

CHAPTER 7

ON THE TRANSFORMATION OF WAVE STATISTICS DUE TO SHOALING

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ABSTRACT

The results of a 1:40 scale physical model investigation into the shoaling process are described. The model simulated a nearly constant slope of 1:40 with wave measurements made at a depth of approximately 25 and 9 m. Two hundred individual tests were undertaken, with four offshore significant wave heights as the only test variant. The results indicate that the most severe nearshore wave conditions do not occur with the worst offshore conditions. There is evidence of a significant increase in low frequency wave energy in the nearshore zone.

INTRODUCTION

For the laboratory design of coastal structures in depth-limited situations, it is customary to simulate a pre-specified sea state in the offshore region of a model and let the shoaling take its natural course on a bathymetry similar to that found in nature. The final design of the structure is then related to that offshore sea state and its probability of occurrence. Coastal engineers usually design their structures to withstand 1 in 100 year storm conditions.

The transformation of wave statistics due to shoaling has been an active field of research for a number of years. However, the complex physics of this phenomenon is not thoroughly understood. An extensive series of model investigations were carried out recently, for the purpose of gaining a better understanding of this phenomenon. This study included only one bathymetry and only one offshore water depth. Only one spectral shape with only one peak frequency was used. Waves were only recorded at two locations. In this sense, the investigation was simply a case study.

This research was motivated by the discovery of certain unexplained inconsistencies during a commercial model study. It was therefore undertaken to obtain a better insight into

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the shoaling phenomenon and to quantify the wave parameter statistics that may result from shoaling. Some of the results have previously been reported by Readshaw et al (1987). Further results are presented here. Therefore, this paper complements the information contained in Readshaw et al (1987) and is intended to provide the basis for further comprehensive research.

The model investigation has shown that, for a given offshore sea state described solely by the variance spectral density, the severity of the nearshore wave climate can be grossly misjudged. Although this research has led to several useful conclusions, which are stated below, it must be expected that other test conditions for wave period, water depth, bathymetry and perhaps wave direction may lead to different results than those reported here. Nevertheless, it is believed that the conditions for this investigation are typical and the results will give a general indication of what may be expected for other situations.

EXPERIMENTAL SET-UP AND TEST WAVE CONDITIONS

Figure 1 shows a sketch of the experimental set-up. Waves were allowed to shoal from a depth of approximately 25.2 m (full scale units) to a depth of 8.6 m, where a breakwater was to be constructed. For convenience, the deeper region will be referred to here as the offshore zone and the other as the nearshore zone. The bathymetry that was used in this study had a nearly constant slope of approximately 1:40. This corresponded to the conditions in nature after which the study was modelled. The scale factor of the model was 1:40.

Waves were generated from numerically synthesized data derived from a JONSWAP spectral density with a peak period of 16.6 s and a γ value of 3.3. The method of wave synthesis was the "Random Complex Spectrum" method³ described by Funke & Mansard (1984). This method of synthesis creates wave trains from a Gaussian distributed white noise complex spectrum which is filtered by the specified target spectrum. It is known that it produces wave trains which mimic the variability of the natural sea state. Fifty different time series of 20 minute duration (full scale) were synthesized from a common JONSWAP target spectrum. Each of these was rescaled in four different ways, creating therefore four different sets of wave generator command signals, one for each offshore significant wave height of 4.8 m, 6.3 m, 7.8 m and 9.3 m. The rescaling was adjusted for each individual wave record to ensure that the desired variance was achieved near the wave board. As a result, 200 command signals were synthesized originating from a common spectral shape with common peak frequency, but differing only in their variance. Each set contained therefore 50 time series which were identical to those in the other sets in every respect except the amplitude scale. This is an important point, because the only parameter which was varied for the purpose of this study was the offshore significant wave height H_{m0} .

³ Miles and Funke (1988) refer to this method as the "Random Fourier Coefficient" method. A description of its statistical characteristics is also provided in their publication.

ANALYSIS OF WAVE DATA

All wave data was subjected to spectral density and zero-crossing analysis. From the spectral density the following parameters were obtained:

- f_{pD} - the peak frequency according to the Delft method (IAHR List of Sea State Parameters, 1986). It was computed as the centroid of the part of the spectrum which is delimited by its first and its last crossing of a threshold that is 80% of the spectral peak value.
- H_{m0} - the estimate of the significant wave height which was computed as $4 \cdot \sqrt{m_0}$, where m_0 was obtained by integration of the spectral density from $0 < f < 0.5$ Hz. This corresponded, in this case, to an upper limit of $8.5f_p$ and was for all practical purposes an upper limit of infinity. For reasons to be explained later, this parameter was also evaluated for the integration limits of $0.03 < f < 0.5$ and $0.03 < f < 0.15$ Hz.
- $m_{0,LW}$ - the variance of the long waves which was computed by numerical integration of the spectral density from $0 < f < 0.03$ Hz. This corresponded, in this case, to an upper limit of $0.5f_p$.

