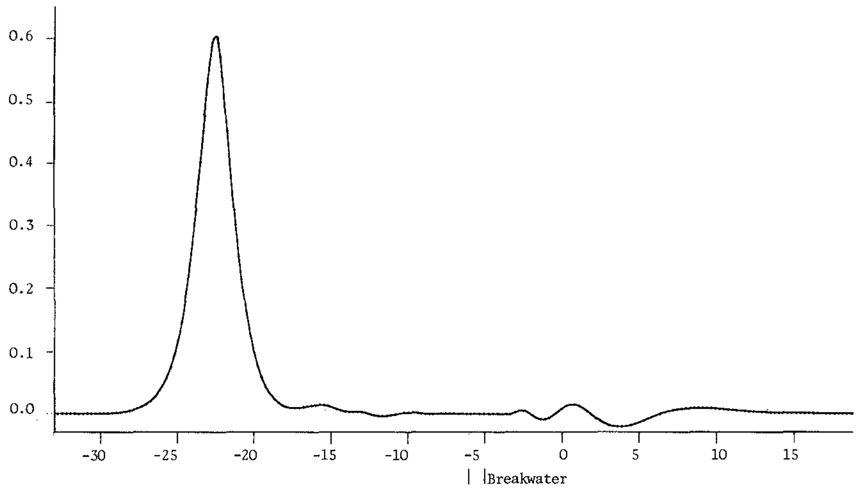

Figure 4: H = 0.6, R = 0.5. Dispersive waves trailing transmitted wave. Note right-travelling reflected waves. Vertical exaggeration = 40.

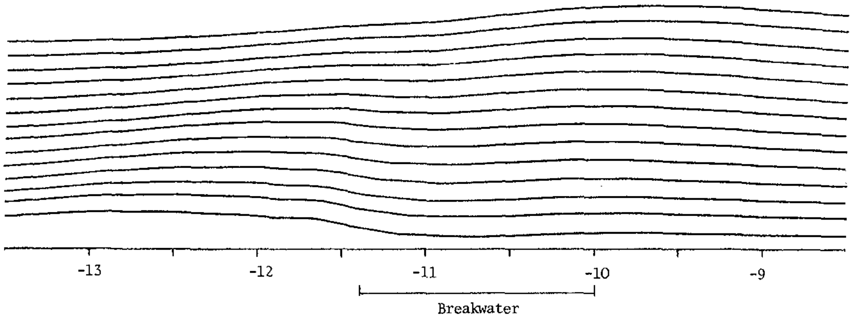

Figure 5: H = 0.22, R = 0.7. Exchange of crests across breakwater. Times 9, 9.2, ... 9.6. Natural scaling.

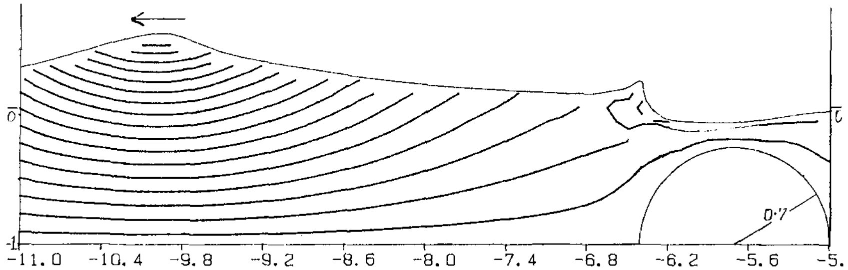

Figure 6: The backwards breaker for H = 0.46 and R = 0.7. The instantaneous streamlines are also shown. Natural scaling.

