CHAPTER 41

A NUMERICAL MODEL OF WAVE DEFORMATION IN SURF ZONE

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Abstract

A numerical model is presented for nearshore wave deformation due to shoaling and breaking, and to decay and recovery in the surf zone. The model is based on a set of time-dependent mild slope equations including a term of wave energy dissipation caused by breaking. Its applicability is demonstrated by comparisons between the computations and the measurements of cross-shore distributions of the wave height and potential energy over typical beach configurations.

1. INTRODUCTION

Upon arriving at the nearshore zone, waves come to play an important role in various coastal processes. It is in this area that the waves undergo some drastic changes due to shoaling, diffraction, refraction and breaking, and that they gain the ability to affect sea bottom configuration, to damage man-made structures or to cause difficulties in the handling of ships. A conventional approach to describing nearshore waves is through using the wave energy equation. Although the method has been widely employed in coastal engineering, it is not a general one in the sense that reflection, refraction and diffraction of the waves should be separately calculated. In addition in case of boundaries with complicated geometry, the method faces severe computational difficulties. A more general and new approach is available through using the mild-slope equation first derived by Berkhoff (1972). The equation, expressed in an elliptic form, describes waves under combined diffraction and refraction on a slowly varying bottom. Its solution, however, often involves

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considerable computational trouble, and the treatment of boundary conditions is in general diffieult. To overcome these difficulties, simplified parabolic forms of this equation have been derived, which reduce the computation involved, but instead we eannot easily deal with reflected waves in general. Watanabe and Maruyama (1986) have proposed a set of time-dependent mild-slope equations, which has the advantages of reduced eomputation time as eompared with the elliptic form equation and of simpler treatment of boundary eonditions for open boundaries as well as for boundaries with arbitrary reflectivity. In addition, their model incorporates wave breaking and decay in the surf zone. However its validity has not yet been fully examined.

The time-dependent mild-slope equations will be applied in the present paper to computing cross-shore ehange in a one-dimensional wave field; namely, deformation of waves with normal incidenee on a straight parallel-contour eoast. The selection of such simple eonditions will avoid the involvement of wave diffraetion and refraction, and enable us to study in details about the wave deformation due to breaking. The formulation for energy dissipation in the previous study will be modified to more properly express the wave decay and recovery processes. Some numerical eomputations through the new equations will be conducted on eross-shore distributions of the wave height and potential energy over three kinds of typical beach configurations, and compared with the experimental data.

2. Time-Dependent Mild-Slope Equations for Waves in the Surf Zone

Watanabe and Maruyama (1986) have presented a set of the time-dependent mild-slope equations and a numerical model for nearshore waves under combined refraction, diffraetion, and breaking. This model is applied here to a cross-shore one-dimensional wave field, and is improved for the behavior in the surf zone.

The equations are expressed in terms of the water surfaee elevation ζ and the flow rate Q into the eross-shore direction as:

$$\frac{\partial Q}{\partial t} + c^2 \frac{\partial \zeta}{\partial x} + f_D Q = 0 \tag{1}$$

$$\frac{\partial \zeta}{\partial t} + \frac{1}{n} \frac{\partial (nQ)}{\partial x} = 0 \tag{2}$$

where t is time, x is the horizontal eoordinate normal to the shoreline, c is the phase veloeity, and

$$Q = \int_{-h}^{o} u\, dt, \qquad n = \frac{1}{2}\left(1 + \frac{2kh}{\sinh 2kh}\right) \tag{3}$$

in which u is the x-component of the orbital velocity, h is the water depth, and k is the wave number. Equations (1) and (2) may be regarded as vertically integrated equations of motion and of continuity, respectively. In Eq. (1), f_D is the energy dissipation factor, and has been expressed by Watanabe and Maruyama (1986) as:

$$f_D = \alpha_D \tan\beta \sqrt{\frac{g}{h}\left(\frac{Q_m}{Q_r} - 1\right)} \tag{4}$$