From zero crossing analysis the following parameters were obtained:

- $H_{1/3}$ - the significant wave height, computed as the average of the highest one-third of all zero down and zero up-crossing waves.
- $H_{1/10}$ - the average one-tenth of all wave heights from zero down and zero up-crossing analysis.
- $H_{1/20}$ - the average one-twentieth of all wave heights from zero down and zero up-crossing analysis.
- H_{max} - the larger of the maximum zero up- or down-crossing wave.
- $\bar{s}_{H1/3,d}$ the average steepness of the significant waves. This is the average of the ratios of wave heights and wave lengths for those waves which belong to the highest one-third of all zero down-crossing waves.

RESULTS AND DISCUSSION

The results presented here correspond to waves measured at the two locations shown in the Figure 1. Four pairs of example wave records, as measured by the two probes, are shown in Figure 2. It will be noticed that the offshore wave record is given below the corresponding nearshore wave record for each of the four significant wave heights given in this example. Closer comparison between the four offshore wave records will reveal that they originate from the same time history which was scaled to yield the desired wave heights $H_{m0} = 4.8, 6.3, 7.8$ and 9.3 m. Evidently, for the higher wave heights, the offshore wave crests become more accentuated and some distortions occur because of breaking. Nevertheless, except for scaling, the inputs to the four tests were the

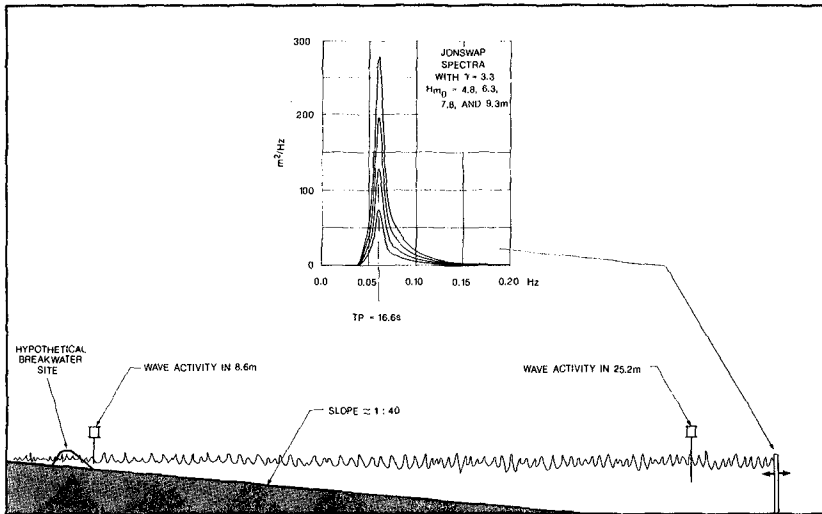


FIGURE 1
DEFINITION SKETCH OF THE EXPERIMENTAL SET-UP

same and hence this figure illustrates the influence of just the variance of the offshore wave activity on the transformation of wave profiles from deep to shallow water.

The nearshore wave records in Figure 2 were synchronized to the offshore wave trains. The arrows in the wave records point approximately to corresponding points in the two recordings, making allowance for the propagation delay. From observations during the tests, it was apparent that the wave trains on the left hand side of the figure describe the shoaling situation before breaking in the nearshore zone, whereas the right hand wave records illustrate the nearshore post-breaking state.

Figure 2 shows, as one would expect, that the nearshore wave profiles have sharper crests and flatter troughs when compared to the nearly sinusoidal profiles in deep water. Even for the lowest significant wave height, it is difficult to identify a similarity between the nearshore and the corresponding offshore wave record. However, with some stretch of the imagination, one can see a correspondence between wave groups. But, for the largest significant wave height, all meaningful relationship seems to have vanished. It can be speculated that, as the variance of the offshore sea state increases, the individual nearshore wave profiles steepen up much more quickly during shoaling and become unstable. This results then in the breaking of waves and their subsequent reconstitution before they reach the hypothetical breakwater site.

