CHAPTER 29

THE MEASURED PROPERTIES OF IRREGULAR WAVE BREAKING AND WAVE HEIGHT CHANGE AFTER BREAKING ON THE SLOPE

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ABSTRACT

Wave height change of the zero-down-cross waves on uniform slopes were examined experimentally. The properties of shoaling, breaking and decay after breaking for a total of about 4,000 irregular waves of the Pierson-Moskowitz type on 4 different slopes (1/10, 1/20, 1/30 and 1/50) were investigated. The shoaling property of the zero-down-cross waves can be approximated by the linear wave theory. However, the properties of breaking and decay after breaking differ considerably from those for periodic waves. The wave height water depth ratio (H/d) at the breaking point for the zero-down-cross waves is about 30% smaller than that for periodic waves on average despite the slopes. Wave height decay after breaking also differs from that for periodic waves and can be classified into three regions, i.e. shoaling, plunging and bore regions. Experimental equations for the breaking condition and wave height change after breaking are proposed in the study. A new definition of water depth for the zero-crossing wave analysis which can reduce the fluctuation in the plotted data is also proposed.

1.INTRODUCTION

Several models for irregular wave propagation in a surf zone have already been proposed by researchers such as Collins(1970), Battjes(1972), Kuo and Kuo(1972) and Goda(1975a). However, these models attempt to build a theory combining the theoretical or experimental properties of periodic waves i.e. breaking condition, energy dissipation, wave set-up and -down, etc. There have been few models which apply only the properties of the zero-(down or up)-crossing waves to complete the theory. There is a group of researchers, on the other hand, who have studied about the dynamic properties of irregular waves determined by the wave by wave method. The present author(1977), Isobe et al.(1980) and Sawaragi et al. (1980), for example, have examined experimentally the breaking properties of zero-crossing waves and showed that the averaged breaking wave height water depth ratio for zero-crossing irregular waves is smaller than that for periodic wave. This study also attempts to investigate the dynamic properties of zerocrossing irregular waves, especially their shoaling, breaking and decay after breaking, and tries to establish experimental equations for these properties.

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2.EXPERIMENTS

The wave tank used was a 29m long, 0.5m wide and 0.75m deep glass sided tank. At one end of this wave tank, a hydro-electric type irregular wave generator was set up (Kimura et al., 1987). At the other end, a uniform slope was installed. The slopes investigated were 1/10, 1/20, 1/30 and 1/50. The wave profile was measured by wave gauges installed with an equal interval on the slope, and also taken by two video cameras from the side of the wave tank. The data recorder and the video-cameras were synchronized using a clock unit. The spectra of the irregular waves were of the Pierson-Moskowitz type with different peak frequencies from f_p=0.7Hz to 1.0 Hz. The experiments for breaking, shoaling and decay after breaking were carried out separately. Individual measurements were carried out for about 1 hour. The wave profiles recorded by the wave gauges were discretized and stored in floppy discs. The records taken by the video cameras were read off using a function of the super-impose unit in a micro-computer system and stored also in floppy discs. Throughout this study, wave height was determined by the zero-down-cross method for the wave profile in the time domain.

3. BREAKING

The picture taken by the video-camera and the coordinates were superimposed on the CRT(Cathode-Ray-Tube) of the micro-computer. Reading the video-tape frame by frame the vertical line is drawn on the CRT at the breaking point of the wave which broke within the sights of the videocameras. Moving the video-tape back and forth, the times when the wave profile crossed the still water level at the vertical line and the maximum and minimum water levels within the zero-down-cross intervals at this line were read off on the CRT. The wave height and period can be, therefore, determined as if there were a wave gauge just at the breaking point. For further investigations, the zero-up-cross wave definition method was also applied for the same wave. The definition of breaking applied in this study was that of Stokes' i.e. the instance when a water particle start to come out from it's crest. Sometimes temporal white forms were observed around wave crests which were left behind the wave movement. Those were not considered to be breaking in this study.

Case	Slope	f _p (Hz)	H _{1/3} (cm)	T _{1/3} (s)	Data Total
1	1/10	0.7,0.9	8.2,8.0	1.33,1.01	800
2	1/20	0.7,1.0	8.2,7.9	1.33,0.95	1010
3	1/30	0.8	7.5	1.11	1004
4	1/50	0.8	7.5	1.11	1007

Table-1 Experimental conditions

Table-1 shows the slope, peak frequency, significant wave height and period, and total number of waves investigated. Figure 1 shows the energy spectra of the wave profiles measured at the horizontal part (h=45cm) of the wave tank (f_p =0.7Hz and 0.9Hz; tan θ =1/10). Since the saw shape pattern on the spectrum, which is inherent when reflected and incident waves are composed, is not prominent, the reflection coefficient of the slope may be negligibly small.