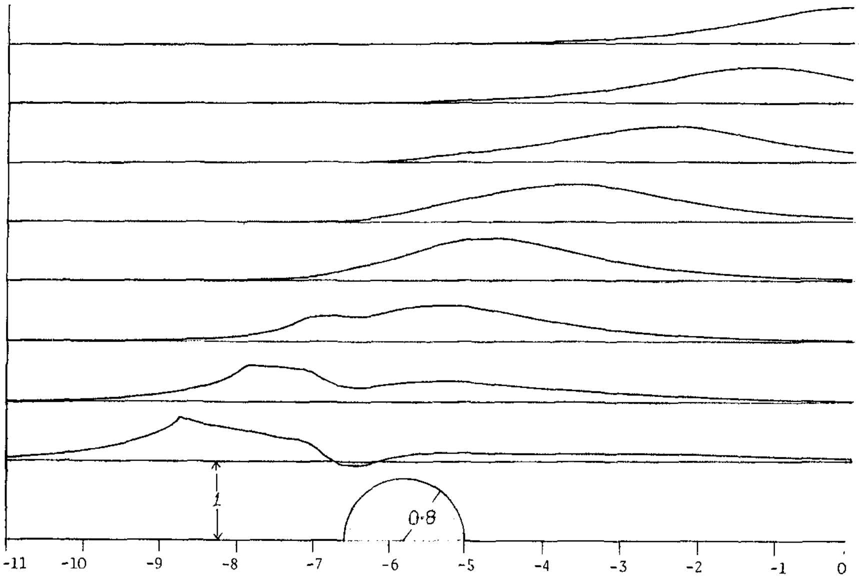

Figure 7: H = 0.46, R = 0.8. Wave breaks forward, downstream of cylinder. Times 0, 1, ..., 7.

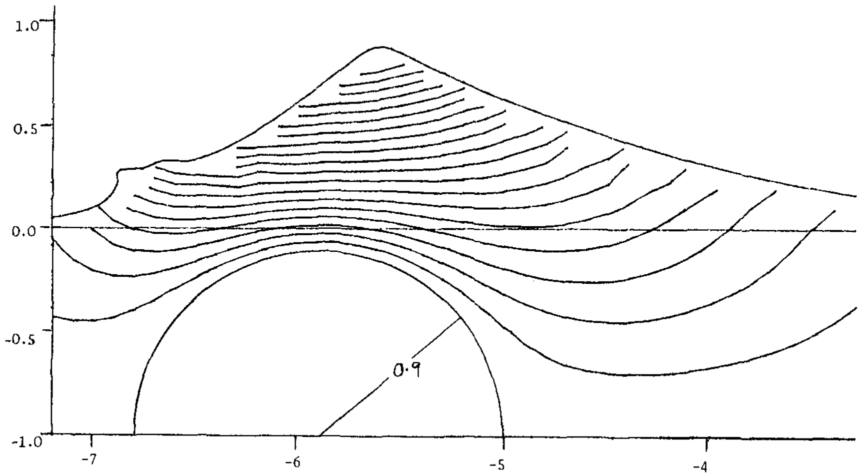

Figure 8: H = 0.8, R = 0.9. Instantaneous streamlines. Natural scaling.

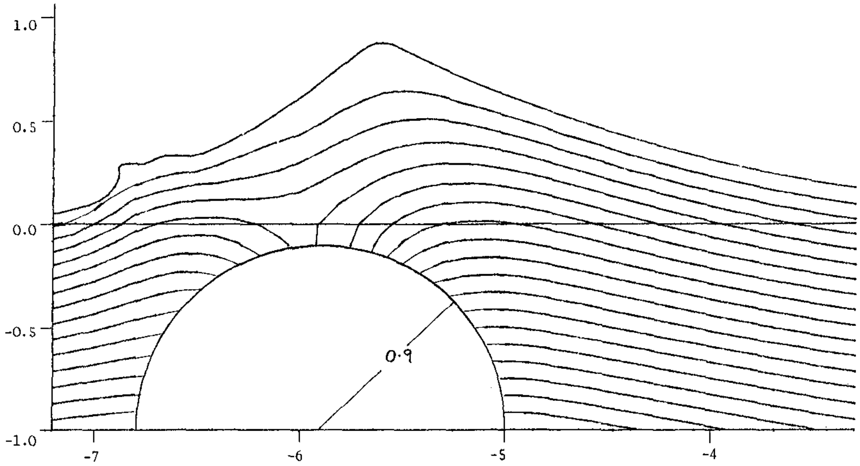

Figure 9: H = 0.8, R = 0.9. Pressure contours. Natural scaling.