Another interesting observation is the skewness of waves in the nearshore zone. This is more pronounced for large offshore wave heights. It is speculated that this is the result

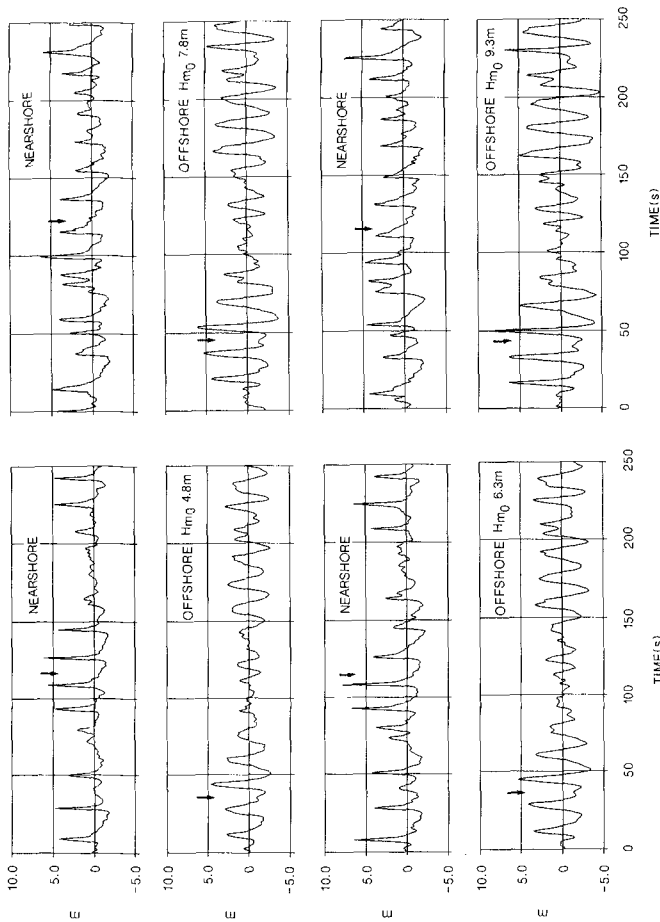


FIGURE 2
SAMPLE TIME SERIES MEASURED BY THE TWO PROBES
FOR THE FOUR SIGNIFICANT WAVE HEIGHTS (H_{m0})
OF THE TARGET SPECTRUM

of an interaction between the shoaling waves and the return currents from the beach.

The spectral densities of the eight wave trains in Figure 2 are shown in Figure 3 with each corresponding pair of spectra for a nearshore and an offshore case superimposed on the same graph. From this it can be seen that, in spite of a substantial increase in the variance of the offshore sea state, there is not an equivalent increase in the variance in the nearshore sea state, evidently because of energy loss due to breaking. On the other hand, as a result of shoaling, the nearshore spectra indicate a significant increase in the variance for the low frequency range (i.e. $0 < f < 0.03\text{Hz}$). It can also be noted, that the peak frequencies of the nearshore spectra tend to be lower than those of the offshore spectra. This frequency shift can be partly attributed to possible transfer of energy from high to low frequencies during the