Fig.1 Power spectra

Figures 2 (a), (b), (c) and (d) show the relation between the relative wave height (H_b/h_b) and relative water depth (h_b/L_0) at the breaking point, in which ${\rm H}_{
m b}$ and ${\rm h}_{
m h}$ are wave height and water depth at the breaking point respectively and ${\rm L}_{\rm O}$ is the deep water wave length. The solid lines in individual figures show the breaking condition for the periodic waves (Goda,1970). The fluctuation of the plotted data in individual figure is considerably large, especially in figure (a). The major reason for the large fluctuations may exist in the insufficient definition of irregular wave properties in the analysis. Several researchers have investigated the effects of additional wave parameters. Herbers (1984), Sawaragi et al.(1980) and Isobe et al.(1980), for example, examined the effect of local wave profile or irregularity of the former and latter waves. The present authors also investigated these effects (some are very difficult in practical application), but no significant property which can reduce the fluctuation has been detected. Therefore, no more wave properties other than those determined by the ordinary zero-down-cross method are applied in this study except for the newly introduced definition for water depth.

3.1 A new definition of water depth

Figure 3 shows applications of the zero-down-cross method for two different waves. If the wave height H and period T for these two waves are the same, and if no additional property such as wave crest height, for example, is used, these waves are considered to be the same in the ordinary definition. In this study, to distinguish the difference between these waves without any additional properties to those used in Fig.2, water depth is determined as,

$$d = h + (a_c - a_+) / 2$$
(1)





Fig.3 Definition of water depth d and h

in which a_c and a_t are the zero-crossing crest height and trough excursion (positive) respectively.

The data used in Fig.2 are reanalysed using the new definition of water depth in Fig.4. Considerable reductions in scattering are achieved for all cases. The reductions are prominent when the slope is mild. The solid lines in the figures are the experimental equations proposed in this study.

3.2 The breaking condition for irregular waves

Two types of breaking conditions for periodic waves have been proposed, which apply (1) the critical wave steepness and (2) the critical wave height water depth ratio respectively. Since the present experiments were carried out in the region $d_b/L_0 > 0.01$, the formulation following the former condition (1) was applied in this study. Following Goda's formulation for periodic waves (1970), we assume the breaking condition in the form,

$$H_{\rm b}/d_{\rm b} = A \left\{ 1 - \exp\left[-B \pi \left(d_{\rm b}/L_{\rm f} \right) \left(1 + 15 \tan \theta \, \frac{4/3}{3} \right) \right] \right\} \left(L_{\rm f}/d_{\rm b} \right) + C$$
(2)

in which θ is an angle of slope , and A=1.7, B=1.5 and C=0. In Goda's equation water depth is measured from the bottom to the still water level. Applying the least square method for the data in Fig.4, optimum values for A, B and C were calculated as follows.

$$H_{b}/d_{b}=0.16 \{ 1-\exp[-0.8 \pi (d_{b}/L_{0})(1+15\tan\theta^{4/3})] \} (L_{0}/d_{b}) -0.96\tan\theta +0.2$$
(3)

The solid lines in Fig 5 (and in Fig.4) show the experimental equations (eq.3) and the broken lines are Goda's breaking condition for periodic waves. Since the difference between the equations for the 1/30 and 1/50 slopes is very small and the equation for the 1/50 slope agrees well with the measurement on the 1/60 slope by Kuo and Kuo (1972) ($\rm H_b/h_b=0.63$), the equation for 1/50 may be applicable for further mild slopes as in the case for periodic waves (Goda, 1970).

The above equations were introduced using experimental data in the range $0.01 < d_b/L_0 < 0.3$, the application of these equations may be restricted to waves within the above range.

The breaking types were also investigated. However, no recognized





Fig.5 Experimental equations (eq.3; solid lines)

property was found other than the property that the portion of plunging breaker increases with an increasing angle of the slope. The clarifications with other factors such as the surf-similarity parameter were not successful.

