CHAPTER 13

Wave Groups on Wave Records Measured Along Spanish Shores

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Abstract

A large amount of wave records measured at four different locations of the Spanish shores have been analysed from the point of view of wave group formation. The influence of wave climate and depth on the results is studied. The results are compared with existing theories for runs of waves, SIWEH and envelope functions. Characteristics of extreme groups are also considered.

1 Introduction

It is generally recognized the importance that the presence of wave groups can have in the behaviour of coastal and offshore structures and consequently many studies have been carried out trying to determine the grouping characteristics of ocean waves.

As a consequence of these studies, different theories have been proposed to explain the formation of groups among wind generated waves. These theories try to relate the groupiness characteristics observed in the waves with other better known wave properties (spectral parameters, zero crossing statistics and others).

All of them could be divided into two main groups: those which defend that the formation of groups can be explained by pure linear interactions and those which propose the contrary. No definitive conclusion has been reached yet (Marón, 1986).

Among the linear methods we have selected those corresponding to the statistics of runs of high waves (Goda, 1976 and Kimura, 1980), the SIWEH function (Funke & Mansard, 1980) and the envelope theory (Longuet-Higgins, 1984) as well as different modifications introduced by several authors. Most of the studies have deal with simulated records and only a few have analysed measured records and generally not in a systematic basis.

For this study we have implemented a computer program which calculates all the parameters for wave groupiness proposed in the previously mentioned works as well as others relating to extreme events which we think that can be of

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interest in design. This program was run on a large amount of wave records measured in four different locations of the Spanish shores by means of moored waverider buoys.

We intended in this way to evaluate the ability of the different theories to predict the observed results. At the same time we tried to see if the groupiness characteristics were affected directly by local maritime climate factors or in the contrary they only depend on spectral and/or statistical wave properties independently of site.

Finally, one of the buoys was moored in relatively shallow waters and therefore it can give us some indication on how shoaling could affect the group behaviour.

### Description of wave data

The main characteristics of the four buoys used in the analysis are presented in Table I.

<table>
<thead>
<tr>
<th>Buoy</th>
<th>Net</th>
<th>Sea</th>
<th>Depth</th>
<th>No. of points</th>
<th>No. of records</th>
</tr>
</thead>
<tbody>
<tr>
<td>BILBAO</td>
<td>REMRO</td>
<td>Cantabric</td>
<td>50</td>
<td>5120</td>
<td>2605</td>
</tr>
<tr>
<td>CARTAGENA</td>
<td>REMRO</td>
<td>Mediterranean</td>
<td>70</td>
<td>5120</td>
<td>4420</td>
</tr>
<tr>
<td>SEVILLA-3</td>
<td>Special</td>
<td>Gulf of Cádiz</td>
<td>10</td>
<td>x</td>
<td>613</td>
</tr>
<tr>
<td>SEVILLA-4</td>
<td>Special</td>
<td>Gulf of Cádiz</td>
<td>38</td>
<td>x</td>
<td>1327</td>
</tr>
</tbody>
</table>

* Variable for each record between 675 to 1500.*

Their locations can be seen in figure 1. Two of the buoys belong to the REMRO Net (Spanish Net for Wave Recording and Measurement) owned by the Spanish Port and Shore Authority. The other two buoys were deployed by the Sevilla Harbour Authority for a special campaign. The records from the first two buoys are specially adequate for group analysis because of their long length of 5120 points at a sampling rate of 2 Hz.

Regrettfully the other two sets of records are of very short and variable length which increases the variance and the uncertainty in the results. Although they are not the best buoys for group analysis, they were the only available
case of two buoys in different depths but at very near positions.

The BILBAO buoy is located in a place characterized by strong storms and frequent presence of long swell coming from the North Atlantic. The CARTAGENA buoy is located in the Mediterranean Sea and therefore the waves are shorter and steeper than in the previous one, also the possibilities of swell are very reduced. The two SEVILLA buoys are moored in the Atlantic Ocean but due to their positions, they are protected from most of the swell and bigger storms.