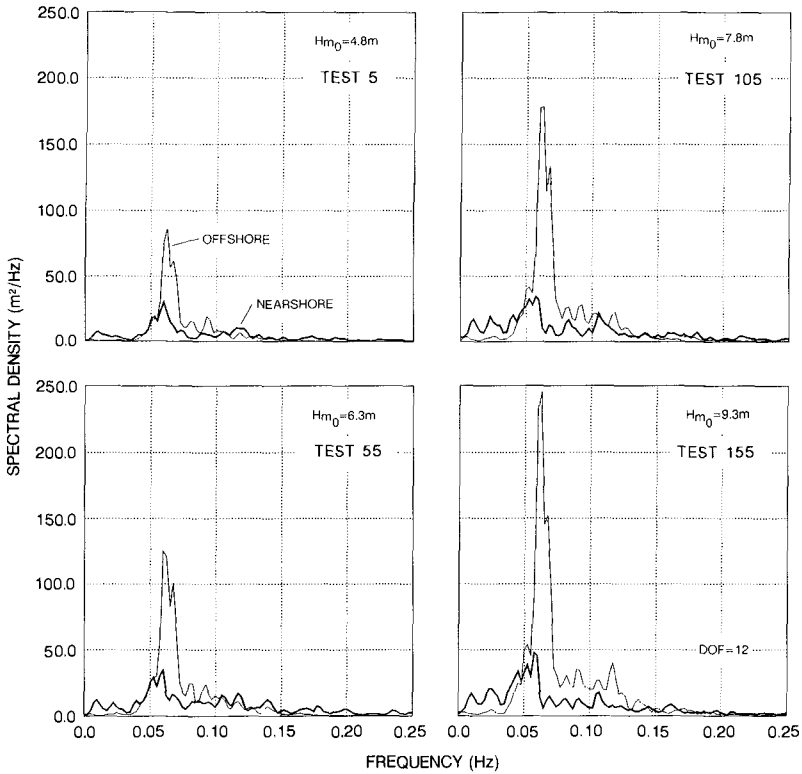


FIGURE 3
SPECTRAL TRANSFORMATION ON A 1:40 SLOPE
FOR THE FOUR SIGNIFICANT WAVE HEIGHTS (H_{m0})
OF THE TARGET SPECTRUM

breaking process. In general, the nearshore spectra are much broader than their offshore origins.

Figure 4 shows the results of the analysis for the offshore significant wave height of 9.8 m. The time series given in this figure, are the various wave parameters from the fifty offshore and nearshore wave records placed in sequential order.

It can be seen from this figure, that there is no significant correlation between offshore and nearshore wave parameters. Except for the peak frequency and the long wave energy, the variability of wave parameters is approximately the same. It is also evident that there is generally a significant increase in long wave energy as a result of

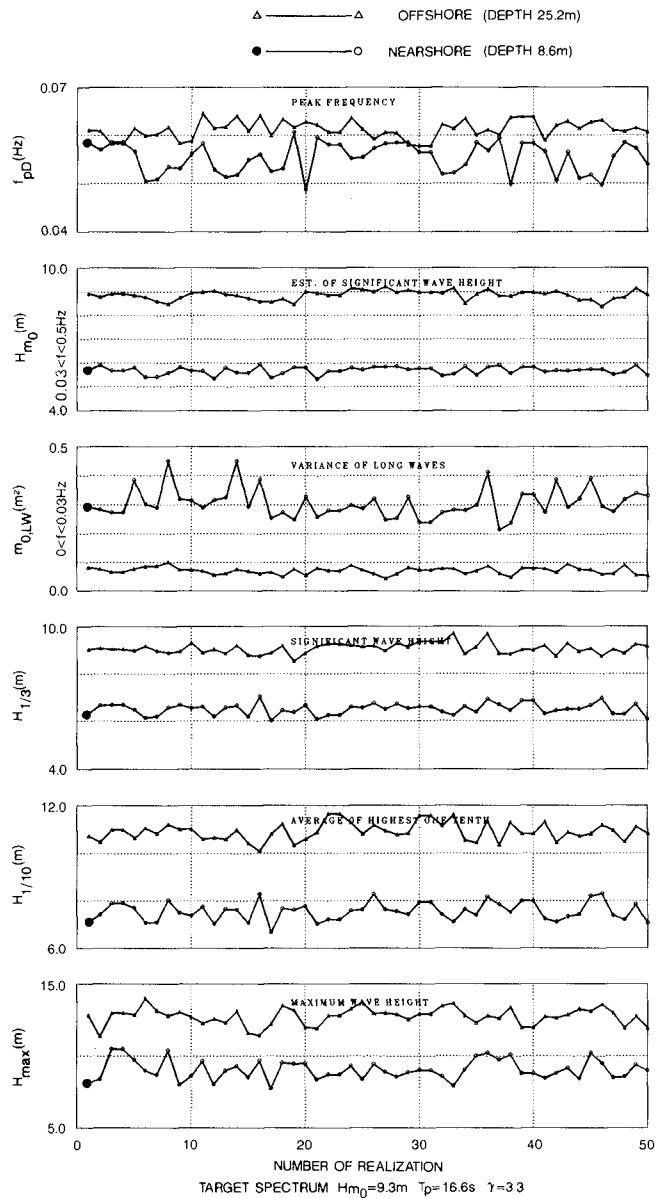


FIGURE 4
SELECTED WAVE PARAMETERS , OFFSHORE AND NEARSHORE ,
FOR 50 DIFFERENT REALIZATIONS OF THE TIME SERIES

shoaling. Although the command signals to the wave machine were rescaled to eliminate the variability of the offshore significant wave height, it is apparent that at the probe location 11 m from the wave board some variability has re-established itself. This is partly due to breaking and non-linear effects.