4. WAVE SHOALING

The measurement system applied was almost the same as that explained in the former section. Wave gauges were shifted to the offshore-side on the slope. The breaking point of waves were read off, at first, from the pictures recorded in the video-tapes. The corresponding zero-downcross wave heights and periods at the wave gauge points were calculated from the wave profiles stored in the floppy discs. The figures in Fig.6 show the relation between the measured wave height and relative water depth d/L_O on the slope of (a) 1/10, (b) 1/20, (c) 1/30 and (d) 1/50 respectively, in which H_0' and L_0 are wave height and period in deep water conditions, which were calculated substituting the wave height and period, measured by the most off-shore side wave gauge, in the small amplitude wave theory. In individual figures, data of about 1000 waves are plotted. The solid lines are the shoaling coefficient $K_{\rm s}$ of the small amplitude wave theory. Since the scatterings in the data are considerably large, the averaged relations of the above properties were compared with the theory. The figures in Fig.7 show the averaged relation between H/H_0' and d/L_0 . The circles are the averages of about 100, which distribute around them, and plotted individual mean positions. The vertical line attached to each circle shows the range of its standard deviation. The top figure (a) is for the data of $0.015 < H_0'/L_0 < 0.025$. The solid line shows the shoaling coefficient of the linear wave theory and the chain line also shows the coefficient of Shuto's theory (1974) which is introduced on the basis of the K-dV equation. The dotted line is the breaking condition for irregular waves newly proposed in section 3. The breaking condition for the 1/30 slope is representatively shown in the figure. Other breaking conditions for different slopes are very close to this line. The middle (b) and bottom





Fig.7 Averaged relations between ${\rm H/H}_{\rm O}{\rm '}$ and ${\rm d/L}_{\rm O}$

(c) figures are for the data of $0.035 < H_0'/L_0 < 0.045$ and $0.055 < H_0^{-1}/L_0 < 0.065$ respectively. The dotted lines are also the new breaking conditions for the slope of 1/30. No averaged data is plotted left of the dotted lines despite the slopes and H_0'/L_0 . For all slopes, a decreasing nature of plotted data with increasing wave steepness is observed. Furthermore data on the mild slope are slightly larger than those on the steep slope, especially in figure (a). However, the differences between the data and the theory are small (about 10% of the linear wave theory at most). Wave steepness may give a small effect on K_g. Especially, in the case of middle figure (b) $(H_0'/L_0=0.04)$, in which the major portion of the data is included in this study, agreement between data and the theory is good despite the slope. The nonlinearity in the wave height change is compared with Shuto's theory in this study. His theory branches from the linear wave theory at Urcell number 30. The nonlinearity in the wave shoaling becomes, therefore, prominent from this point. However, the branch point and the intersections between the new breaking condition and solid and chain lines are very close to each other. This means that irregular waves break and start to decay before the difference between the linear and non-linear wave theories becomes prominent. For the waves with small steepness, however, the interval between the branch point and intersections becomes wider. For example, the distance is wider in figure (a) than in (c). However, the steepness of a significant wave in a fully developed sea condition distributes around 0.03 (Goda,1975b). Thus, we may conclude that the shoaling property of the zero-down-



(a) 1/10





Fig.8 Wave height change after breaking

cross waves can be approximated by the linear wave theory as far as the present definition of wave breaking is applied and where the sea condition is a fully developed one. However, for waves with a smaller steepness than $\rm H_0'/L_0=0.01$ the non-linear property in the shoaling may not be neglected in a very small region of $\rm d/L_0$. New investigations on the wave shoaling for zero-down-cross waves may be necessary.

5. WAVE HEIGHT DECAY AFTER BREAKING

The same measurement system as used in the former sections was applied in the experiments, although the whole system was shifted onto the shore side on the slope. The breaking points, at first, were read off from the picture recorded in the video-tape. The corresponding zerodown-cross wave heights and periods at the wave gauge points were also read off from the wave profiles stored in the floppy discs. The figures in Fig.8 show the wave height after breaking on the slope of (a) 1/10, (b) 1/20 (c) 1/30 and (d) 1/50, in which H_b and d_b are the wave height and water depth at the breaking point. Measured wave height decay of about 1000 are plotted in individual figures. The new definition of water depth also had a significant effect in reducing the scattering of the plotted data. To examine the mean tendency in detail, the data of every 0.1 interval of d/d_b were averaged and plotted in their mean position. Figure 9 shows the averaged relation between H/H_b and d/d_b on the slope of 1/30. The plotted data increase even after the breaking point. This is mainly due to the definition applied in this study since no practical energy dissipation takes place and wave height keeps increasing nature until it reaches the plunging point. There have been many studies which have adopted the maximum wave height to be the breaking wave height. However, this definition is difficult for practical measurements and the wave property at this point has not been clarified yet.