A quality control program has been applied to all the records in the study. Any record considered incorrect or suspected to be so was rejected for the analysis. A correction to allow for the transfer function of the buoy electronics was applied to the records. This transfer function affects mainly the phases of the Fourier components of the wave record and therefore it can have a great influence in the results as far as groupiness is concerned.

3 Statistics of runs of high waves

The length of a run of high waves (R) is the number of consecutive waves that are higher than a given threshold. The corresponding length of group (L) is the number of consecutive waves from the beginning of a run to the beginning of the next one. The wave height is defined by the zero upcrossing method rejecting waves lower than 10% of the standard deviation or shorter than one second.

Goda (1976), assuming no correlation between consecutive wave heights, proposed the following distributions for R and L:

\[
R(i) = Pr(R = i) = P_0^{t-1}Q_0 \quad \text{and} \quad L(i) = Pr(L = i) = \frac{P_0Q_0}{P_0 - Q_0}\left(P_0^{-1} - Q_0^{-1}\right)
\]

where \(P_0\) and \(Q_0\) are the exceedence and nonexceedence probabilities for the given threshold. As a consequence the respective mean values are given by:

\[
R = \frac{1}{Q_0} \quad \text{and} \quad L = \frac{1}{P_0} + \frac{1}{Q_0}
\]

The exceedence probabilities can be estimated theoretically by assuming a Rayleigh distribution for the wave heights.

As an example, figure 2 compares these theoretical results with the measured mean distributions for R and L for the threshold at the mean wave height and for records with wave height correlations below 0.1 and between 0.4 and 0.5. Here and in the following the curves give the theoretical values, the solid dots are the results for BILBAO, the open
dots the results for CARTAGENA, the solid squares are those of SEVILLA-3 and the open squares those of SEVILLA-4.

*Fig. 2.* Mean distributions of group and run length, compared with Goda's theory.

It can be seen that the theory is quite good for records with low or no correlations but the lengths are underpredicted for records with high correlations, results which are coincident with those from other authors. The comparisons when other thresholds are considered are very similar.

It should be kept in mind that these and the following results are in general accompanied by a big dispersion about the mean, specially for the shorter records of the SEVILLA buoys, but these dispersions are not considered in this study for simplicity.

*Fig. 3.* Probability of exceedence as a function of wave height correlation, compared with Rayleigh distribution.
between successive wave heights is high. In figure 3, it can be seen that the exceedence probabilities are overpredicted for the mean height and underpredicted for the significant wave height at high correlation values.

Kimura (1980) introduced the correlation between successive wave heights in the formulation, resulting in the following equations:

\[ R(i) = (1 - P_{22})P_{11}^{i-1} \]
\[ L(i) = (1 - P_{11})(1 - P_{22})^{i-1}P_{22}^{i-1} \]

Here the probabilities \( P_{11} \) and \( P_{22} \) are defined as:

\[ P_{11} = Pr(H_i \leq H_0 | H_{i-1} \leq H_0) \]
\[ P_{22} = Pr(H_i > H_0 | H_{i-1} > H_0) \]

and \( H_0 \) is the threshold considered. These probabilities can be calculated assuming a two dimensional Rayleigh distribution for the joint distribution of successive wave heights. This distribution depends on a shape parameter \( k \) which can be related to the statistical or spectral properties of the records in different ways. In this study three different formulations have been investigated: Uhlenbeck (1943) equation relating the correlation between successive wave heights and the shape parameter, Longuet-Higgins (1984) relation between \( k \) and the spectral shape and Battjes (1983) equation based in the correlation between the squares of the wave heights. The comparisons between them have shown some superiority for the first approach in predicting the results although the differences are not very important. In the following the first equation is used.

Figures 5 and 6 compare Kimura's theory with the measurements for two different thresholds. Here two theoretical curves are plotted for each distribution, corresponding to the lower and upper bound of the
correlation. The agreement is very good except for the SEVILLA's buoys at high correlation but this is due to the fact that only a small number of records has been found with this correlation values and therefore the statistical variability is very high.

Fig. 5.- Mean distributions of group and run length for $H_{\text{mean}}$, compared with Kimura's theory. Upper plot is for high correlation and lower plot is for low correlation values.