NON-LINEARITIES

The Laplace equation with proper boundary conditions is most often used for describing waves. Solutions of this equation can be obtained by the perturbation technique and waves are therefore of infinite order. However, for most practical applications, the first order solution to this equation is found to be quite satisfactory. But in shallow water, where the boundary conditions are principally non-linear, this equation would have to be solved to a higher order and, during this, two important phenomena will appear. These are the bounded sub- and the super-harmonics (Mansard et al 1988). The wave components resulting from these non-linearities travel with velocities bound to their fundamentals, while other components satisfy the linear dispersion relation. Barthel et al (1983) and Sand & Mansard (1986) show that classical first order wave generation theory does not properly satisfy the boundary conditions needed for the correct reproduction of these waves and therefore some spurious components of similar frequencies will appear in the simulations. This theory can be used to estimate the frequencies and their amplitudes as a consequence of shallow water boundary conditions, and permits therefore the prediction of spectral distortions.

Figure 5 shows two arbitrarily selected sample spectra measured by the first probe when using the target spectrum with a significant wave height $H_{m0} = 7.8$ m. On the left hand side of this figure, a comparison is shown between the expected spectral density for this particular realization and the one which was actually measured. The expected spectral density corresponds here to the one synthesized by the random complex spectrum, which is of course based on linear superposition of frequency components. It can be seen from this figure, that the measured energy content in the two frequency ranges of $0 < f < 0.03$ Hz and $0.1 < f < 0.5$ Hz are distinctly higher than expected. However, by applying second order wave and wave generation theory to the prediction of the wave spectra at this site, the agreement with the measured spectra is much improved as is shown on the right side of Figure 5.

RELATIONSHIP BETWEEN OFFSHORE AND NEARSHORE WAVE PARAMETERS

Examples of the relationship between offshore and nearshore wave heights are illustrated in Figures 6 and 7 for the four different offshore wave height parameters. In these illustrations a nearshore wave height parameter is plotted against the same offshore parameter.

In order to reduce the variability of wave height estimates as computed from 20 minute long records, the data were also analyzed by joining them first into blocks of 3 and 6 to give 1 and 2 hour averages of wave parameters. Figures 6 and 7 provide therefore the results for each of four different wave height parameters, calculated for three different averages.

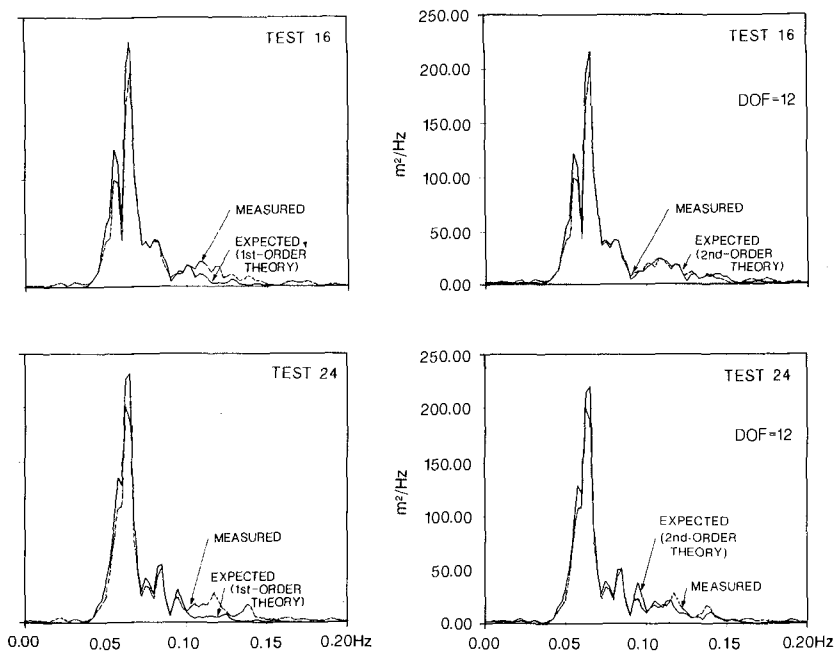


FIGURE 5
NON-LINEAR PREDICTION OF WAVE SPECTRA

$H_{m0}=7.8\text{m}$ $T_p=16.6\text{s}$ $h=25.2\text{m}$

DISTANCE FROM PADDLE = 11m

As one would expect, it can be seen from these results that the variability does decrease with increasing record lengths. The 2 hour averages illustrate quite clearly the general trend in the shoaling process. It will be seen that the nearshore wave heights do not increase with increasing offshore wave height. There is also an indication that for three of these parameters there is a slight reverse trend in as much as the nearshore wave height seems to decrease for very large offshore wave heights.