where $\tan\beta$ is a representative bottom slope around the breaking point, Q_m is the amplitude of Q, and Q_r is the flow rate amplitude of the broken waves recovered in an area of uniform depth of h and expressed as $Q_r = \gamma' \sqrt{gh^3}$. Values of 2.5 and 0.25 have been proposed by them for the coefficients, α and γ', respectively. The dissipation factor, f_D, is set equal to zero outside the surf zone and in any region in which $Q_m < Q_r$. Under this condition, $f_D = 0$, Eqs. (1) and (2) reduce to the mild-slope equation proposed by Berkhoff (1972).

In order to improve the behavior of the model for the wave decay and recovery processes, we now redetermine the expression for the energy dissipation factor f_D. A proper modeling of wave transformation in the surf zone depends strongly on appropriate evaluation of this factor. Several models for wave motion in the surf zone have been proposed. A brief summary of the works up to the present is given by Horikawa (1988). However, the wave breaking process and the subsequent breaking-induced turbulence have not yet been fully clarified and much remains to be done. In the present work a semi-empirical general expression for f_D is given through using some new concepts and experimental results.

If we assume purely progressive waves, combination of Eqs. (1) and (2) yield the following wave energy equation.

$$\frac{d}{dx}(E c_g) = -n f_D E \tag{5}$$

where E is the wave energy density per unit horizontal area, and c_g is the group velocity. Now assuming long waves over a uniformly sloping beach and a constant ratio of the wave height to the water depth, we obtain the following expression for f_D from Eq. (5).

$$f_D = \alpha_D \tan\beta \sqrt{\frac{g}{h}} \tag{6}$$

For a general bottom topography, Eq. (6) needs some modification. To do this we should introduce two parameters. The first one is the amplitude of the wave-induced flow rate inside the surf zone of a uniformly sloping beach, which will be denoted by Q_s. According to experimental data (Isobe, 1986), Q_s can be safely formulated as:

$$Q_s = \gamma_s c h, \quad \gamma_s = 0.4 \ (0.57 + 5.3 \tan\beta) \qquad (7)$$

For the second parameter, we will adopt the flow rate of recovered waves in a similar way to Watanabe and Maruyama (1986). Let us assume waves coming into an area of uniform depth h after breaking as shown in Fig. 1. It is well known that upon arriving at this area, broken waves start to recover and after a certain distance shown as the transient zone, they will find a stable form and will no more lose their energy. Considering the experimental results (Maruyama & Shimizu, 1986), the amplitude of the flow rate of recovered waves, Q_r, can be expressed as:

$$Q_r = \gamma_r c h, \quad \gamma_r = 0.4 \ (a / h)_b \qquad (8)$$

where $(a/h)_b$ is the ratio of the wave amplitude to the water depth at the breaking point.

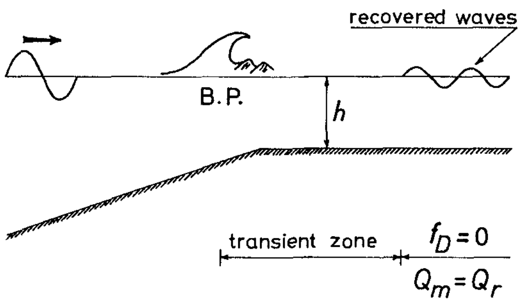

Figure 1 Wave recovery zone with constant depth.