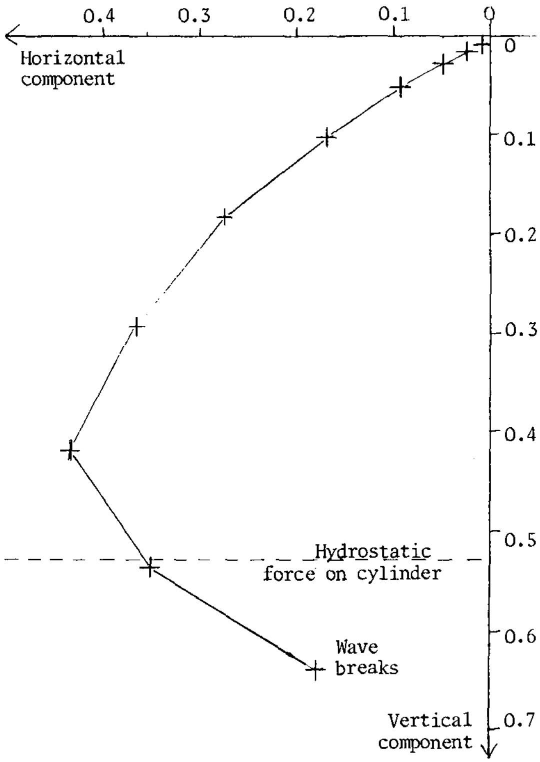

Figure 10: The dynamic force on a cylinder H = 0.8, R = 0.9. Times 0, 0.5, ..., 4.5.

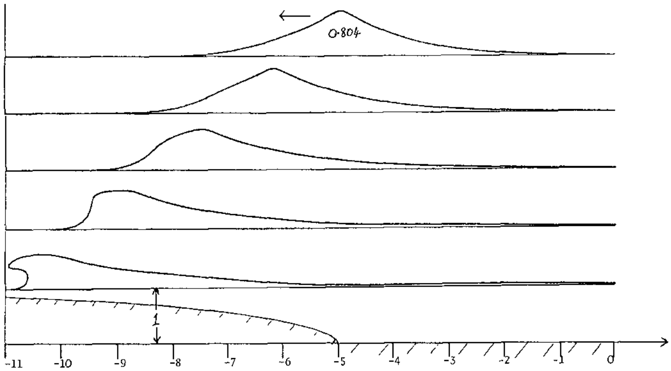

Figure 11: Wave breaking on an elliptical shoal where minimum depth is 0.1, H = 0.8. Times 4, 5, 6, 7, 8. Natural scaling.

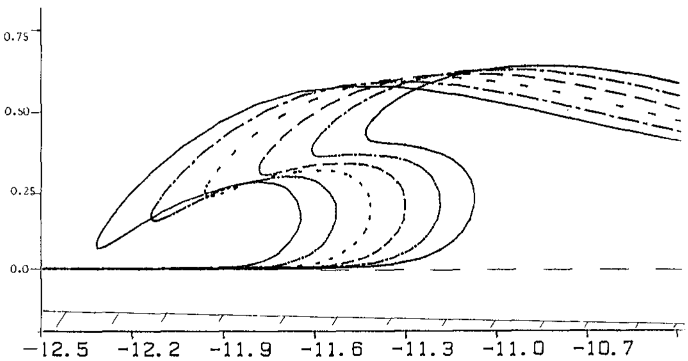

Figure 12: H = 0.8. Elliptical shoal. Times 8.0, 8.1, ..., 8.5. Natural scaling. Continuation of figure 11.

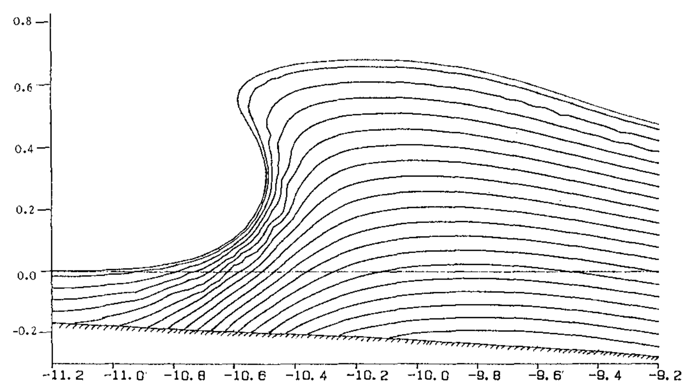

Figure 13: As figure 11. Pressure contours at time 7.5. Natural scaling.