Figure 8 presents the data in a different mode. Following a statistical analysis of nearshore wave height parameters obtained from 20 min records, the extrema, the average as well as the average $+1$ and -1 standard deviation were plotted against the offshore significant wave height H_{m0} . This was also done for the variance of the long waves and the steepness of the significant waves.

RELATIONSHIP BETWEEN FREQUENCY AND TIME DOMAIN ESTIMATION OF SIGNIFICANT WAVE HEIGHT

In deep water, the significant wave heights derived either by zero crossing analysis or by spectral density analysis are

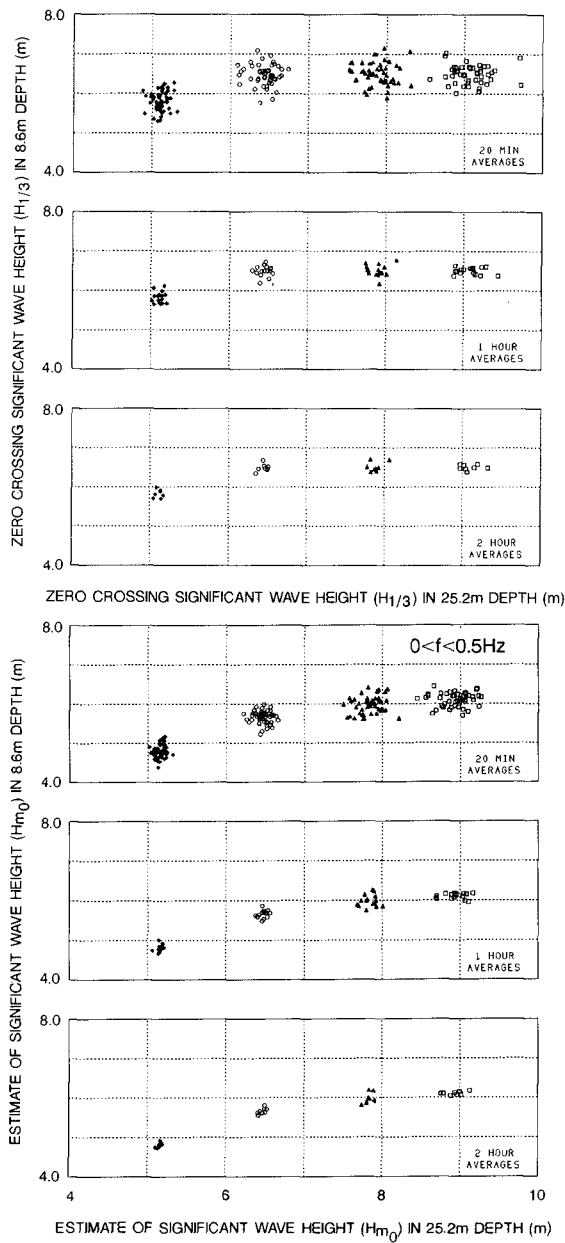


FIGURE 6
VARIABILITY OF SIGNIFICANT WAVE HEIGHTS

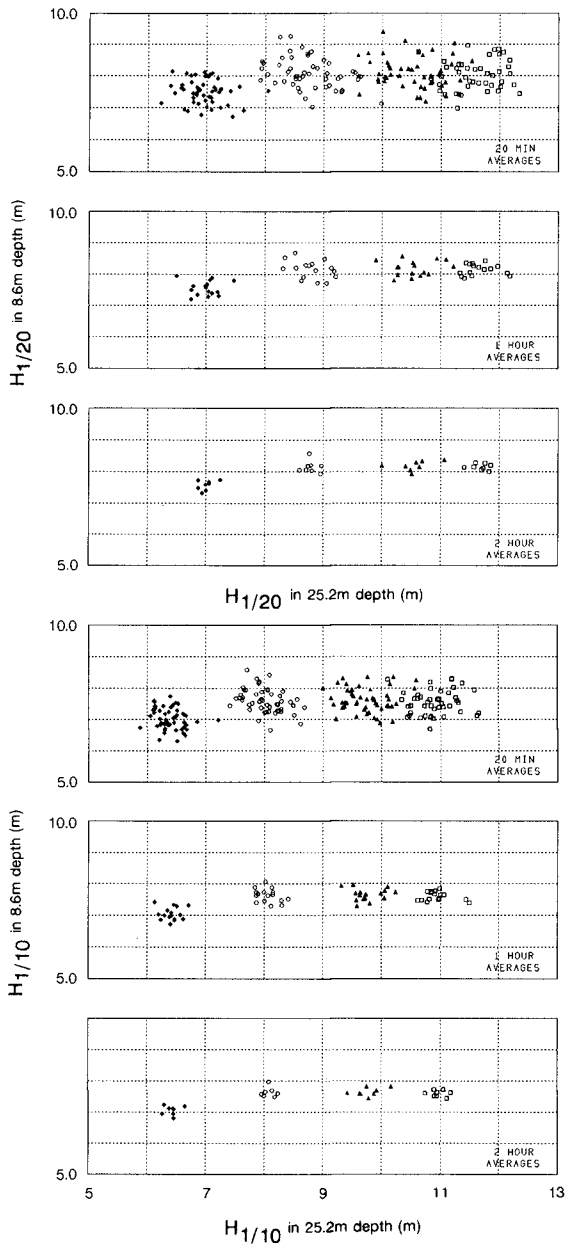


FIGURE 7
VARIABILITY OF $H_{1/10}$ AND $H_{1/20}$

proportional to each other. This is because deep water waves are, for all practical purposes, Gaussian in character. However, in the nearshore zone, this assumption is not valid and the estimation formula for the significant wave height, $H_{1/3} \sim 4 \cdot \sqrt{m_0}$, no longer applies. There is also a real difficulty in deciding over what band of frequencies the spectral moment m_0 should be evaluated, because the nearshore spectrum contains significant energy below and above the original first order spectrum.

The left side of Figure 9 gives the average over 50 wave records of the ratio between the two nearshore significant wave heights, one as computed from zero crossing and the other as estimated from spectral analysis. For the latter, the variance of the nearshore sea state was computed with three different bandwidths, i.e.:

- $0 < f < 0.5$ Hz, which for all practical purposes, covers the range from 0 to ∞ ,
- $0.03 < f < 0.5$ Hz, which excludes the long wave frequency range, and
- $0.03 < f < 0.15$ Hz, which represents the typical first order frequencies contained between $0.5f_p < f < 2.5f_p$.

The reasons for choosing these integration limits are as follows. Waverider buoys cannot give outputs for wave periods longer than 30 seconds. Therefore, wave data recorded by them do not contain meaningful energy in the low frequency band. Furthermore, because of communication interference, it is generally presumed that spectral energy beyond $f > 2.5f_p$ must be considered background noise. By applying the same limits to this data collected in the laboratory, which are usually imposed implicitly or explicitly on natural wave data, Figure 9 may serve a useful purpose in the interpretation of some wave spectra recorded in nature.