The fact that the value of f_D should become equal to zero for recovered waves suggests that at any water depth, the value of Q_r may be regarded as the critical flow rate below which there is no dissipation of energy. Therefore we assume the following form of expression for f_D.

$$f_D = A \ (Q_m - Q_r)^m \qquad (9)$$

where at any depth Q_m is the amplitude of the actual flow rate and Q_r is the flow rate amplitude of the broken waves virtually recovered in the same constant depth. The power m should be less than unity in order to get a finite distance to recovery. The proportionality constant A can be determined by requiring that when $Q_m = Q_s$

then the value of f_D should be equal to that for uniform slope, *i.e.* Eq. (4). Assuming $m = 1/2$ for simplicity we finally obtain Eq. (10) for general bottom topography.

$$f_D = \alpha_D \tan\beta \sqrt{\frac{g}{h}\left(\frac{Q_m - Q_r}{Q_r - Q_s}\right)} \tag{10}$$

The location of the breaking point is calculated with the generalized breaker index diagram proposed by Watanabe *et al.* (1984). Change in the mean water elevation is evaluated through the distribution of radiation stresses, which are calculated from time histories of ζ and Q by using the formulas presented by Watanabe and Maruyama (1986).

3. Boundary Conditions

For the computation of a cross-shore one-dimensional wave field, there are two boundaries: the offshore boundary and the shoreline boundary.

The offshore boundary where incident waves are prescribed is treated as an open boundary in order to let the reflected waves, if any, to go out of the region freely. For this, assuming a locally constant depth region at the offshore, we express the boundary condition in terms of the water surface elevation as:

$$\zeta(x_0) = \zeta(x_0 + \Delta x) + a_I \{\sin(k x_0 - \sigma t) - \sin[k(x_0 + \Delta x) - \sigma(t - \tau)]\} \tag{11}$$

where $\tau = \Delta x / c_0$, Δx is the grid length in the finite difference scheme, a_I and σ are the amplitude and angular frequency of the incident waves, respectively, the subscript o denotes quantities at the offshore boundary and the x-axis is taken shoreward.

At the shoreline, previous computations have usually assumed a hypothetical constant depth region to avoid infinity in the value of wave height. However, the solution of the time-dependent mild-slope equations gives a time history of wave propagation so that the breaking point can be determined contemporarily with the computation of wave propagation and the wave height will decay thereafter. This makes it possible to impose the shoreline boundary condition simply as $Q = 0$. The receding of the shoreline due to change in the mean water level is also included using a moving boundary technique.

4. Results and Discussion

The nearshore wave model based on Eqs. (1), (2), and (7) to (10) is here applied to computing wave deformation for three kinds of typical beach profiles: uniform slope, step-type, and bar-type. The equations are solved by a finite difference method with a staggered mesh scheme.

The results are compared with measurements reported by Nagayama (1983) in Figs. 2 to 4, where H is the wave height, E_p is the potential energy, ρ is the water density, and the x-axis is taken here in the offshore direction with the origin at the shoreline in still

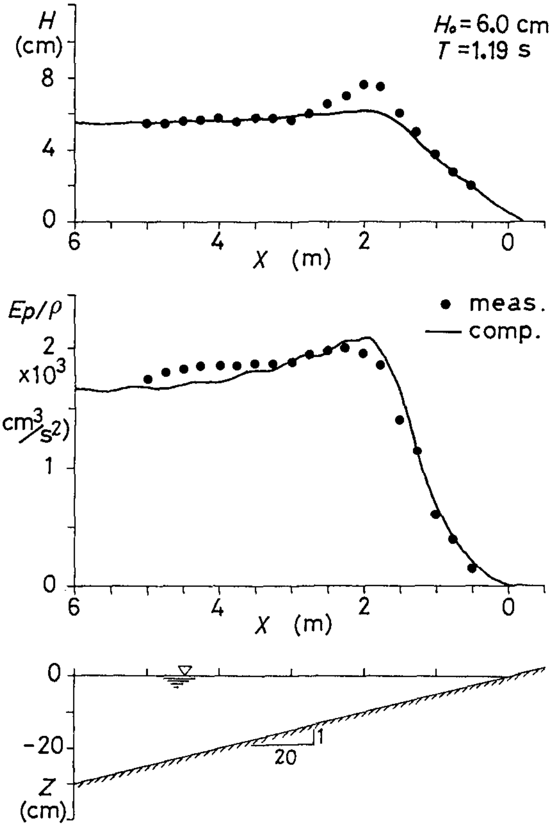

Figure 2 Wave height and potential energy on a uniform slope.