Fig. 6.- Same as figure 5 but for $H_{\text{m0}}$. 
The mean run and group length can be calculated from the given distributions:

\[
\frac{1}{1 - P_{22}}
\]

\[
\frac{1}{1 - P_{11}} \cdot \frac{1 - P_{22}}{1 - P_{11}}
\]

These are compared with the results in figures 7 and 8. The agreement is very good except for a small overprediction of the mean group length at the significant level. This could be due to an overprediction of P11 at this level. The error is in any case below 10%.

Ewing (1973) gave another formulation for the mean run length based in the envelope theory and whose main result can be written as:

\[
R = \sqrt{\frac{m_2}{2\pi \mu_2 H_0}}
\]

where the \( m_i \) are the spectral moments of order \( i \), \( \mu_2 \) is the second order central moment of the spectrum and \( H_0 \) is the threshold level. Here the length of run is counted in a continuous basis and should be corrected for discrete counting in order to be comparable with the zero crossing measurements. This can be done by using the following relationship:

\[
R = \frac{1}{1 - \exp\left(-\frac{1}{\mu_2}\right)}
\]

From the previous equation an asymptotic expression valid for narrow banded spectra can be found:

\[
R = Q \sqrt{\frac{2m_0}{H_0}}
\]
where $Q_p$ is the peakedness parameter proposed by Goda.

Figure 9 gives the measured values plotted against the first factor of Ewing's equation. The results compare very bad with the theory and are quite different for each buoy. This can be explained by the fact that the parameter considered is uncorrelated with the wave height correlation as can be seen in figure 10. Another evidence of this is that if one computes this factor for a Jonswap spectrum the result is independent of the peak enhancement factor while it is clear that more peaked spectra give longer runs.

If one filters the records (see the envelope section later in the paper) the correlation between the measurements and Ewing's factor is much better but still far from the theory (figure 11).
The asymptotic equation gives acceptable results only for high values of Qp (figure 12). Here the Qp calculated from the raw spectra have been corrected to account for the bias in the estimator (dividing by 2). It seems from the results that Ewing's theory will only be valid for spectra narrower than those found in nature.

Anyway, the peakedness parameter is a good indicator of groupness as can be deduced from figure 13, where a good relationship is observed between it and the wave height correlation.

This is specially so for the BILBAO and CARTAGENA buoys which are the more reliable ones. The relation for these two buoys can be approximated by:

\[ y = 0.28 \ln Q_p + 0.10 \]

4 Analysis of SIWEH function

Funke & Mansard (1980) proposed the SIWEH function (Smoothed Instantaneous Wave Energy History) as a mean to recognize the presence of groups in the wave records. This function is obtained by convoluting the square of the signal with a triangular window of length equal to the spectral peak period. In the present study the convolution is actually made in the frequency domain.

The same authors have proposed two parameters obtained from the SIWEH in order to characterize the group formation.
They are the groupiness factor and the mean SIWEH period which are given by:

\[ GF = \frac{1}{m_0} \sqrt{\frac{1}{T_0} \int_0^{T_p} [E(t) - \bar{E}]^2 dt} \]

\[ T_{SIWEH} = \frac{1}{T_p} N_s \sum_{i=1}^{N_s} T_{Si} \]

where \( E(t) \) is the SIWEH function, \( \bar{E} \) its mean value, \( T_0 \) is the length of the records, \( T_p \) is the spectral peak period, \( N_s \) is the number of zero upcrossings of the SIWEH and \( T_{Si} \) are the corresponding zero upcrossing periods.

Figure 14 presents the observed relation between these two parameters. The result is the same for the four buoys and can be approximated by the straight line:

\[ T_{SIWEH} = 3.92GF + 2.55 \]

Fig. 14.- SIWEH period/groupiness factor.

Also the mean period of the SIWEH is related to the wave height correlation (figure 15) in a similar way for the four buoys which can be approximated by:

\[ T_{SIWEH} = 8.7y^2 - 0.4Sy + 4.3 \]

On the contrary, figure 16 indicates that the relationship between the groupiness factor and the correlation varies with the buoy considered. This relation approximates straight lines with the same slope but different intercepts.
Fig. 17. SIWEH period/mean group length.