The right hand side of Figure 9 gives the shoaling coefficient as a function of the offshore wave height. This coefficient is computed as the ratio of the average nearshore to the average offshore wave height. Once again, various estimates of the significant wave height are displayed. In case of the significant wave height estimated from spectral integration, this is done with the three different cut-off frequencies.

CONCLUSIONS AND RECOMMENDATIONS

For the conditions which prevailed during the test series, the study concluded that:

- The significant wave height in the nearshore zone derived from zero-crossing analysis is generally larger than the nearshore significant wave height estimated from the zeroth spectral moment function (see also Thompson & Vincent 1985). This is apparent from Figure 9.
- A particular nearshore wave height can be realized by a wide range of offshore sea states. This information is contained in Figure 8.
- It is possible for the nearshore zero-crossing significant wave height to be larger than the offshore significant wave

height. This is the case here for the group of offshore wave heights of 4.8 m, as shown in Figure 9.

- On the average, it may be predicted that the worst nearshore wave conditions may not be produced by the worst offshore sea states. The offshore wave conditions of moderate wave height may lead to the worst nearshore significant wave heights. In this case the worst conditions arise probably at an offshore wave height of 7.8 m (c.f. Figures 6 and 7).
- As a result of the shoaling process, there is a significant increase of long wave energy in the nearshore zone. Figures 3 and 8 attest to this.
- The nearshore spectra tend to have a larger peak period than the corresponding offshore spectra. Otherwise, there is no simple relationship between offshore and nearshore peak periods. This is indicated in Figure 3.

The results presented so far illustrate the fact that for a given offshore sea state a large combination of nearshore sea states, some severe and some less severe, can be obtained. This is a consequence of statistical variability, and it implies that in every model study of depth limited situations one cannot be absolutely sure that the worst nearshore wave conditions have been simulated for a specified offshore sea state. One obvious solution to this problem is to simulate very long wave records in physical model studies. However, this may become expensive and time consuming since design optimization of a given structure usually requires many repetitions of the same test conditions in order to optimize several design parameters.