Fig.9 Averaged relation between H/H_h and d/d_h (1/30)

After reaching its maximum, the wave height starts to decay. The mean relations between H/H_b and d/d_b shore-side from this point are not uniform but have bends in all cases at around $d/d_b<0.6$. Around the bend, the appearance of stable bores were observed in the experiments. Therefore, the process after breaking is divided into three different regions in this study. They are the shoaling region (from the breaking point to the point of maximum wave height), the plunging region (from the point of maximum wave height to the bend) and the bore region (from the bend to the shore). Although, the shoaling region reduces with decreasing slope angle, the data lies on straight lines approximately in individual regions despite the slopes. The wave height changes in these regions are approximated by straight lines in this study. Figures 10 (a) 1/10 and (b) 1/30 show the approximated regression lines for different wave steepnesses classified in the figure. Wave steepness brings a difference in the regression lines. This property is prominent in figure (a), but reduces in (b) considerably and becomes negligibly small in the case of 1/50 (although the figure is not shown in here). As explained in section 4, zero-down-cross wave steepnesses distribute around 0.03 when the sea condition is a fully developed one. In the region between $H_0'/L_0=0.01$ and 0.06, the difference in wave height change with respect to the steepness is small even in the case of 1/10. Therefore we may conclude that the effect of wave steepness on the wave height change after breaking is small if the sea condition is a fully developed one. Figure 11 shows the regression lines and averaged data for all cases. The regression lines are given by the equation,

$$H/H_{\rm h} = A \left(d/d_{\rm h} \right) + B \tag{4}$$

in which A and B are the constants given as follows in individual regions. (Subscripts S,P and B show that the values are for the shoaling, plunging and bore regions respectively.)



Fig.10 Effect of the wave steepness

Shoaling region:

$$A_{\rm S} = -0.01 \tan \theta - 0.37$$

 $B_{\rm S} = 1.0 - A_{\rm S}$
(5)

Plunging region: ($(H/H_b)_{bore}$, $(d/d_b)_{bore}$; see eq.(9))

$$A_{\rm P} = 1.65 \exp(-2.3 \tan \theta)$$

$$B_{\rm P} = ({\rm H}/{\rm H}_{\rm b})_{\rm bore} - A_{\rm P}({\rm d}/{\rm d}_{\rm b})_{\rm bore}$$
(6)

Bore region:

$$A_{\rm B} = 0.72 \exp(6.11 \tan \theta)$$

 $B_{\rm B} = 0.02$ (7)

The water depth and the wave height at the boundary between the shoaling and plunging regions (waves reach their maximum around this point) are given by

$$(d/d_b)_{peak} = (B_p - B_S)/(A_S - A_p)$$

(H/H_b)_{peak} = (A_p B_S - A_S B_p)/(A_p - A_S) (8)

The water depth and wave height at the bend (the bore appears around this point) are given by

$$(H/H_b)_{bore} = 0.67 \exp(-3.1\tan\theta)$$

$$(d/d_b)_{bore} = \{ (H/H_b)_{bore} - B_B \} / A_B$$
(9)
$$(d/d_b)_{bore} = \{ (H/H_b)_{bore} - B_B \} / A_B$$
(9)
$$H/H_b$$

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Fig.11 Regression lines for the wave height change after breaking

In the definition of eq.(5), data from a slightly offshore-side of the breaking point are applied in the calculation.

On the shore-side of the breaking point, wave height change is affected by the mean water level change such as wave set-up and surf beat. The new definition applied in this study therefore determines the water depth as a sum of the still water depth and the (long period) mean water level displacement(s), consequently those effects of the (long period) mean water level changes are not included in these equations. In the practical application of these equations, the long period water level changes (Goda, 1975b) should be introduced, although their properties are not investigated in this study.

6. CONCLUDING REMARKS.

The properties of zero-down-cross irregular waves on uniform slopes are examined experimentally. About 4,000 irregular waves of the Pierson-Moskowitz spectrum are analyzed in the study. The followings results were obtained:

- (1) The shoaling characteristics can be approximated by the linear wave theory.
- (2) $H_{\rm b}/d_{\rm b}$ is about 30% smaller than that for periodic waves.
- (3) The process after breaking can be divided into three regions i.e. the shoaling, plunging and bore regions.
- (4) The experimental equations throughout the process are given.
- (5) A new definition of water depth which can largely reduce the fluctuation in the plotted data is proposed.

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