Figure 17 indicates that a linear relation exists between these two quantities but again it depends on the buoy considered. The two SEVILLA buoys behave exactly in the same way and the same happens with BILBAO and CARTAGENA buoys.

5 The envelope function

The envelope of all the records has been computed by using the complex pre-envelope function. Following a recommendation by Longuet-Higgins (1978) the same has been made for filtered records obtained by suppressing all the frequencies in the raw signal below .5 the spectral peak frequency and above 1.5 the same frequency.

In the first place we looked at the effect of this filtering in the characteristics of the signal and we observed that while the total variance is only slightly reduced and other spectral and statistical parameters are not significantly changed, the spectral width suffers a drastic change.

This will result in completely different group behaviours as predicted by the envelope theory. In figure 18 the bandwidth (measured by , defined later) after filtering is compared with the raw record value.

Fig. 18. Comparison between the bandwidth after and before filtering.
Both bandwidths are uncorrelated or even negatively correlated for the BILBAO buoy.

A level crossing analysis of the envelope has been performed at different levels. From it a mean run and group length can be computed. The theory predicts their values as given by:

\[ R = \frac{1}{\sqrt{2\pi}} \sqrt{\frac{1 + \nu^2 2\sqrt{m_0}}{\nu H_0}} \]

\[ L = \frac{1}{\sqrt{2\pi}} \sqrt{\frac{1 + \nu^2 2\sqrt{m_0}}{\nu H_0}} \exp\left(\frac{H_0^2}{8m_0}\right) \]

Here \( H_0 \) is two times the level considered and \( \nu \) is a measure of the spectral bandwidth given by:

\[ \nu = \sqrt{\frac{m_2 m_0}{m_1^2} - 1} \]

The results obtained for the raw and filtered records can be seen in figures 19 and 20. The continuous lines correspond to the level which is near the mean height and the dashed lines are for which is the significant height. The measurements agree very well with the theory except maybe in some cases for the SEVILLA buoys but this could be most probably due to the statistical variability.

Only the mean group length at the significant height level seems to be overestimated by the theory for this two buoys.

A comparison of the results obtained with this procedure and those corresponding to the zero crossing
analysis was made. It was found that no relation is apparent between both results when the raw records are considered but some correspondence is found if the filtered record envelopes are considered (figure 21) although it depends on the buoy selected. This fact seems to support the need of the filtering for the envelope analysis.

Fig. 21. - Envelope/zero-crossing run length.

6 Longest group and run

Assuming that the Kimura's distribution is correct, one can easily compute the theoretical distribution of the longest run and group in a record. These will be dependent on the number of runs present in the record and therefore on the mean group length and mean wave period for records of a fixed length.

Fig. 22. - Longest group and run for $H_{\text{mean}}$ and $Hm_{0}$ and different correlations.
From these distributions the expected values can be estimated. Figure 22 presents the results obtained for the means of the longest run and group adimensionalized by the respective means compared with the values predicted by the theory. Two threshold levels have been considered. The theoretical values have been numerically approximated on the computer because no closed form was found for the resulting expressions. As the distribution depends on the wave correlation, the records have been grouped into classes of correlation and two cases are presented in the figure representative of low and high correlations. The two theoretical curves correspond to the lower and higher values of each class.

The agreement with the theory is quite good although the observed scattering is somewhat higher than in the previous results, but this is always to be expected when dealing with extreme order statistics.

7 Conclusions

* The theory developed by Kimura predicts very well the group behaviour of waves. The correlation between successive wave heights seems to be the best indicator of groupiness. Some discrepancies are found between the observations and the Rayleigh distribution for wave heights.

* The formalations proposed by Ewing are not correct for the range of values observed in nature.

* The groupiness factor and mean period of the SIWEH are good indicators of group characteristics but the results depend in some cases on the buoy considered and this is probably due to differences in the wave correlations at the different sites. Therefore, it seems that some information on groupiness is lost with these parameters.

* The envelope theory performs quite well. A previous filtering of the signal seems to be needed in order to get results coherent with other analysis methods.

* No significant difference was found between the buoys that could not be explained by differences in the correlation and therefore it can be concluded that the group formation is independent of the local maritime climate and depth variation for the depths considered here.

8 Acknowledgments

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References