To overcome this difficulty, the technique currently used at the Hydraulics Laboratory of the NRCC, consists of first determining experimentally the worst nearshore wave climate by testing a large number of time domain realizations for each offshore sea state. This can be done by building a structure, say a preliminary design of a breakwater, at the projected site and then by identifying which of the time domain realizations results in the maximum number of breaking waves impacting on the structure. If this approach is not practical, the various nearshore wave records obtained during the preliminary tests can be subjected to frequency and time domain analyses. For example, at the NRCC Hydraulics Laboratory the following nearshore wave parameters are computed:

- the estimate of the significant wave height, H_{m0} ,
- the variance of long waves, $m_{0,LW}$,
- the zero crossing significant wave height, $H_{1/3}$,
- the average of the highest 1/10th zero crossing waves, $H_{1/10}$,
- the average crest front steepness of the significant waves, $\bar{s}_{H1/3}$, and
- the Groupiness factor value, GF.

Those offshore time series which give consistently the worst nearshore wave conditions for most of these six parameters are then selected as being the most suitable for the design study.

None of the above techniques are entirely satisfactory because the decisions made during this approach are very

subjective and are often based on intuition. It is speculated that there is no single wave parameter, but rather a combination of several parameters which may form a relevant criterion in the selection process. Furthermore, every structure is different and may therefore require different criteria for testing.

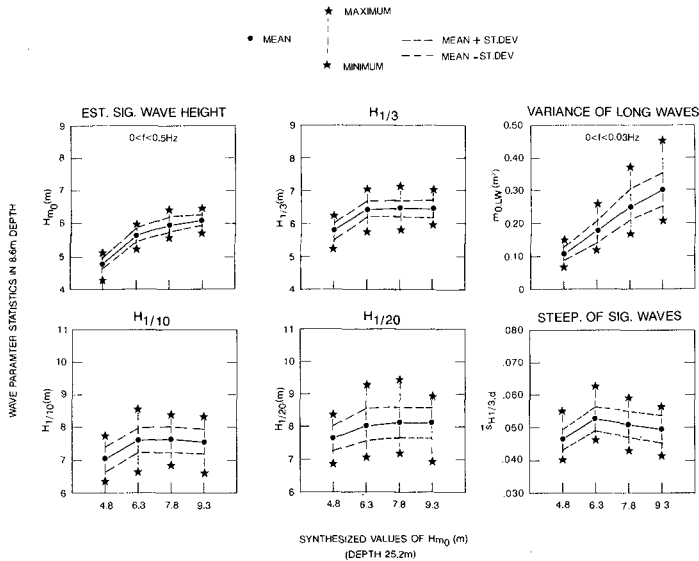


FIGURE 8
EFFECT OF OFFSHORE WAVE HEIGHTS ON NEARSHORE WAVE STATISTICS

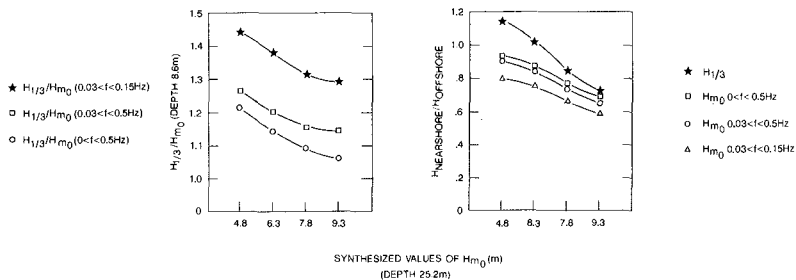


FIGURE 9
EFFECT OF OFFSHORE WAVE HEIGHTS ON THE RATIO OF
NEARSHORE $H_{1/3}/H_{m0}$ AND ON THE SHOALING COEFFICIENT

Evidently, the best approach is to strive towards a better understanding of the relevance of various wave parameters, such as grouping, crest front steepness etc., to the stability of a structure and to establish their relationship to the offshore sea state through a better understanding of the shoaling mechanism. However, because of the complexity of the problem, a major research effort with international cooperation will undoubtedly be required.

Last, but not least, it should be apparent from information contained in Figure 8, that the distribution functions for the probability of occurrence of offshore wave parameters require a transformation in order to estimate from them the return period of a nearshore design wave. It can be seen, for example, that a nearshore significant wave height of, say, 6.5 m can be produced by all offshore sea states with significant wave heights equal to or greater than 6.3 m. This fact alone places a new perspective on the design of coastal structures.

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