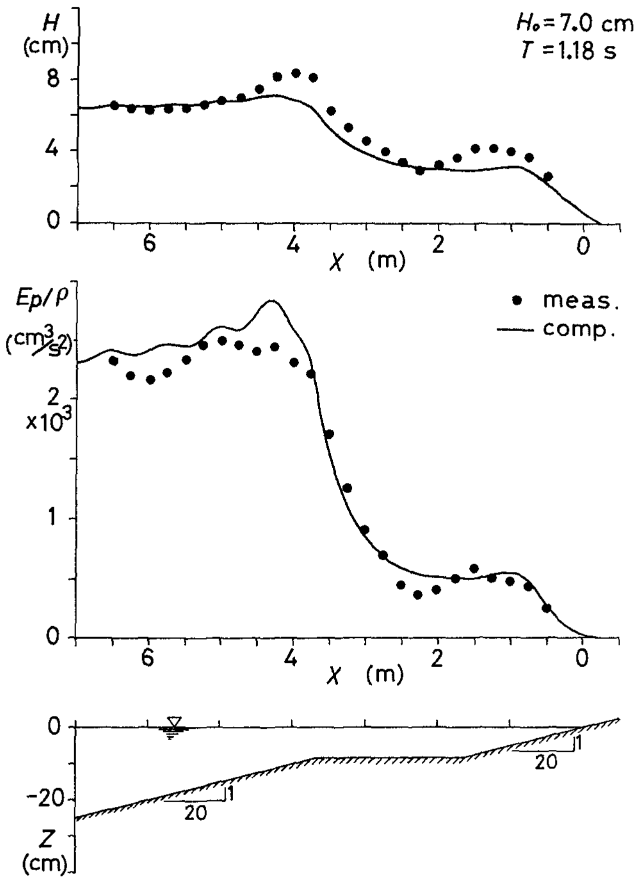

Figure 3 Wave height and potential energy on a step-type beach.

water. The potential energy in the measurements has been evaluated from the mean square values of $\zeta(t)$ at each location.

The wave decay, recovery, and secondary breaking are well reproduced in the computations. Although the wave height is slightly underestimated around the breaking

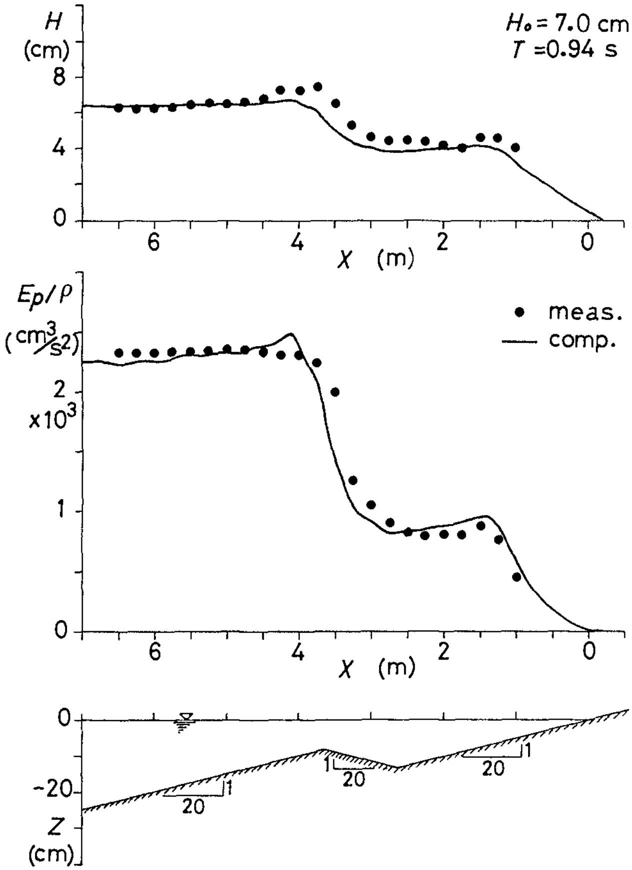

Figure 4 Wave height and potential energy on a bar-type beach.

point as is anticipated whenever linear theories are used, the distributions of the potential energy which represents overall intensity of the wave motion over one wave period are very well estimated. The small disagreement in the potential energy on the step-type beach before the first breaking point is attributed to the generation of cross waves in the wave channel.

Figure 5 shows the cross-shore variations of some other quantities calculated for the case of the bar-type beach. Figure 5(a) gives the variations of the energy dissipation factor f_D and of the energy dissipation rate per unit mass $n f_D E / \rho$. It is seen that after the first breaking the energy dissipation becomes zero as the water depth behind the bar increases and it is kept as zero until the secondary breaking occurs. In Fig. 5(b) the variation of the mean water level is shown. Unfortunately experimental data of the mean water level change are not available. However, considering the results for wave energy, we can expect that the wave set-up/down are well predicted, as we have found in further applications of the model. Figure 5(c) shows the variations of the flow rate amplitude Q_m and Q_r.

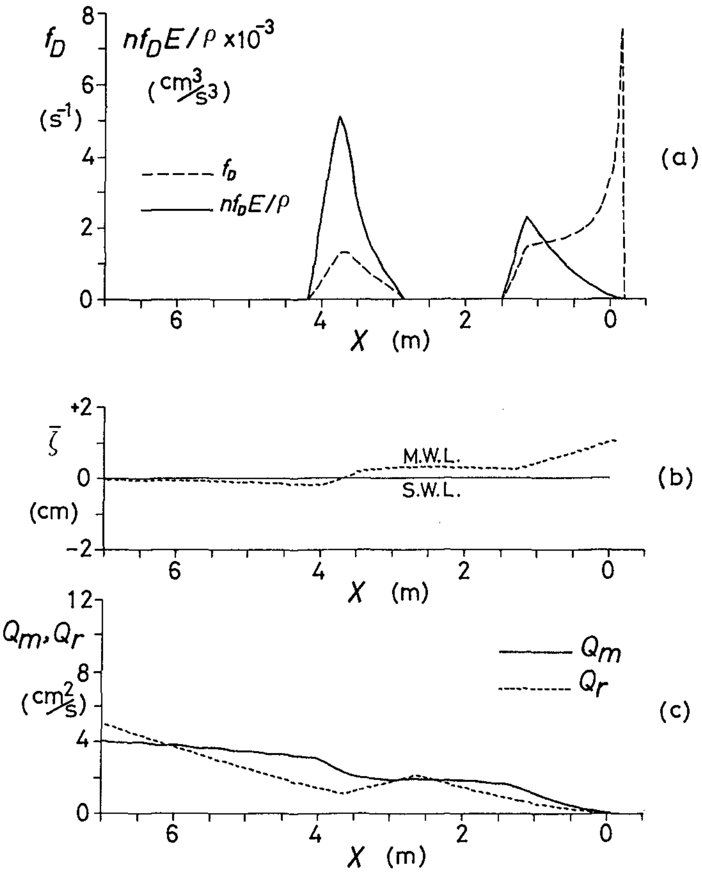

Figure 5 Energy dissipation factor, dissipation rate, mean water level and flow rate amplitude.

5. Concluding Remarks

A numerical model for nearshore waves based on the time-dependent mild-slope equations have been presented, with modification of the term for the energy dissipation due to breaking. It has been shown that the model can reproduce very well cross-shore wave deformation due to shoaling, breaking, decay, and recovery.

Generalization of the present model to two-dimensional wave fields will be rather straightforward and easy to conduct in a similar way to that described by Watanabe and Maruyama (1986). The two-dimensional model thus obtained will be applicable to computing wave deformation due not only to shoaling, breaking and recovery but also to refraction and diffraction.

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